CALCULATOR PROGRAMS FOR
ENGINEERING ANALYSIS OF
FLUVIAL SYSTEMS

HYDROLOGY PACKAGE

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CALCULATOR PROGRAMS FOR ENGINEERING ANALYSIS OF FLUVIAL SYSTEMS

HYDROLOGY PACKAGE

By

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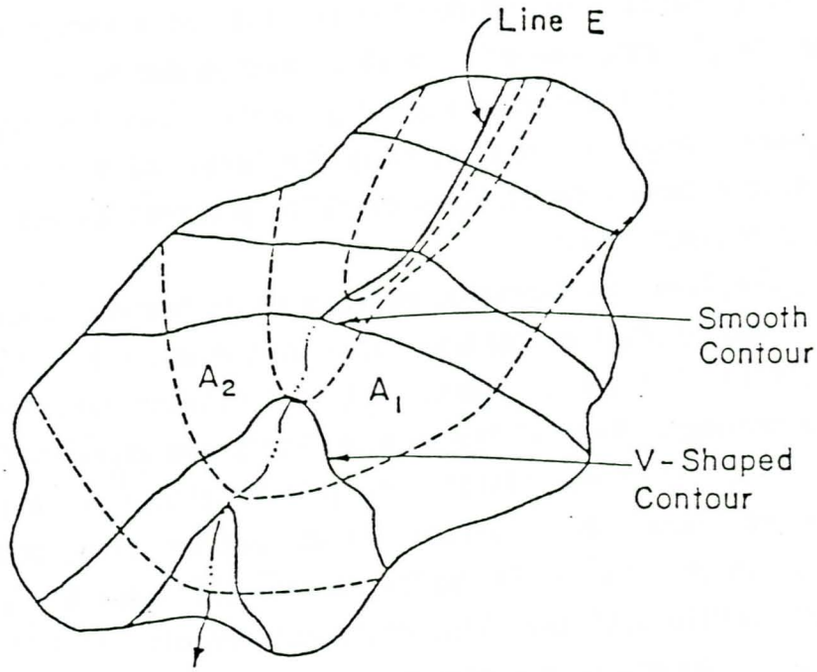
Hydrology Package

The Hydrology program package consists of a series of programs to estimate the hydrologic response of a small watershed due to a rainfall event. A watershed runoff hydrograph may be generated from the user's design rainfall hyetograph. Programs incorporating the Bureau of Reclamation techniques for estimating rainfall intensities are also provided to assist in estimating the probable maximum flood.

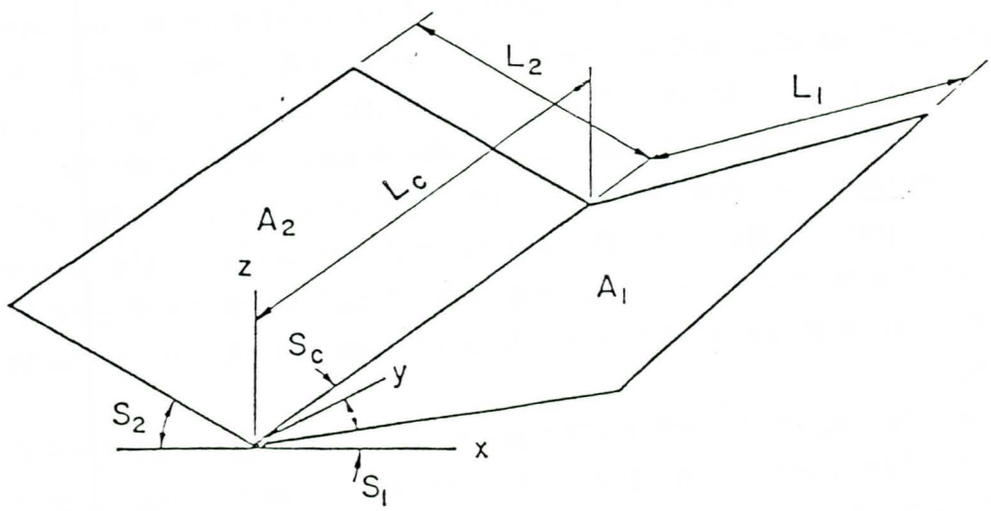
To ultilize the Hydrology package in estimating runoff from a watershed, the user must form a conceptual model of the watershed which subdivides it as a collection of plane surfaces. Figure 1 illustrates a conceptual model of a small watershed. Each of the sloped planes are drained along one edge by a channel. At any time, rainfall and infiltration occur all over the plane at a constant rate and excess rainfall flows down the plane towards the channel. Discharge to the channel is uniform across the plane but varies in time as different portions of the plane contribute runoff and as the soil becomes saturates. Kinematic-wave routing is used to calculate the discharge to the channel, and to estimate the propagation of the hydrograph downstream through the watershed.

The sequence of steps to develop a runoff hydrograph from a rainfall hyetograph is described below. Detailed information on the theory and use of each program follow. Infiltration is calculated using the Green-Ampt infiltration equation and requires the user to specify the soil infiltration properties. Where field determinations of these properties are unavailable, the tabulated values in the Appendix may be useful in estimating these properties. The rainfall excess, which is calculated by the program, is conceptually a uniform depth of rainfall throughout the plane, which contributes to runoff. The time sequence of excess rainfall is used as input to a kinematic wave-routing program to calculate the arrival of the runoff at the channel. To establish the hydraulic properties of the plane for this overland-flow routing, a program is provided to evaluate flow resistance and depth relationships.

Since the rainfall and infiltration is assumed to be uniformly distributed over the plane at any time, the kinematic-wave routing is performed on a unit-width basis. Upon routing the overland flow to the channel for a plane on either side of the channel, the user may perform a second routing to collect all the lateral inflows and develop a hydrograph at the watershed



(a) Original Watershed Topographic Map



(b) Wooding Plane Representation

Figure 1. Transformation of topographic map into watershed geometric representation.

outlet. Should the channel-routing effects be insignificant, the user may elect to approximate the watershed outlet hydrograph by simple multiplication of the unit discharge by the channel length and summing inflows from both planes.

Two kinematic wave-routing programs are provided. The method of characteristics program is most suitable for the overland-flow routing and may be used in the channel routing where there is no upstream inflow. A numerical kinematic wave solution is also provided. This program is well suited to the channel-routing effort and allows consideration of upstream inflows, thus, allowing the solution to combine an upstream hydrograph from another contributing watershed. Another desirable feature of the numerical technique is that the solution result is obtained at a uniform time step. The numerical technique is poorly suited to the overland-flow routing since the small flow depths result in a small finite difference grid and require a large number of computations be performed.

Rainfall Hyetographs

Rainfall hyetographs for the probable maximum storm are developed using the widely accepted techniques at the U.S. Bureau of Reclamation (USBR). Two programs are provided to accomplish this. The first program computes a reduction factor for converting maximum-point rainfall depths to average depths over a given area. The second program produces rainfall durations of a specified duration using a given storm volume and the reduction factor from the first program. These rainfall intensities may be arranged in time by the user to produce a design hyetograph.

The USBR methodology (1974) divides the country into zones for which typical normalized 6-hour cumulative rainfall functions have been developed. For the United States, west of the 105° meridian, probable maximum 6-hour volumes may be read from a map such as Figure 2 (USBR, 1974). As these volumes represent point maximums, a reduction factor must be determined for converting the point maximums to average volumes over the watershed area under consideration. This is accomplished in Program RAIN where the curves in Figure 3 from the USBR (1974) have been programmed. Rainfall intensities for durations shorter than six hours can be determined from curves such as shown in Figure 4 (USBR, 1974). These curves have been entered into Program HYET. Given a point maximum rainfall and area reduction factor, this program calculates the design and rainfall intensities for a specified time increment. The rainfall intensities are determined such that a range of intensities are produced consistent with the appropriate curves on Figure 4 and the total storm volume is conserved. These rainfall intensities may then be sequenced in time by the user to produce a design rainfall hyetograph that would produce the maximum response from the basin, considering basin shape and the critical storm pattern.

The curves entered into Programs RAIN and HYET are based on the USBR methodology for developing 6-hour probable maximum storm events. Using the programs for total storm durations of other than six hours or events less than the probable maximum rainfall may not be valid.

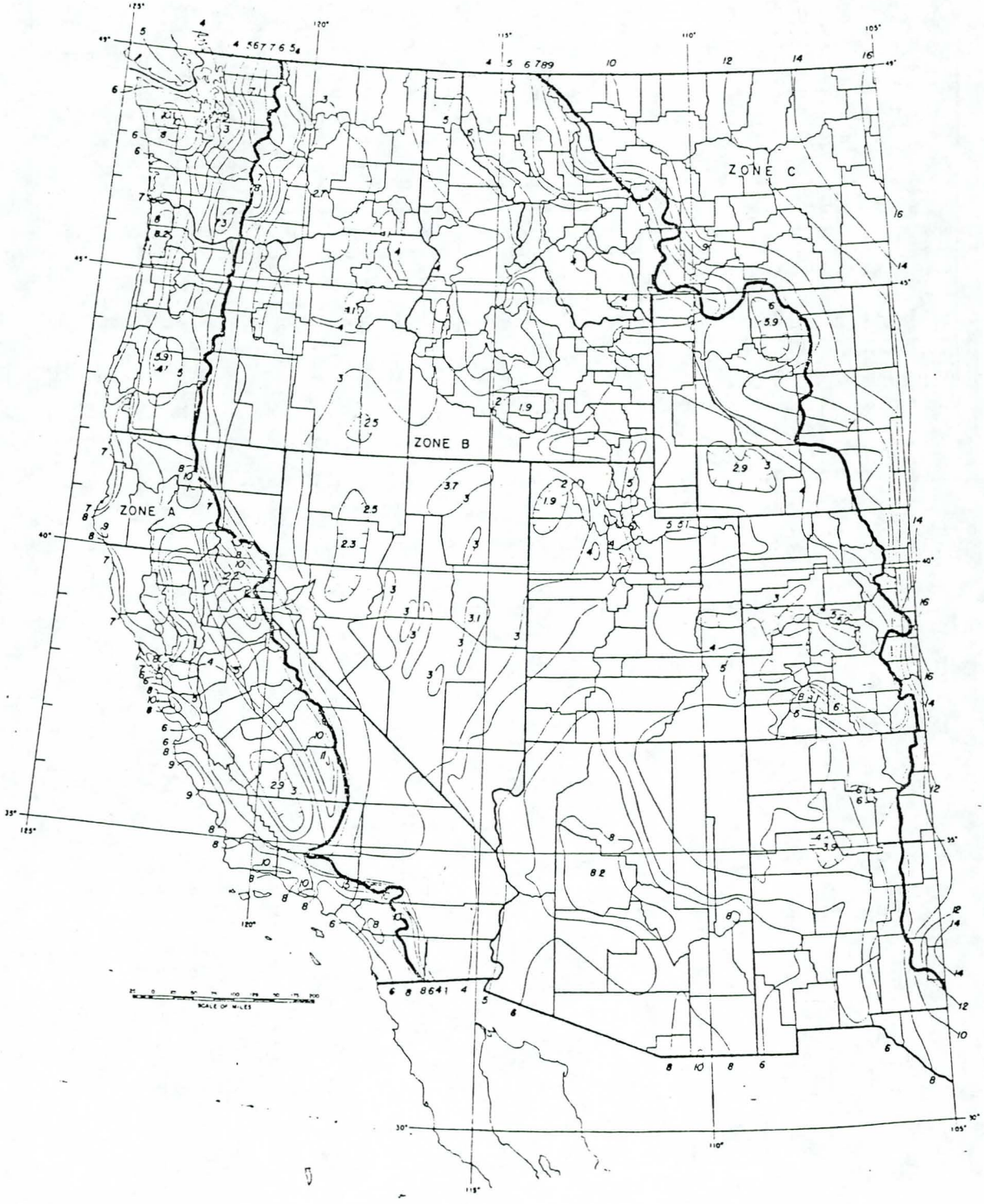


Figure 2. Probable maximum 6-hour point rainfall values in inches for general-type storms west of the 105° meridian (after USBR, 1974).

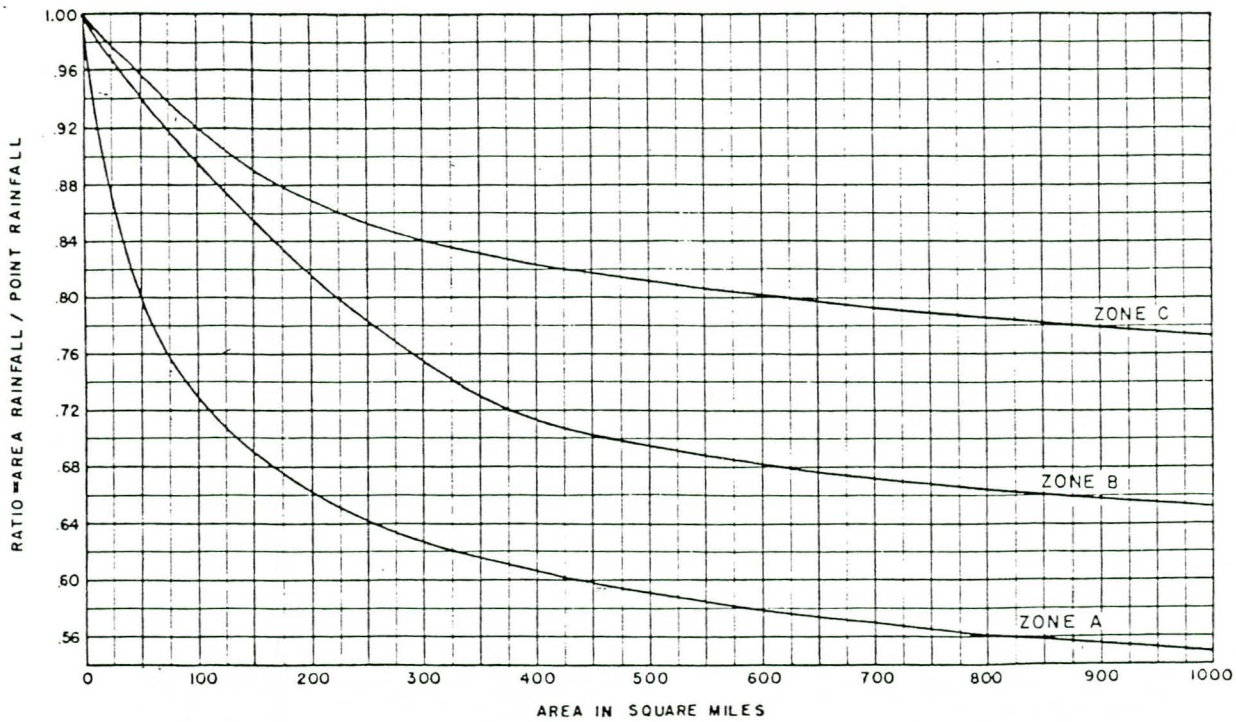


Figure 3. General storm: conversion ratio from 6-hour point rainfall to 6-hour area rainfall for area west of 105° meridian (after USBR, 1974).

WHAT ARE THE ZONE TYPES?

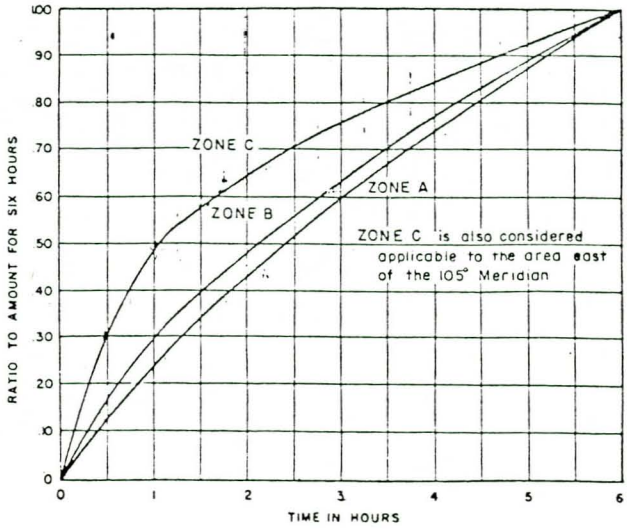


Figure 4. Temporal distribution of 6-hour rainfall for area west of 105° meridian.

Program RAIN User Instructions.

Step	Instructions	Input	Function	Display
1	Load program (USER mode)		[USER]	
2	Begin execution		[XEQ]RAIN	ZONE ?
3	Enter zone from Figure 2		A, B, or C	AREA ?
4	Enter area (mi^2)	area	[R/S]	ZONE X
	and compute reduction			
	factor (F)		[R/S] *	AREA=
			[R/S]*	FACT=
5	For new area in same zone		D	AREA ?
	execute D			
6	Go to 4			

* Note: If printer is connected to calculator and turned on, executing [R/S] is not required here.

WHAT ARE THE SOIL TYPES FOR HARQUAHALA?

Program HYET Users Instructions.

Step	Instructions	Input	Function	Display
1	Load program (USER mode)		[USER]	
2	Begin execution		[XEQ]HYET	ZONE ?
3	Enter zone from Figure 2		A, B, or C	VOLUME ?
4	Enter storm volume (in)	volume	[R/S]	RED FACT ?
5	Enter area reduction factor	F	[R/S]	DURATION ?
6	Enter storm duration (hr)	duration	[R/S]	TIME STEP ?
7	Enter hyetograph time step (min)	Δt	[R/S]	ZONE X
	and compute total storm		[R/S]*	VOL=
	volume (in) and rainfall		[R/S]*	INT=
	intensities (in/hr)		[R/S]*	INT=
				ect.

* Note: If printer is connected to calculator and turned on, executing [R/S] is not required here.

Example: Bureau of Reclamation Rainfall Hyetograph Generation

This example illustrates the use of Programs RAIN and HYET. Consider a watershed with the following data:

Zone C

Area = 100 mi^2

Point Rainfall Volume = 2.5 inches

Duration = 6 hours

Δt = 30 minutes

Using the Program RAIN:

```
                    XEQ "RAIN"
ZONE ?
                         XEQ C
AREA ?
          100.0000    RUN
ZONE C
AREA=100.0000
FACT=0.9200
```

Using Program HYET:

```
              XEQ "HYET"
ZONE ?
                    XEQ C
VOLUME ?
          2.5000    RUN
RED FACT ?
           .9200    RUN
DURATION ?
          6.0000    RUN
TIME STEP ?
         30.0000    RUN
```

```
ZONE C
VOL=2.3000

INT=1.3800

INT=0.8280

INT=0.4140

INT=0.3220

INT=0.3220

INT=0.2300

INT=0.2300

INT=0.1840

INT=0.1840

INT=0.1840

INT=0.1840

INT=0.1380
```

The rainfall intensities can be arranged to form a hyetograph as shown in Figure 5.

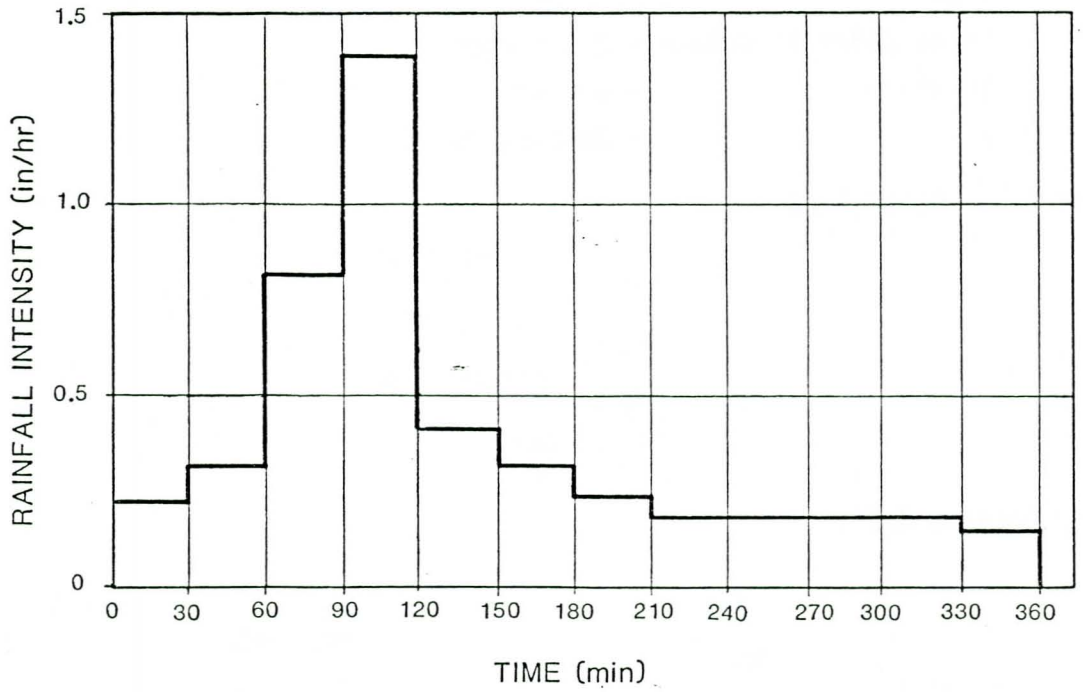


Figure 5. Design rainfall hyetograph.

Infiltration

Excess rainfall is computed using the Green-Ampt infiltration equation. Input of a rainfall hyetograph yields an excess rainfall histogram over the same time period.

The Green-Ampt infiltration equation is expressed as

$$\frac{F}{\delta} - \ln(1 + \frac{F}{\delta}) = \frac{Kt}{\delta} \qquad (1)$$

in which F is the infiltrated volume, K is the hydraulic conductivity of the soil in the wetted zone, t is the time, and δ is the potential head parameter and defined as

$$\delta = (\theta_w - \theta_i)\ \psi_{ave} \qquad (2)$$

in which θ_w is the moisture content of the soil after wetting, θ_i is the antecedent moisture of content, and ψ_{ave} is the average suction head across the wetting front.

A time explicit solution to Equation 1 is used in Program GREAMP (see for example, Eggert, 1976). The principal equation used is given as

$$\Delta F = \frac{(2F - K\Delta t)}{2} + \frac{[2F - K\Delta t)^2 + 8K\Delta t(\delta+F)]^{1/2}}{2} \qquad (3)$$

where ΔF is the potential infiltrated volume during time Δt. If during time Δt, the rainfall intensity is less than $\frac{\Delta F}{\Delta t}$, no excess occurs. If the rainfall intensity is greater than $\frac{\Delta F}{\Delta t}$, then excess occurs and is computed as

$$e = i - \frac{\Delta F}{\Delta t} \qquad (4)$$

where e is the excess rate and i is the rainfall intensity.

The Green-Ampt formulation assumes the following:

1. Homogenious soil properties.
2. A distinct piston wetting front exists.
3. The hysteresis effects in the soil properties are negligable.
4. The pressure of ponded water is negligible.

Program GREAMP Users Instructions.

Step	Instructions	Input	Function	Display
1	Load program			
2	Begin execution		[XEQ]GREAMP	SUCTION ?
3	Enter avg sucion head (in)	ψ_{ave}	[R/S]	THETA W ?
4	Enter wetted soil moisture content	θ_w	[R/S]	THETA I ?
5	Enter initial soil moisture content	θ_i	[R/S]	HYD COND ?
6	Enter hydraulic conductivity (in/hr)	K	[R/S]	TIME ?
7	Enter time at end of constant rainfall (min)	t	[R/S]	RAIN INT ?
8	Enter rainfall intensity (in/hr)	i	[R/S]	DTIME=
	and compute excess in		[R/S]*	TIME=
	in/hr and total runoff		[R/S]*	RAIN=
	volume in inches		[R/S]*	EXCESS=
			[R/S]*	TIME?
	Repeat steps 7 and 8 for each constant rainfall in the storm hyetograph			

* Note: If printer is connected to calculator and turned on, executing [R/S] is not required here.

Example: Computation of Rainfall Excess Histogram

This example illustrates the use of Program GREAMP. Consider the following data:

ψ_{ave} = 2.0 in
θ_w = 0.36
θ_i = 0.26
K = 0.63 in/hr

Time (min)	Rain Intensity (in/hr)
15	0.5
30	2.0
45	2.5
60	0.6

Using Program GREAMP:

```
          XEQ "GREAMP"
SUCTION ?
          2.0000    RUN
THETA W ?
           .3600    RUN
THETA I ?
           .2600    RUN
HYD COND ?
           .6300    RUN

TIME ?
         15.0000    RUN
RAIN INT ?
           .5000    RUN

DTIME=15.0000
TIME=15.0000
RAIN=0.5000
EXCESS=0.0000
TOT VOL=0.0000

TIME ?
         30.0000    RUN
RAIN INT ?
          2.0000    RUN

DTIME=15.0000
TIME=30.0000
RAIN=2.0000
EXCESS=0.8919
TOT VOL=0.2230

TIME ?
         45.0000    RUN
RAIN INT ?
          2.5000    RUN

DTIME=15.0000
TIME=45.0000
RAIN=2.5000
EXCESS=1.6237
TOT VOL=0.6289

TIME ?
         60.0000    RUN
RAIN INT ?
           .6000    RUN

DTIME=15.0000
TIME=60.0000
RAIN=0.6000
EXCESS=0.0000
TOT VOL=0.6289
```

Resistance to Overland Flow and Overland Flow Depth

A program is included here to determine resistance factors for overland flow and compute overland-flow depths. The program also determines coefficients and exponents in power relationships that relate depth and unit discharge. These relationships are used in the kinematic wave-routing programs.

The resistance to overland flow in natural watersheds may be regarded as a function of the ground cover density. Ground cover can include not only vegetation but also rock and litter, which all contribute to flow resistance. The Darcy-Weisbach resistance equation may be expressed as

$$S_f = f \frac{q^2}{8gy^3} \tag{5}$$

where S_f is the friction slope, g is the acceleration due to gravity, q is the discharge per unit width and f is the Darcy-Weisbach resistance factor. Numerous authors, e.g., Chen, 1976, have used expressions for f of the form

$$f = \frac{K}{Nr} \tag{6}$$

where K is a constant related to ground cover and Nr is the flow Reynolds number. Normally this form is only used in the laminar flow range; however, extension of the expression well into the turbulent ranges are common in watershed simulation.

K has been shown to be a function of the fraction of ground cover. One possible fit is of the form

$$K = K_\ell + (K_h - K_\ell)C_g^2 \tag{7}$$

where K_ℓ and K_h are lower and upper bounds on K and C_g is the ground cover density. K_ℓ is regarded as the value of K associated with a smooth surface and is usually given the value of 100.

Equation 5 may be rewritten using Equation 6 and noting that $Nr = q/\nu$ where ν is the kinematic viscosity of water. This relation when solved for depth has the following form

$$y = \left(\frac{K_{\nu q}}{8gS_f}\right)^{1/3} \tag{8}$$

Discharge is often assumed to be a power function of depth and is expressed as

$$q = \alpha y^{\beta} \tag{9}$$

For the overland flow case, β is always equal to three. α is given as

$$\alpha = \frac{8gS_o}{K\nu} \tag{10}$$

Equation 9 is used in the method of characteristics solution for kinematic waves. For the numerical solution to the kinetic wave problem, depth, and discharge are related as

$$y = \alpha' q^{\beta'} \tag{11}$$

Here:

$$\alpha' = \left(\frac{1}{\alpha}\right)^{1/3}$$

$$\beta' = 1/3$$

Program DWFRIC Users Instructions.

Step	Instructions	Input	Function	Display
1	Load program			
2	Begin execution		[XEQ]DWFRIC	COVER ?
3	Enter ground cover density	C_g	[R/S]	HIGH K ?
4	Enter lower bound for K	K_h	[R/S]	LOW K ?
5	Enter lower bound for K and	K_ℓ	[R/S]	K=
	Calculate K		[R/S]*	SLOPE ?
6	Enter overland flow slope	S_f	[R/S]	ALPHA=
	and calculate α , β, α'		[R/S]*	BETA=
	and β'		[R/S]*	ALPHA*=
			[R/S]*	BETA*=
			[R/S]*	UNIT Q ?
7	Enter unit discharge (cfs/ft) and calculate depth (ft)	q	[R/S]	DEPTH=

* Note: If printer is connected to calculator and turned on, executing [R/S] is not required here.

Example: Overland Flow Resistance

This example illustrates the use of program DWFRIC. Consider an overland flow slope with the following data:

$C_g = 0.35$
$K_h = 10{,}000$
$K_l = 100$
$S_f = 0.02$
$q = 0.01$ cfs/ft

Using Program DWFRIC:

```
                XEQ "DWFRIC"
COVER ?
                .3500    RUN
HIGH K ?
        10,000.0000    RUN
LOW K ?
           100.0000    RUN

K=1,312.7500

SLOPE ?
                .0200    RUN

ALPHA=360.0537
BETA=3.0000

ALPHA*=0.1406
BETA*=0.3333

UNIT Q ?
                .0100    RUN

DEPTH=0.0303
```

Kinematic Wave Routing - Method of Characteristics Approach

A program is provided here for performing kinematic wave routing using the method of characteristics approach. This program can be used for overland flow or channel flow with no upstream inputs. Upstream inputs cannot be used with the method of characteristics approach due to the possibility of kinematic shock. A second program in the package which uses a numerical solution can be used with channels that have upstream inputs.

The analytical solution to the kinematic wave approximation is based on the continuity and momentum equations in the absence of all dynamic interaction. The continuity equation for overland flow is

$$\frac{\partial q}{\partial x} + \frac{\partial y}{\partial t} = e \tag{12}$$

where q is the discharge per unit width, x is the downstream dimension, y is the depth, t is time and e is the lateral inflow. For overland flow, e represents excess rainfall and for channel flow it represents lateral flow from overland-flow units. The same equation applies the channel flow when Q (discharge) is substituted for q; A (area) is substituted for y; and q_ℓ (lateral inflow) is substituted for e. The remaining derivations given here will be written for the overland-flow case.

By neglecting dynamic effects, the momentun equation for a kinematic wave becomes:

$$S_f = S_o \tag{13}$$

where S_f is the friction slope and S_o is the bed slope.

Discharge is often assumed to be a power function of depth:

$$q = \alpha y^\beta \tag{14}$$

For overland flow α and β can be determined directly from known friction factors (see Program DWFRIC). For channel flow, α and β can be empirically derived using known channel properties (a program is included in the hydraulics package to accomplish this).

Substituting Equation 14 into Equation 12 yields

$$\alpha\beta y^{\beta-1} \frac{\partial y}{\partial x} + \frac{\partial y}{\partial t} = e \tag{15}$$

This partial differential equation may be solved by the method of characteristics. The total differential of $y(x,t)$ is given by

$$dy = \frac{\partial y}{\partial t}\, dt + \frac{\partial y}{\partial x}\, dx \tag{16}$$

Equations 15 and 16 form a system of two equations in two unknowns, and may be written in matrix form as

$$\begin{bmatrix} \alpha\beta y^{\beta-1} & 1 \\ dx & dt \end{bmatrix} \begin{bmatrix} \frac{\partial y}{\partial x} \\ \frac{\partial y}{\partial t} \end{bmatrix} = \begin{bmatrix} e \\ dy \end{bmatrix} \tag{17}$$

Characteristic paths along which the solution is valid are found by determining the loci of indeterminacy of the solution. These loci are obtained by equating the determinant of the coefficient matrix with zero. The resulting characteristic equation is

$$\frac{dx}{dt} = \alpha\beta y^{\beta-1} \tag{18}$$

Integrating Equation 18 with respect to time yields

$$x - x_0 = \alpha\beta \int_{t_0}^{t} y^{\beta-1}\, dt' \tag{19}$$

Invariants of this solution are found by substituting the right-hand side of Equation 17 for each column of the coefficient matrix and equating the determinant of the resulting matrix equal to zero. The invariants are

$$\frac{dy}{dt} = \frac{dq}{dx} = e \tag{20}$$

or by integrating Equation 20

$$y - y_0 = \int_{t_0}^{t} e\, dt' \tag{21}$$

or

$$q - q_0 = \int_{x_0}^{x} e\, dx' \tag{22}$$

By substituting Equation 21 into Equation 19, the following expression for the characteristic path is obtained

$$x - x_o = \alpha\beta \int_{t_o}^{t} \left[\int_{t_o}^{t'} e d\sigma + y_o \right]^{\beta-1} dt' \tag{23}$$

Theoretically, if the functional form of $e(t)$ is know, then Equation 23 will give the characteristics in the x-t plane. When $e(t)$ is a histogram, then with reference to Equation 21, Equation 23 may be evaluated in pieces.

The procedure that calculates characteristic paths is illustrated in Figure 6. From any point (x_i, t_i) on characteristic C, the value of x_{i+1} on C corresponding to time t_{i+1} may be calculated by Equation 23 expressed as

$$x_{i+1} = x_i + \alpha\beta \int_{t_i}^{t_{i+1}} [e_{i+1}(t' - t_i) + y_i]^{\beta-1} dt' \tag{24}$$

From Equation 21, it may be seen that

$$y_{i+1} = y_i + e_{i+1}(t_{i+1} - t_i) \tag{25}$$

Integrating Equation 24 yields

$$x_{i+1} = x_i + \frac{\alpha}{e_{i+1}} [(y_i + e_{i+1}(t_{i+1} - t_i))^{\beta} - y_i^{\beta}] \tag{26}$$

For a given plane of length L, values of x_{i+1} are calculated until $x_{i+1} \geq L$. Time of arrival of the characteristic at the downstream boundary t_L is found by solving Equation 26 for t_L in terms of t_j and x_j.

$$t_L = t_j + \frac{1}{e_{j+1}} \left\{ \left[\frac{e_{j+1}(L-x)_j}{\alpha} + y_i^{\beta} \right]^{\frac{1}{\beta}} - y_j \right\} \tag{27}$$

For cases where $e_{j+1} = 0$, Equation 26 and 27 may be rederived from Equation 23 yielding

$$t_L = \frac{L - x_j}{\alpha\beta y_j^{\beta-1}} + t_j \tag{28}$$

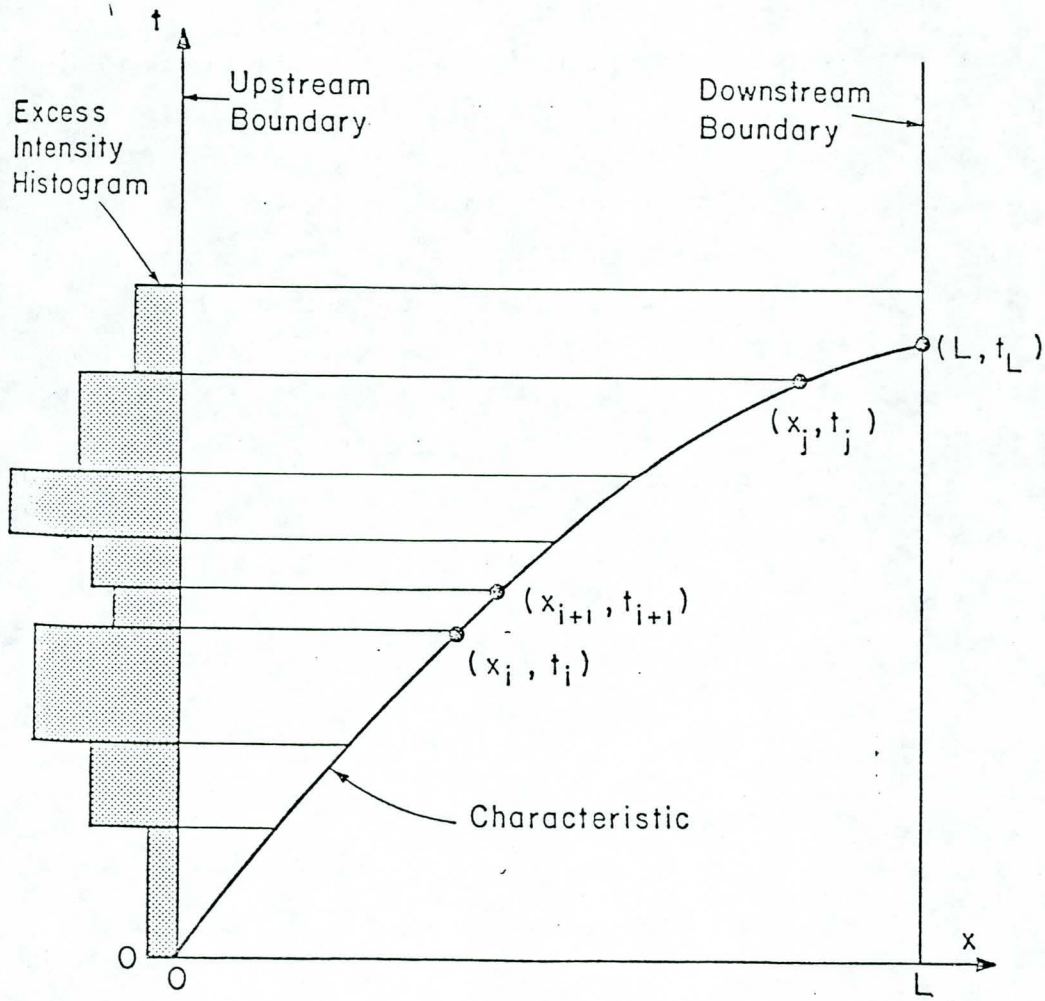


Figure 6. Method of characteristics solution domain illustrating calculation of characteristics in downstream direction.

$$x_{i+1} = x_i + \alpha\beta y_i^{\beta-1}(t_{i+1} - t_i) \tag{29}$$

Equations 26 through 29 are used serially, until the storm ends and the discharge is arbitrarily small, to calculate the characteristics lines beginning at the starting time of each block of excess rainfal (Figure 7).

A specific limitation of the method of characteristics solution to the kinematic wave problem is that characteristics computed from the end points of the excess intensity histogram intersect the downstream boundary at uneven time intervals (Figure 7). For specific cases, the solution points at the downstream boundary may be insufficient to define the outflow hydrograph accurately. This problem can be circumvented in practice by breaking up the excess histogram into smaller time steps where several time steps may contain the same excess.

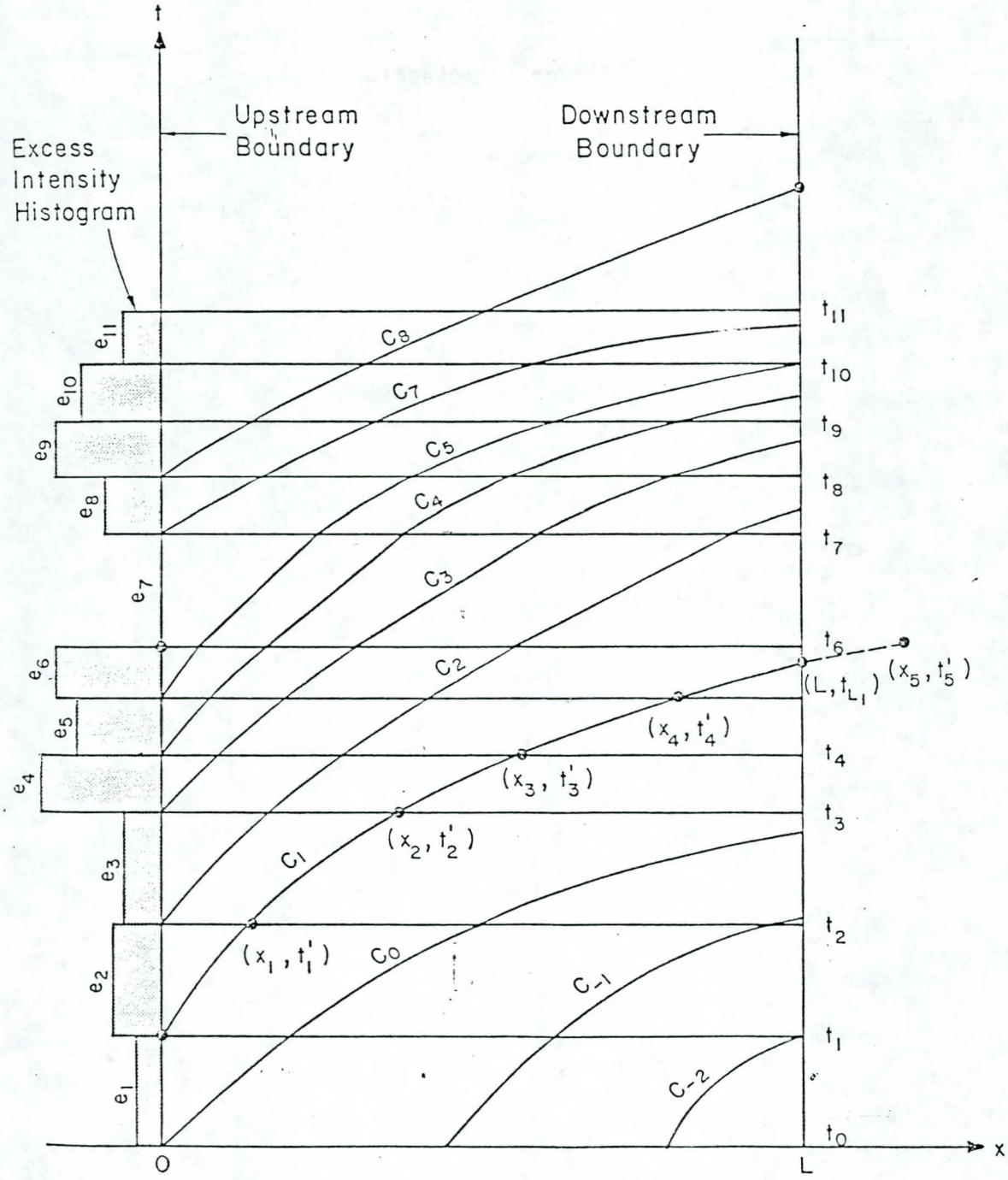


Figure 7. Method of characteristics solution domain illustrating a set of characteristics for an arbitrary excess rainfall histogram.

Program KWR Users Instructions.

Step	Instructions	Input	Function	Display
1	Load program			
2	Begin execution		[XEQ] KWR	OF OR CH ?
3	Enter OF for overland flow or CH for channel flow (calculator automatically in alpha mode)	OF or CH	[R/S]	ALPHA ?
	The following is for the overland flow case:			
4	Enter α in $q = \alpha y^{\beta}$	α	[R/S]	BETA ?
5	Enter β in $q = \alpha y^{\beta}$	β	[R/S]	LENGTH ?
6	Enter overland flow length(ft)	L	[R/S]	NO OF PTS ?
7	Enter number of points in excess rainfall histogram (first point must be 0.0)	N	[R/S]	EXCESS ?
8	Enter rainfall excess intensity (in/hr)	e	[R/S]	TIME ?
9	Enter ending time of constant rainfall excess (min)	t	[R/S]	EXCESS ?
0	Repeat steps 8 and 9 until total storm hyetograph is input (Note: last point should have zero excess and a large time). Program will output discharge (cfs/ft) and times (min)		[R/S]* [R/S]*	Q= TIME= ect.
	The following is for the channel case:			
4	Enter α in $Q = \alpha A^{\beta}$	α	[R/S]	BETA ?
5	Enter β in $Q = \alpha A^{\beta}$	β	[R/S]	LENGTH ?
6	Enter channel length (ft)	L	[R/S]	NO OF PTS ?
7	Enter the number of points in the lateral flow hydrograph (first point must equal 0.0)	N	[R/S]	QLAT ?
8	Enter laterial inflow (cfs/ft)	e	[R/S]	TIME ?
9	Enter ending time of lateral inflow (min)	t	[R/S]	QLAT ?
10	Repeat steps 9 and 10 until lateral inflow hydrograph has been read in (Note: last point should have zero flow and a large time). Program will output discharge (cfs) and time (min).		[R/S]* [R/S]*	Q= TIME= ect.

* Note: If printer is connected to calculator and turned on, executing [R/S] is not required here.

Example: Kinematic Wave Routing Using the Method of Characteristics Approach

This example illustrates the use of program KWR for an overland flow case. Consider the following data:

$\alpha = 400$

$\beta = 3.0$

$L = 4000$

Time (min)	Excess (in/hr)
0	0
10	.8640
15	.7776
16.67	1.2960
33.33	.8640
41.67	.6480
45	.5616
55	.9072
61.67	.4320
66.67	.0432
1×10^{18}	0

Using Program KWR:

```
            XEQ "KWR"
OF OR CH ?
OF                  RUN
ALPHA ?
        400.0000    RUN
BETA ?
          3.0000    RUN
LENGTH ?
      4,000.0000    RUN
NO OF PTS ?
         11.0000    RUN
EXCESS ?
          0.0000    RUN
TIME ?
          0.0000    RUN
EXCESS ?
           .8640    RUN
TIME ?
         10.0000    RUN
EXCESS ?
           .7776    RUN
TIME ?
         15.0000    RUN
EXCESS ?
          1.2960    RUN
TIME ?
         16.6700    RUN
EXCESS ?
           .8640    RUN
TIME ?
         33.3300    RUN
EXCESS ?
           .6480    RUN
TIME ?
         41.6700    RUN
EXCESS ?
           .5616    RUN
TIME ?
         45.0000    RUN
EXCESS ?
           .9072    RUN
TIME ?
         55.0000    RUN
```

```
EXCESS ?
             .4320    RUN
TIME ?
           61.6700    RUN
EXCESS ?
             .0432    RUN
TIME ?
           66.6700    RUN
EXCESS ?
            0.0000    RUN
TIME ?
              1+18    RUN

Q=4.0000E-88
TIME=0.0000

Q=6.9120E-4
TIME=10.0000

Q=2.1072E-3
TIME=15.0000

Q=3.3989E-3
TIME=16.6700

Q=2.6372E-2
TIME=33.3300

Q=4.3972E-2
TIME=41.6700

Q=5.1519E-2
TIME=45.0000

Q=7.1284E-2
TIME=49.5817
```

```
Q=6.4721E-2
TIME=50.6501

Q=4.9976E-2
TIME=51.4826

Q=4.1522E-2
TIME=53.6877

Q=7.8779E-3
TIME=94.5366

Q=2.9657E-3
TIME=158.5264

Q=1.9314E-3
TIME=204.3665

Q=3.1847E-5
TIME=3,006.9206

Q=1.0800E-8
TIME=617,287.2840

PROGRAM STOP
```

Kinematic Wave Routing - Numerical Solution Approach

A program is provided here for performing kinematic-wave routing using a numerical-solution approach. Although this program has been made general to include both overland flow or channel flow, it is most useful for routing in channels. For the channel case, this program can handle both lateral and upstream inputs. The method of characteristics approach cannot include upstream inputs due to the possibility of kinematic shock.

Numerical solution to the kinematic-wave problem is based on the same governing equation as the method of characteristics approach.

The water continuity equation for a channel can be expressed

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = q_\ell \tag{30}$$

in which Q is water discharge, x is the downslope distance, A is the flow area, t is time, and q_ℓ is the lateral inflow. The same equation applies to overland flow when q (unit discharge) is substituted for Q; y (depth) is substituted for A; and e (excess rainfall) is substituted for q_ℓ. The remaining derivations given here will be for the channel case.

becomes

$$S_e = S_o \tag{31}$$

Derivation for the discharge-area relationship is slightly different than that used for the method of characteristics. Here it is expressed as

$$A = \alpha' Q^{\beta'} \tag{32}$$

The finite difference form of Equation 30 can be represented as

$$\frac{Q_{j+1}^{n+1} - Q_j^{n+1}}{\Delta x} + \frac{A_{j+1}^{n+1} - A_{j+1}^{n}}{\Delta t} = \frac{1}{2}\left(q_{\ell\,j+1}^{\;n+1} + q_{\ell\,j+1}^{\;n}\right) \tag{33}$$

in which Q_j^n is the quantity Q at grid point $x = j\Delta x$ and $t = n\Delta t$, Δx is the space increment, and Δt is the time increment. The rectangular network shown in Figure 8 illustrates the x-t grid system.

Unknowns in Equation 33 are Q_{j+1}^{n+1} and A_{j+1}^{n+1}, but the discharge bears a definite relation with the flow area as indicated by Equation 32. With

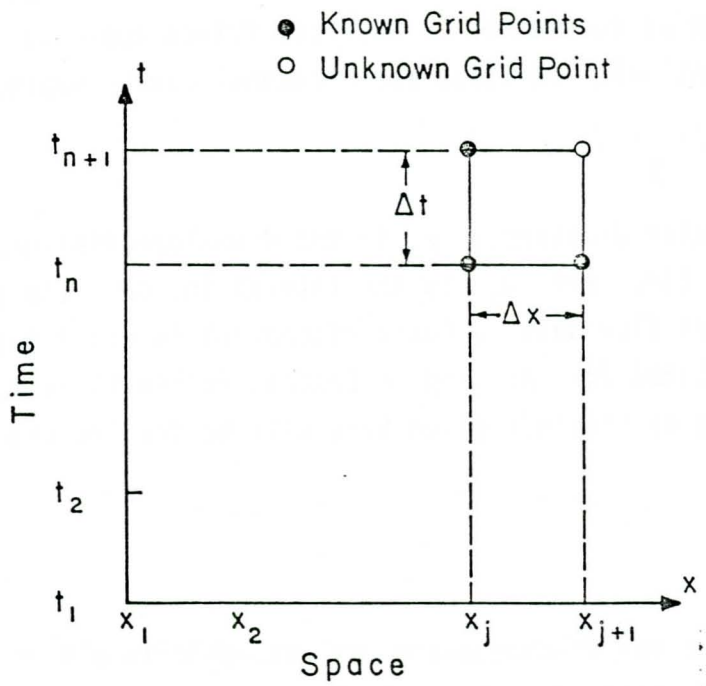


Figure 8. Definition sketch for channel water and sediment finite difference routing scheme.

these two equations, the values of the two unknowns can be obtained. This is accomplished by using a nonlinear scheme with an interactive procedure. A linear scheme is used to obtain the initial estimate of the unknown discharge for the nonlinear scheme. The linear scheme may be used with no iterations to solve for the unknown discharge if the accuracy is satisfied.

Substitution of Equation 32 into Equation 33 yields

$$\frac{\Delta t}{\Delta x} q_{j+1}^{n+1} + \alpha' (Q_{j+1}^{n+1})^{\beta'} = \frac{\Delta t}{\Delta x} Q_j^{n+1} + A_{j+1}^n + \frac{\Delta t}{2} (q_{\ell_{j+1}}^{n+1} + q_{\ell_{j+1}}^{n}) \qquad (34)$$

The right side of Equation 34 contains known quantities and is denoted by Ω which can be expressed as

$$\Omega = \frac{\Delta t}{\Delta x} Q_j^{n+1} + A_{j+1}^n + \frac{\Delta t}{2} (q_{\ell_{j+1}}^{n+1} + q_{\ell_{j+1}}^{n}) \qquad (35)$$

If $r = Q_{j+1}^{n+1}$ and $\theta = \frac{\Delta t}{\Delta x}$, then the left side of Equation 34 is expressed as

$$f(r) = \theta r + \alpha' f^{\beta'} = \Omega \qquad (36)$$

The solution of Equation 34 satisfies the condition

$$f(r) = \theta r + \alpha' r^{\beta'} = \Omega \qquad (37)$$

An approximate solution to this nonlinear equation is obtained by the following interactive scheme.

If r^k is the value of r at the kth interation, Taylor Series expansion of the function $f(r)$ around r^k is

$$f(r) = f(r^k) + (r - r^k) f'(r^k) + \frac{1}{2} (r - r^k)^2 f''(r^k) \qquad (38)$$

$$+ \frac{1}{6} (r - r^k)^3 f'''(r^k) + \ldots$$

in which $f'(r^k)$ and $f''(r^k)$ are values of the first and second derivatives of the function at r^k. Dropping the terms higher than the second order, one obtains

$$f(r) \simeq f(r^k) + (r - r^k) f'(r^k) + \frac{1}{2} (r - r^k)^2 f''(r^k) \qquad (39)$$

The purpose of iteration is to force $f(r^{k+1})$ to approach the value of Ω or

$$\Omega \simeq f(r^k) + (r^{k+1} - r^k)\, f'(r^k) + \frac{1}{2}(r^{k+1} - r^k)^2\, f''(r^k) \tag{40}$$

The solution of Equation 40 is

$$r^{k+1} = r^k - \frac{f'(r^k)}{f''(r^k)} \pm \sqrt{\frac{f'(r^k)^2}{f''(r^k)} - \frac{2(f(r^k) - \Omega}{f''(r^k)}} \tag{41}$$

in which

$$f(r^k) = \theta r^k + \alpha'\,(r^k)^{\beta'} \tag{42}$$

$$f(r^k) = \theta + \alpha'\beta'(r^k)^{\beta'-1} \tag{43}$$

and

$$f''(r^k) = \alpha'\beta'(\beta'-1)\,(r^k)^{\beta'-2} \tag{44}$$

There are two solutions to Equation 43. It is advisable to choose the solution that gives the smaller value of $\left| f(r^{k+1})-\Omega \right|$. The above iteration is continued until the absolute error $\left| f(r^{k+1})-\Omega \right|$ is less than a preassigned tolerance η. The termination criteria can be written as

$$\left| f(r^{k+1}) - \Omega \right| \leq \eta \tag{45}$$

An appropriate value for η is 0.01 Ω. However, it may be changed according to the purpose of individual problems.

The scheme represented by Equation 33 has proved to be unconditionally stable and can be used with a wide range of time to space increment ratios without loss of significant accuracy (Li, Simons, and Stevens, 1975). However, the initial guess r° is the key to the speed of convergence to the correct numerical solution. The best way of determining r° is to use a linear scheme.

The term $\frac{\partial A}{\partial t}$ in Equation 30 can be expressed as

$$\frac{\partial A}{\partial t} = \frac{\partial A}{\partial Q}\frac{\partial Q}{\partial t} \tag{46}$$

Also, from Equation 32

$$\frac{\partial A}{\partial Q} = \alpha'\beta' Q^{\beta'-1} \tag{47}$$

Substitution of Equations 46 and 47 into Equation 30 yields

$$\frac{\partial Q}{\partial x} + \alpha'\beta' Q^{\beta'-1} \frac{\partial Q}{\partial t} = q_\ell \tag{48}$$

The finite difference form of Equation 49 is given by the expression

$$\frac{Q_{j+1}^{n+1} - Q_j^{n+1}}{\Delta x} + \alpha'\beta' \left(\frac{Q_{j+1}^{n} + Q_j^{n+1}}{2} \right)^{\beta'-1} \frac{Q_{j+1}^{n+1} - Q_{j+1}^{n}}{\Delta t} = \frac{1}{2} \left(q_{\ell_{j+1}}^{n+1} + q_{\ell_{j+1}}^{n} \right) \tag{49}$$

so that

$$r^\circ = Q_{j+1}^{n+1} = \frac{\theta Q_j^{n+1} + \alpha'\beta' Q_{j+1}^{n} \left(\frac{Q_{j+1}^{n} + Q_j^{n+1}}{2} \right)^{\beta'-1} + \frac{\Delta t}{2} \left(q_{\ell_{j+1}}^{n+1} + q_{\ell_{j+1}}^{n} \right)}{\theta + \alpha'\beta' \left(\frac{Q_{j+1}^{n} + Q_j^{n+1}}{2} \right)^{\beta'-1}} \tag{50}$$

Equation 51 uses a linear scheme to provide the best initial estimate of r° for the nonlinear scheme. However, Equation 51 is not applicable if both Q_{j+1}^{n} and Q_j^{n+1} are zero. When both Q_{j+1}^{n} and Q_j^{n+1} are zero, $\beta' = 1$ is used in Equation 37 and then

$$r^\circ = \frac{\Omega}{\theta + \alpha'} \tag{51}$$

Although the numerical solution presented here has been shown to be stable over a wide range of time to space increment ratios, the absolute magnitude of these terms must be considered carefully to preserve an adequate level of accuracy. The time step (Δt) should be chosen small enough to accurately define the rate of change of flow conditions. A Δt 1/10th to 1/20th the time of rise at the simulated hydrograph is generally sufficient. The most accurate solution will occur when $\Delta x/\Delta t$ is near the travel speed of the flood wave. Usually sufficient choices at the time and distance incremently can be made with one or two trial runs.

This program has been made general to include both overland flow and flow in channels. Due to the small depths and slow travel speeds of waves on overland-flow unit, however, very small time and distance steps are required for the equations to converge on a reasonable number of iterations and produce accurate results. For this reason, the analytical method of characteristics approach would generally be appropriate for overland flow. The

method of characteristics approach may also be preferable for flow in channels where there is no upstream input.

Program NKWR Users Instructions.

Step	Instructions	Input	Function	Display
1	Load program			
2	Begin execution		[XEQ] NKWR	OF OR CH ?
3	Enter OF for overland flow or CH for channel flow (calculator automatically in alpha mode)	OF or CH	[R/S]	ALPHA* ?
	The following is for the overland flow case:			
4	Enter α' in $y = \alpha' q^{\beta'}$	α'	[R/S]	BETA* ?
5	Enter β' in $y = \alpha' q^{\beta'}$	β'	[R/S]	LENGTH ?
6	Enter the overland flow length (ft)	L	[R/S]	NO OF DX ?
7	Enter number of space increments ($L/\Delta x$)	N	[R/S]	DELTA T ?
8	Enter time step (min)	Δt	[R/S]	EXCESS ?
9	Enter rainfall excess during current time step (in/hr) and complete discharge (cfs/ft)	e	[R/S]	Q=
	Repeat step 8 and 9 for each time step of the simulation			
	The following complete the input for the channel case:			
4	Enter α' in $A = \alpha' Q^{\beta'}$	α'	[R/S]	BETA* ?
5	Enter β' in $A = \alpha' Q^{\beta'}$	β'	[R/S]	LENGTH ?
6	Enter the channel length (ft)	L	[R/S]	NO OF DX ?
7	Enter number of space increments ($L/\Delta x$)	N	[R/S]	DELTA T ?
8	Enter time step (min)	Δt	[R/S]	QLAT ?
9	Enter the lateral inflow during the current time step (cfs/ft)	q_ℓ	[R/S]	QUP ?
10	Enter the upstream input during the current time step (cfs) and compute the downstream discharge (cfs)	Q_{up}	[R/S]	Q=
	Repeat steps 8, 9, and 10 for each time step of the simulation			

Example: Kinematic Wave Routing Using the Numerical Solution Approach

This example illustrates the use of Program NKWR for a channel with upstream and lateral inputs. Consider the following data:

$\alpha' = 0.56$

$\beta' = 0.74$

$L = 2600$ feet

$\Delta x = 1300$ feet

$\Delta t = 4.0$ min

Time (min)	Lateral Inflow (cfs/ft)	Upstream Inflow (cfs)
4	0	2
8	0	39
12	.001	140
16	.013	200
20	.035	205
24	.022	125
28	.009	70
32	.003	39
26	.001	23

Using Program NKWR:

```
                XEQ "NKWR"
OF OR CH ?
CH                    RUN
ALPHA* ?
              .5600   RUN
BETA* ?
              .7400   RUN
LENGTH ?
        2,600.0000    RUN
NO OF DX ?
             2.0000   RUN
DELTA T ?
             4.0000   RUN
QLAT ?
             0.0000   RUN
QUP ?
             2.0000   RUN

Q=0.0040

QLAT ?
             0.0000   RUN
QUP ?
            39.0000   RUN

Q=1.5671

QLAT ?
              .0010   RUN
QUP ?
           140.0000   RUN

Q=24.8854

QLAT ?
              .0130   RUN
QUP ?
           200.0000   RUN

Q=96.9719
```

```
QLAT ?
              .0350   RUN
QUP ?
           205.0000   RUN

Q=199.9175

QLAT ?
              .0220   RUN
QUP ?
           125.0000   RUN

Q=224.9318

QLAT ?
              .0090   RUN
QUP ?
            70.0000   RUN

Q=191.5109

QLAT ?
              .0030   RUN
QUP ?
            39.0000   RUN

Q=143.9387

QLAT ?
              .0010   RUN
QUP ?
            23.0000   RUN

Q=103.0270
```

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APPENDIX A

Infiltration Parameters

Five soil parameters are required as input data for the Green-Ampt infiltration equation. They are:

K_W = hydraulic conductivity in the wetted zone (in./hr)

ψ = capillary suction head (in.)

ϕ = soil porosity

S_I = initial degree of soil saturation

S_W = final degree of soil saturation

Since soils are generally anisotropic in their physical properties, significant variations in these parameters may occur over a very small area. The program assumes these five parameters to be constant over a given unit. The user must strive to obtain representative values for each unit.

The most reliable way of obtaining these parameters is to conduct infiltration tests on samples of the soil. If data from an infiltration test are available, the following method by Eggert (1976) can be used to obtain estimates of K_W and the storage suction factor γ, where γ is defined as:

$$\gamma = (S_W - S_I)\ \phi\ \psi \qquad (A.1)$$

1. Plot the infiltration rate and the infiltrated volumes as a function of time. Figure A.1 is an example of plotted infiltrometer data from a drainage basin in Arizona.

2. Plot the infiltration rate as a function of the reciprocal of the infiltrated volume. Figure A.2 shows a curve of this type based on the data given in Figure A.1.

3. The curve of Figure A.2 may be approximated as a straight line, to the extent that the Green and Ampt equation represents the actual soil performance. If a straight line is fitted to these data, the Y intercept is K_W and the slope is γK_W. From Figure A.2, the value of K_W may be estimated as 0.95 in./hr and γ as 0.83 in.

When no field data exist, the user must resort to tables and previous field experience to obtain the five parameters. The porosity of soil usually varies between about 0.35 for a fine gravel to 0.50 for a tight clay. The final soil saturation depends upon the antecedent rainfall and can range from essentially zero for a very dry soil, up to the value of S_W.

Values of K_W and ψ are more difficult to obtain. Table A.2, taken from the U.S. Department of Agriculture Soil Conservation Service, presents

Table A.1. Water Holding Capacities for Various Types of Vegetation (Zinke, 1965)

Species	Storage Amount (mm)	Storage Amount (in.)
CONIFERS		
White Fir		
Mature	3.8	0.15*
Pole size	6.6	0.26
California Red Fir	7.6	0.30*
Lodgepole Pine	0.8	0.03
Ponderosa Pine		
Mature	2.3	0.09*
Young (14 ft high)	0.3	0.01*
Monterey Pine	0.3-0.1	0.01-0.4'
Red Pine	0.8	0.03
Red Spruce (30 yr old)	9.1	0.36
White Pine	0.5	0.02
Hemlock	0.8-1.3	0.03-0.05
HARDWOODS		
California Buckeye		
Fall and Winter	0.8	0.03
Spring and Summer	1.3	0.05
Mixed hardwoods		
Maple, Elm, Oak, Birch	0.5-1.8	0.02-0.07
Mixed hardwoods		
Northern Red Oak and Tulip Tree		
Summer	0.5	0.02
Winter	0.2	0.006
Northern hardwoods	2.0	0.08
Sweet Gum Tree	0.8	0.034
All eastern N. American hardwood data	0.03-1.6	0.001-0.064

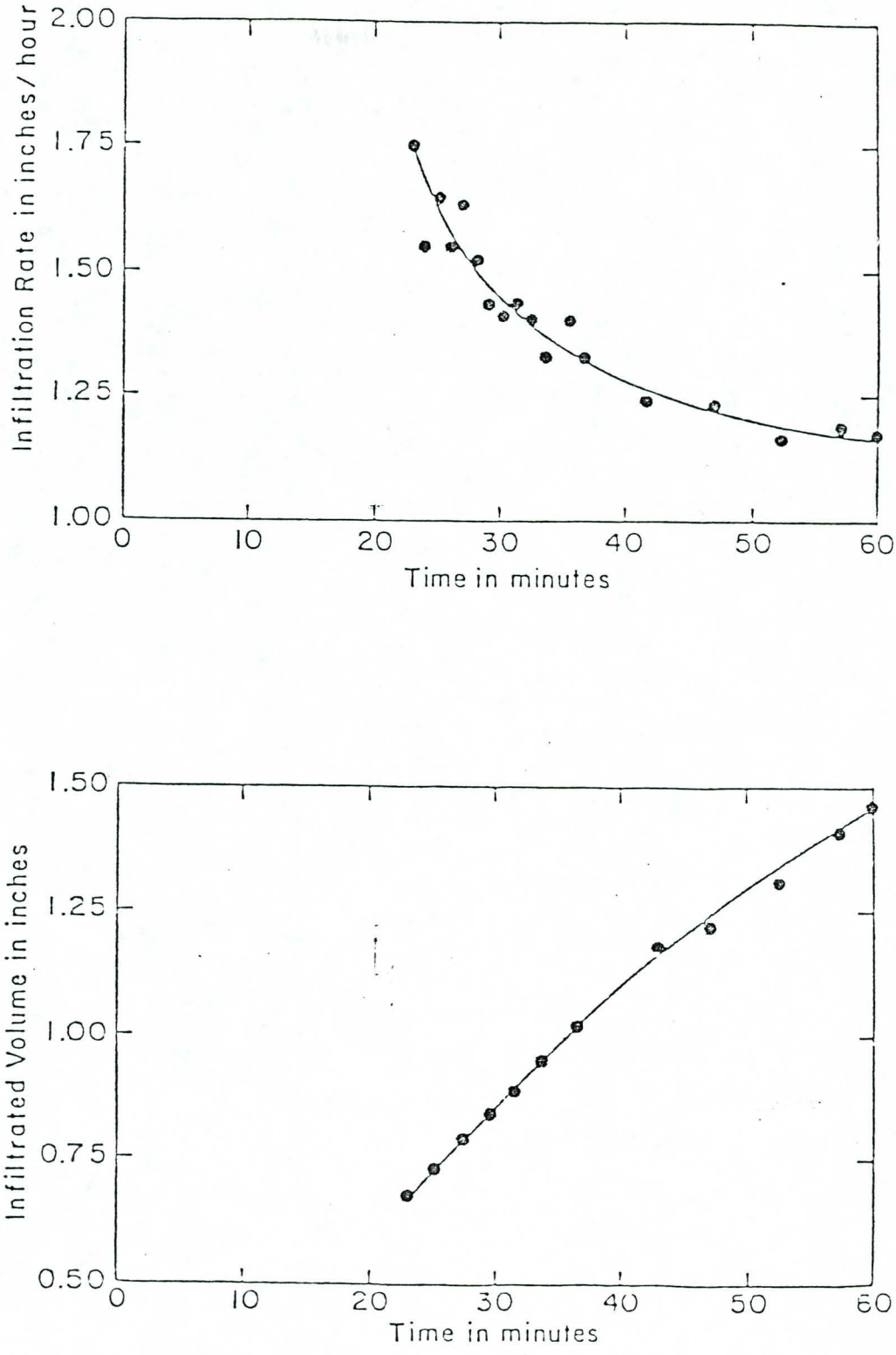

Figure A.1. Infiltrometer data from Walnut Gulch, Arizona.

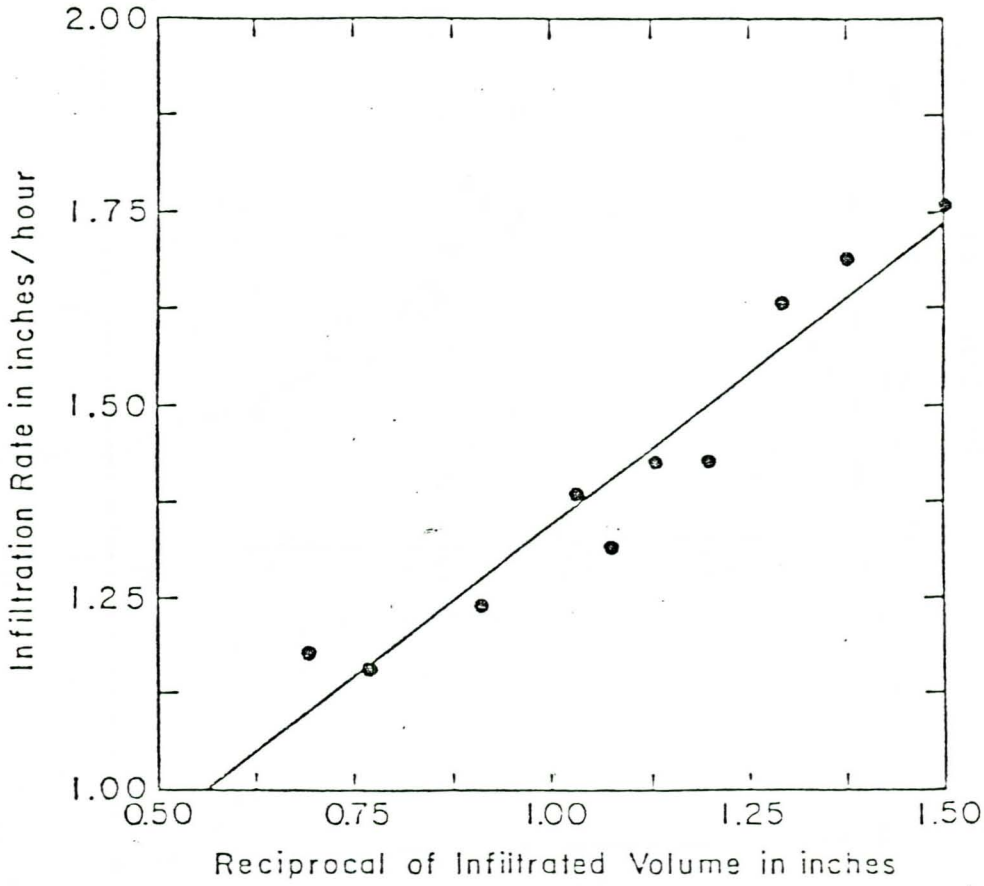

Figure A.2. Technique to determine the infiltration parameters from the infiltrometer data shown in Figure A.1.

Table A.2. Hydraulic Conductivity Classes

(from USDA Soil Conservation Service permeability tests)

Class	K_W Rate (in./hr.)
very slow	< 0.03
slow	0.03-0.1
moderately slow	0.1-0.32
moderate	0.32-1.0
rapid	1.0-3.2
very rapid	> 3.2

Table A.3. Capillary Suction Head (ψ) for Selected Soil Types (Eggert, 1976).

Soil Type	Range of Capillary Suction Head (in.)
Nickel Gravel-Sand Loam	2.0-4.5
Ida Silt Loan	2.0-3.5
Poudre Fine Sand	2.0-4.5
Plainfield Sand	3.5-5.0
Yolo Light Clay	5.5-10.0
Columbia Sandy Loam	8.0-9.5
Guelph Loam	8.0-13.0
Muren Fine Clay	15.0-20.0

Table A.4. Hydraulic Conductivity and Storage Suction Factor Related to SCS Curve Number (Verdin et al. 1980)

SCS Curve Number	K_W	γ	SCS Curve Number	K_W	γ
100.0	0.000	I	50.0	0.466	0.876
99.0	0.003	0.088	49.0	0.481	0.859
98.0	0.006	0.177	48.0	0.497	0.843
97.0	0.010	0.265	47.0	0.512	0.828
96.0	0.013	0.353	46.0	0.528	0.813
95.0	0.016	0.442	45.0	0.543	0.800
94.0	0.019	0.530	44.0	0.558	0.788
93.0	0.022	0.618	43.0	0.574	0.776
92.0	0.025	0.707	42.0	0.589	0.765
91.0	0.029	0.795	41.0	0.605	0.755
90.0	0.032	0.883	40.0	0.620	0.745
89.0	0.035	0.972	39.0	0.635	0.735
88.0	0.038	1.060	38.0	0.651	0.727
87.9	0.041	1.148	37.0	0.666	0.718
86.0	0.044	1.237	36.0	0.687	0.705
85.0	0.048	1.325	35.0	0.719	0.681
84.0	0.051	1.414	34.0	0.751	0.659
83.0	0.054	1.502	33.0	0.784	0.640
82.0	0.057	1.590	32.0	0.816	0.621
81.0	0.060	1.679	31.0	0.849	0.604
80.0	0.063	1.767	30.0	0.881	0.589
79.0	0.067	1.855	29.0	0.913	0.574
78.0	0.070	1.944	28.0	0.946	0.561
77.0	0.073	2.032	27.0	0.978	0.548
76.0	0.076	2.120	26.0	1.011	0.537
75.0	0.081	2.161	25.0	1.043	0.526
74.0	0.096	1.964	24.0	1.075	0.516
73.0	0.112	1.826	23.0	1.108	0.506
72.0	0.127	1.726	22.0	1.140	0.497

Table A.4 (continued).

SCS Curve Number	K_W	γ	SCS Curve Number	K_W	γ
71.0	0.143	1.652	21.0	1.173	0.488
70.0	0.158	1.595	20.0	1.205	0.480
69.0	0.173	1.552	19.0	1.237	0.473
68.0	0.189	1.519	18.0	1.270	0.466
67.0	0.204	1.494	17.0	1.302	0.459
66.0	0.220	1.474	16.0	1.335	0.452
65.0	0.235	1.421	15.0	1.367	0.446
64.0	0.250	1.352	14.0	1.399	0.441
63.0	0.266	1.292	13.0	1.432	0.435
62.0	0.281	1.238	12.0	1.464	0.430
61.0	0.297	1.190	11.0	1.497	0.425
60.0	0.312	1.147	10.0	1.529	0.420
59.0	0.327	1.108	9.0	1.561	0.416
58.0	0.343	1.073	8.0	1.594	0.411
57.0	0.358	1.040	7.0	1.626	0.407
56.0	0.374	1.011	6.0	1.659	0.403
55.0	0.389	0.984	5.0	1.691	0.399
54.0	0.404	0.959	4.0	1.723	0.396
53.0	0.420	0.935	3.0	1.756	0.392
52.0	0.435	0.914	2.0	1.788	0.389
51.0	0.451	0.894	1.0	1.821	0.386

Table A.5. Resistance Parameters for Overland Flow (Woolhiser, 1975).

Surface	Range of K
Concrete or Asphalt	24-108
Bare Sand	30-120
Gravelled Surface	90-400
Bare Clay-Loam Soil (eroded)	100-500
Sparse Vegetation	1000-4000
Short Grass Prairie	3000-10,000
Bluegrass Sod	7000-40,000

values of hydraulic conductivity in the wetted zone. Table A.3, taken from Eggert (1976), lists values of ψ for selected soil types. A study by Verdin and Morel-Seytoux (1980) has related the hydraulic conductivity and storage suction factor to the Soil Conservation Service curve number. Table A.4 lists the results of this study. Some caution must be used in applying these values since they were developed not for the Green and Ampt model, but for Morel-Seytoux's "ponding time" infiltration model.

Although five parameters are input into the program, S_W, S_I, ϕ and ψ are multiplied together to yield the storage suction factor γ. All of the actual calculations are then based solely on the values of γ and K_W. If the user can confidently select just the two parameters γ and K_W, he can work backwards to obtain values for S_W, S_I, ϕ and ψ. The individual values of S_W, S_I, ϕ and ψ are not important--only their product γ is used.

A.5 Resistance to Flow Parameters

Two different types of resistance parameters are used in the hydrology programs. For the overland flow units, a K value based on the Darcy Weisbach friction factor is used. Table A.5 (Woolhiser, 1975) presents a list of K values. The K value should be reprsentative of areas with ground cover. The program calculates a weighted overall resistance parameter considering the percentage of ground cover. For the turbulent flow in the open channels, Manning's n or Chezy's C values are used. As these values are familiar to most engineers, no tables of values are given.

CALCULATOR PROGRAMS FOR ENGINEERING ANALYSIS OF FLUVIAL SYSTEMS

STREAM HYDRAULICS PACKAGE

By

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Stream Hydraulics Package

The Stream Hydraulics package provides the capability to determine open channel flow characteristics and perform backwater computations on streams with natural or trapezoidal channel cross sections. A channel cross section properties program is provided to develop power curves for area, depth, top width, or wetted perimeter from a series of cross section coordinates. The flow depth, velocity, stream power, top width, area and hydraulic depth of the flow are calculated using the power curves for natural channels. A similar program provides these results directly after specification of the bottom width and side slope in a trapezoidal channel. Backwater computations are performed using the standard step method. For a specified distance interval between two cross sections, the upstream depth and total head are calculated.

Channel Cross Section Properties

Hydraulic calculations in natural channels often require a functional expression of channel geometry. The most common of these relationships are wetted perimeter-flow area, flow area-depth, top width-depth, and wetted perimeter-depth. These equations are usually given in the form of the power curves as

$$P = a_1 A^{b_1} \tag{1}$$

where A is area and P is wetted perimeter;

$$A = a_2 y^{b_2} \tag{2}$$

where y is depth of flow;

$$T_w = a_3 y^{b_3} \tag{3}$$

where T_w is the top width; and

$$P = a_4 y^{b_4} \tag{4}$$

The coefficients a and exponents b are usually statistically fitted. The method usually proceeds by taking the logarithm of both sides of the equation. For example, the wetted perimeter-area expression becomes

$$\ln P = \ln a_1 + b_1 \ln A \tag{5}$$

Using the logarithm of A as an independent variagble and the logarithm of P as the dependent variable, a lest squares fit may be calculated to provide values for a_1 and b_1. In order to provide a good fit, several values of the dependent and independent variables must be supplies from measurements of the channel cross section.

It is assumed that the cross-sectional geometry is known as a series of (x, z) points measured relative to some datum (Figure 1). The programs calculate the values of a_2, b_2, a_3, b_3, a_4, b_4, and present the correlation coefficient of the fit. Values for a_1 and b_1 can easily be determined by:

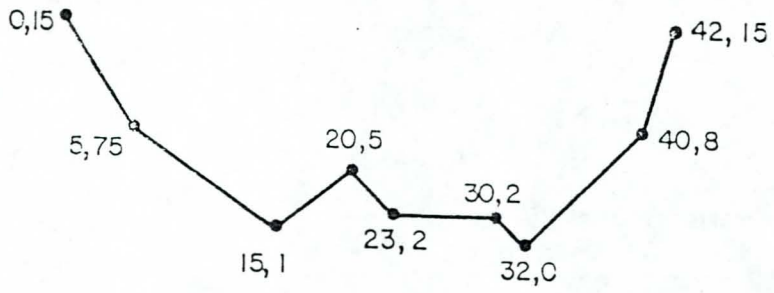

Figure 1. Representative channel cross section.

$$a_1 = \frac{a_4}{a_2^{b_1}} \tag{6}$$

and

$$b_1 = \frac{b_4}{b_2} \tag{7}$$

The HP-41c programs are as follows:

1. Area-detph program (AVSY)
2. Top width-depth program (TVSY)
3. Wetted perimeter-depth program (PVSY)

Program AVSY Users Instructions.

Step	Instructions	Input	Function	Display
1	Load program			
2	Begin execution		[XEQ] AVSY	MAX Z ?
3	Enter maximum elevation	Z_{max}	[R/S]	MIN Z ?
4	Enter minimum elevation	Z_{min}	[R/S]	NO OF PTS ?
5	Enter number of (X,Z) points in cross-section data	N	[R/S]	POINT X ?
6	Enter X coordinent of cross sectional data point	x	[R/S]	ELEV ?
7	Enter elevation of cross-sectional data point	z	[R/S]	POINT X ?
	Repeat steps 6 and 7 until all data has been entered. Program will then list pairs of area versus elevation, the coorelation coefficient of the regression, and the regresson constants.		[R/S]*	AREA=
			[R/S]*	ELEV=
				etc.
			[R/S]*	CORR=
			[R/S]*	A2=
			[R/S]*	B2=

* Note: Executing [R/S] is not required here if printer is connected to calculator and turned on.

Program TVSY Users Instructions.

Step	Instructions	Input	Function	Display
1	Load program			
2	Begin execution		[XEQ] TVSY	MAX Z ?
3	Enter maximum elevation	Z_{max}	[R/S]	MIN Z ?
4	Enter minimum elevation	Z_{min}	[R/S]	NO OF PTS ?
5	Enter number of (X,Z) points in cross-section data	N	[R/S]	POINT X ?
6	Enter X coordinent of cross sectional data point	x	[R/S]	ELEV ?
7	Enter elevation of cross-sectional data point	z	[R/S]	POINT X ?
	Repeat steps 6 and 7 until all data has been entered. Program will then list pairs of top width versus elevation, the coorelation coefficient of the regression, and the regresson constants.		[R/S]*	TW=
			[R/S]*	ELEV=
				etc.
			[R/S]*	CORR=
			[R/S]*	A3=
			[R/S]*	B3=

* Note: Executing [R/S] is not required here if printer is connected to calculator and turned on.

Program PVSY Users Instructions.

Step	Instructions	Input	Function	Display
1	Load program			
2	Begin execustion		[XEQ] PVSY	MAX Z ?
3	Enter maximum elevation	Z_{max}	[R/S]	MIN Z ?
4	Enter minimum elevation	Z_{min}	[R/S]	NO OF PTS ?
5	Enter number of (X,Z) points in cross-section data	N	[R/S]	POINT X ?
6	Enter X coordinent of cross sectional data point	x	[R/S]	ELEV ?
7	Enter elevation of cross-sectional data point	z	[R/S]	POINT X ?
	Repeat steps 6 and 7 until all data has been entered. Program will then list pairs of wetted perimeter versus elevation, the coorelation coefficient of the regression, and the regresson constants.		[R/S]*	WPER=
			[R/S]*	ELEV=
				etc.
			[R/S]*	CORR=
			[R/S]*	A4=
			[R/S]*	B4=

* Note: Executing [R/S] is not required here if printer is connected to calculator and turned on.

Example: Channel Cross-Section Properties

This example illustrates the use of programs AVSY, TVSY, and PVSY. Consider a channel cross section with the following data points.

Horizontal Distance (X)	Elevation (Z)
0	15
5	7.5
15	1
20	5
23	2
30	2
32	0
40	8
42	15

Using Program AVSY:

```
                XEQ "AVSY"
MAX Z ?
          15.0000    RUN
MIN Z ?
           0.0000    RUN
NO OF PTS ?
           9.0000    RUN
POINT X ?
           0.0000    RUN
ELEV Z ?
          15.0000    RUN
POINT X ?
           5.0000    RUN
ELEV Z ?
           7.5000    RUN
POINT X ?
          15.0000    RUN
ELEV Z ?
           1.0000    RUN
POINT X ?
          20.0000    RUN
ELEV Z ?
           5.0000    RUN
POINT X ?
          23.0000    RUN
ELEV Z ?
           2.0000    RUN
POINT X ?
          30.0000    RUN
ELEV Z ?
           2.0000    RUN
POINT X ?
          32.0000    RUN
ELEV Z ?
           0.0000    RUN
POINT X ?
          40.0000    RUN
ELEV Z ?
           8.0000    RUN
POINT X ?
          42.0000    RUN
ELEV Z ?
          15.0000    RUN
```

```
AREA=2.5986
ELEV=1.5000

AREA=21.5769
ELEV=3.0000

AREA=54.8293
ELEV=4.5000

AREA=97.7308
ELEV=6.0000

AREA=146.6250
ELEV=7.5000

AREA=199.8929
ELEV=9.0000

AREA=255.3929
ELEV=10.5000

AREA=313.8357
ELEV=12.0000

AREA=372.8214
ELEV=13.5000

AREA=434.7500
ELEV=15.0000

CORR=0.9895
A2=1.6617
B2=2.1455
```

Using Program TVSY:

```
XEQ "TVSY"
MAX Z ?
        15.0000    RUN
MIN Z ?
         0.0000    RUN
NO OF PTS ?
         9.0000    RUN
POINT X ?
         0.0000    RUN
ELEV Z ?
        15.0000    RUN
POINT X ?
         5.0000    RUN
ELEV Z ?
         7.5000    RUN
POINT X ?
        15.0000    RUN
ELEV Z ?
         1.0000    RUN
POINT X ?
        20.0000    RUN
ELEV Z ?
         5.0000    RUN
POINT X ?
        23.0000    RUN
ELEV Z ?
         2.0000    RUN
POINT X ?
        30.0000    RUN
ELEV Z ?
         2.0000    RUN
POINT X ?
        32.0000    RUN
ELEV Z ?
         0.0000    RUN
POINT X ?
        40.0000    RUN
ELEV Z ?
         8.0000    RUN
POINT X ?
        42.0000    RUN
ELEV Z ?
        15.0000    RUN
```

```
TW=4.3942
ELEV=1.5000

TW=18.5769
ELEV=3.0000

TW=25.7596
ELEV=4.5000

TW=30.6923
ELEV=6.0000

TW=34.5000
ELEV=7.5000

TW=36.2857
ELEV=9.0000

TW=37.7143
ELEV=10.5000

TW=39.1429
ELEV=12.0000

TW=40.5714
ELEV=13.5000

TW=42.0000
ELEV=15.0000

CORR=0.9184
A3=5.2187
B3=0.8572
```

Using Program PVSY:

```
                XEQ "PVSY"
MAX Z ?
          15.0000    RUN
MIN Z ?
           0.0000    RUN
NO OF PTS ?
           9.0000    RUN
POINT X ?
           0.0000    RUN
ELEV Z ?
          15.0000    RUN
POINT X ?
           5.0000    RUN
ELEV Z ?
           7.5000    RUN
POINT X ?
          15.0000    RUN
ELEV Z ?
           1.0000    RUN
POINT X ?
          20.0000    RUN
ELEV Z ?
           5.0000    RUN
POINT X ?
          23.0000    RUN
ELEV Z ?
           2.0000    RUN
POINT X ?
          30.0000    RUN
ELEV Z ?
           2.0000    RUN
POINT X ?
          32.0000    RUN
ELEV Z ?
           0.0000    RUN
POINT X ?
          40.0000    RUN
ELEV Z ?
           8.0000    RUN
POINT X ?
          42.0000    RUN
ELEV Z ?
          15.0000    RUN
```

```
WPER=5.9605
ELEV=1.5000

WPER=22.3566
ELEV=3.0000

WPER=31.7528
ELEV=4.5000

WPER=38.1340
ELEV=6.0000

WPER=43.0077
ELEV=7.5000

WPER=46.5576
ELEV=9.0000

WPER=49.9204
ELEV=10.5000

WPER=53.2832
ELEV=12.0000

WPER=56.6459
ELEV=13.5000

WPER=60.0087
ELEV=15.0000

CORR=0.9477
A4=6.3527
B4=0.8937
```

The coefficients in the equation $P = a_1 A^{b_1}$ are calculated to equal:

$a_1 = 5.1415$

$b_1 = 0.4165$

Normal Depth Calculations

The depth of flow existing under condition of uniform flow is defined as the normal depth. Uniform flow develops when the flow resistance is just balanced by gravitational force. Under these conditions, the slope of the energy grade line, S_E is equal to the bed slope, S_o. The normal depth is frequently of interest, particularly when calculations of the water surface profile are required. The type of water surface profile existing in a given situation depends on the relationship existing between the normal depth, the critical and the existing depth of flow for a given discharge. In this section, normal depth calculations in trapezoidal and natural channels will be discussed. Uniform flow very seldon exists in natural channels; however, in practice, this assumption is frequently made.

Normal Depth in Trapezoidal Channels

The normal depth in trapezoidal channels may be calculated using Manning's equation

$$Q = \frac{1.486}{n} R^{2/3} AS^{1/2} \tag{8}$$

where n is the Manning's resistance parameter, R is the hydraulic radius, A is flow area, S is slope and Q is discharge.

Area and wetted parimeter for a trapezoidal channel may be expressed as a function of depth as follows:

$$A = Z\, y^2 + by \tag{9}$$

where Z describes the side slope as the ratio of horizontal to vertical distance, b is the botton width and y is the depth. Wetted perimeter is given by

$$P = b + 2y\,(1 + z)^{1/2} \tag{10}$$

Therefore,

$$Q = \frac{1.486}{n} \frac{[Z\, y_o^2 + by_o]^{5/3} S^{1/2}}{[b + 2y_o\,(1+Z)^{1/2}]^{2/3}} \tag{11}$$

This equation may be solved for normal depth y_o in terms of the other known parameters by use of Newton's iternation method. The equation actually solved in this case for y_o would be

$$\frac{nQ}{1.486\ S^{1/2}} = \frac{[Zy_o^2 + by_o]^{5/3}}{[b + 2y_o(1+Z)^{1/2}]^{2/3}} \tag{12}$$

In addition, this program also provides the top width cross-sectional area, the mean velocity, and the stream power.

Program NDTR Users Instructions.

Step	Instructions	Input	Function	Display
1	Load program			
2	Begin execution		[XEQ] NDTR	SIDE SLOPE ?
3	Enter side slope (H:V)	Z	[R/S]	BED WIDTH ?
4	Enter channel bed width (ft)	b	[R/S]	FLOW ?
5	Enter discharge (cfs)	Q	[R/S]	SLOPE ?
6	Enter channel bed slope	S	[R/S]	MANNING N ?
7	Enter Manning n	n	[R/S]	INITIAL Y ?
8	Enter initial estimate for depth (ft)	y_o	[R/S]	MAX ITER ?
9	Enter the maximum number of iterations in Newton's iteration method (A maximum of 20 is good; equations will generally converge much faster). Program calculate depth (Y) ft; top width (TW) ft; area (A) ft^2; velocity (V) ft/s; stream power, $\tau_o V$ (S.P.) lb.ft/s	N	[R/S] [R/S]* [R/S]* [R/S]* [R/S]*	Y= TW= A= V= S.P.=

* Note: If printer is connected to calculator and turned on, executing [R/S] is not required here.

Example: Normal Depth is a Trapezoidal Channel

This example illustrates the use of Program NDTR. Consider the following data:

Side slope (H:v)	=	2.0
Bottom width	=	20.0 ft
Discharge	=	500 cfs
Slope	=	0.01
Manning n	=	0.035

Using Program NDTR:

```
          XEQ "NDTR"
SIDE SLOPE ?
          2.0000    RUN
BED WIDTH ?
         20.0000    RUN
FLOW ?
        500.0000    RUN
SLOPE ?
           .0100    RUN
MANNING N ?
           .0350    RUN
INITIAL Y ?
          1.5000    RUN
MAX ITER ?
         20.0000    RUN

Y=2.7524

TW=31.0098

A=70.2010

V=7.1224

S.P.=9.6567
```

Normal Depth in a Natural Channel

When the channel geometry is known through a series of functional expressions, the calculation of normal depth is straight-forward. This program is based on Manning's resistance equation. When the area wetted perimeter equation and the area depth equations, given by

$$P = a_1 A^{b_1} \tag{13}$$

and

$$A = a_2 y^{b_2} \tag{14}$$

where P is wetted parimeter, A is area, y is flow depth, and a_1, a_2, b_1, and b_2 are statistically fitted coefficients and exponents. By using these expressions, hydraulic radius R in Equation 8 may be expressed as a function of y as follows:

$$R = \frac{A}{P} = \frac{a_2 y^{b_2}}{a_1 (a_2 y^{b_2})^{b_1}} = \frac{a_2}{a_1 a_2^{b_1}} y^{(b_2 - b_2 b_1)} \tag{15}$$

Therefore, Equation 8 may be rewritten in terms of depth of flow in a natural channel as

$$Q = \frac{1.486}{n} \left[\frac{a_2}{a_1 a_2^{b_2}} y_o^{(b_2 - b_2 b_1)} \right]^{2/3} a_2 y_o^{b_2} S_o^{1/2} \tag{16}$$

This equation may be solved directly for y_o resulting in

$$y_o = \left[\left(\frac{Qn}{1.49\ S_o^{1/2}} \right)^{3/2} \left(\frac{a_1}{a_2^{(5/2 - b_2)}} \right) \right]^{\frac{2}{(5b_2 - 2b_1 b_2)}} \tag{17}$$

In addition, this program also provides hydraulic depth, top width, area, velocity, and stream power.

Program NDNC Users Instructions.

Step	Instructions	Input	Function	Display
1	Load program			
2	Begin execution		[XEQ] NDNC	A1 ?
3	Enter a_1 in $P = a_1 A^{b_1}$	a_1	[R/S]	B1 ?
4	Enter b_1	b_1	[R/S]	A2 ?
5	Enter a_2 in $A = a_2 y^{b_2}$	a_2	[R/S]	B2 ?
6	Enter b_2	b_2	[R/S]	A3 ?
7	Enter a_3 in $T = a_3 y^{b_3}$	a_3	[R/S]	B3 ?
8	Enter b_3	b_3	[R/S]	FLOW ?
9	Enter channel discharge (cfs)	Q	[R/S]	SLOPE ?
10	Enter channel slope	S	[R/S]	MANNING N ?
11	Enter Manning's n	n	[R/S]	
	Calculator will calculate			HY.D.=
	Hydraulic depth (HY.D.) ft		[R/S]*	N.D.=
	Normal depth (N.D.) ft		[R/S]*	TW=
	Top width (TW) ft		[R/S]*	A=
	Area (A) ft^2		[R/S]*	V=
	Velocity (V) ft/s		[R/S]*	S.P.=
	Stream power, $\tau_0 V$ (S.P.) lb.ft/s			

* Note: If a printer is connected to calculator and turned on, executing [R/S] is not required here.

Example: Normal Depth in Natural Channel:

This example illustrates the use of Program NDNC. Consider a channel with the following data:

$P = 4.89\ A^{0.43}$

$A = 2.15\ y^{1.66}$

$T = 5.22\ y^{0.86}$

Q = 1000 cfs

Slope = 0.005

Manning's n = 0.025

Using Program NDNC:

```
                XEQ "NDNC"
A1 ?
            4.8900    RUN
B1 ?
             .4300    RUN
A2 ?
            2.1500    RUN
B2 ?
            1.6600    RUN
A3 ?
            5.2200    RUN
B3 ?
             .8600    RUN
FLOW ?
        1,000.0000    RUN
SLOPE ?
             .0050    RUN
MANNING N ?
             .0250    RUN

HY.D.=2.7858

N.D.=10.9074

TW=40.7484

A=113.5154

Y=8.8094

S.P.=8.3401
```

Backwater Curve Computation in Natural Channels

Water surface profile computations assume that changes in depth and velocity take place slowly over large distances, resistance to flow dominates and acceleration forces may be neglected. This type of flow is called gradually varied flow. Calculations under these conditions involve; (1) the determination of the general characteristics of the water surface; and (2) the elevation of the water surface or depth of flow.

In gradually varied flow, the actual flow depth y is either larger than or smaller than the normal depth y_o and either larger than or smaller than the critical depth y_c. The water surface profiles which are often called backwater curves, depend on the magnitude of the actual depth of flow y in relation to the normal depth y_o and the critical depth y_c. Normal depth y_o is the depth of flow that would exist for steady-uniform flow as determined using the Manning and Chezy velocity equations, and the critical depth is the depth of flow when the Froude number equals 1.0. Reasons for the depth being different that the normal depth are changes in slope of the bed, changes in cross section, obstruction to flow, imbalances between gravitational forces accelerating the flow and shear forces retarding the flow.

Given channel cross-sectional properties such as the wetted perimeter-depth, area-depth, and top width-depth relations, the standard step method may be modified to provide backwater calculations in natural channels.
Let

$$P = a_4 y^{b_4} \tag{18}$$

$$A = a_2 y^{b_2} \tag{19}$$

where P is wetter perimeter, A is cross-sectional area and y is depth. Using Manning's equation

$$K = \frac{Q}{S^{1/2}} = \frac{1.486}{n} R^{2/3} A \tag{20}$$

where Q is discharge, n is Manning's resistance, S is slope, R is hydraulic radius, and K is known as conveyance

$$K = \frac{1.486}{n} \left[\frac{(a_2 y^{b_2})}{(a_4 y^{b_4})} \right]^{2/3} (a_2 y^{b_2}) \tag{21}$$

$$K = \frac{1.486}{n} a_4^{-2/3} a_2^{+5/3} y^{(5/3\ b_2 - 2/3\ b_4)} \tag{22}$$

Therefore,

$$S = \left(\frac{Q}{K}\right)^2 \tag{23}$$

The total energy equation at a station may be written in terms of a known total head at another station.

$$y + z - \Delta x (2Q)^2 \left[K + C_1 y^{C_2}\right]^{-2} + \frac{Q^2}{2g} (a_2 y^{b_2})^{-2} = T_n \tag{24}$$

where z is the bed elevation, T_n is the known total head at another station and C_1 and C_2 are given by

$$C_1 = \frac{1.486}{n} a_2^{5/3} a_4^{-2/3} \tag{25}$$

$$C_2 = \frac{5}{3} b_2 - \frac{2}{3} b_4 \tag{26}$$

Equation 24 may be solved for depth y using Newton's method.

Program BWC Users Instructions.

Step	Instructions	Input	Function	Display
1	Load program			
2	Begin execution		[XEQ] BWC	DISCHARGE ?
3	Enter discharge (cfs)	Q	[R/S]	Y1 ?
4	Enter depth at Section 1 (ft)	y_1	[R/S]	Z1 ?
5	Enter bottom elevation at Section 1 (ft)	z_1	[R/S]	A2 ?
6	Enter a_2 in $A = a_2 y^{b_2}$	a_2	[R/S]	B2 ?
7	Enter b_2	b_2	[R/S]	A4 ?
8	Enter a_4 in $P = a_4 y^{b_4}$	a_4	[R/S]	B4 ?
9	Enter b_4	b_4	[R/S]	MANNING N ?
10	Enter Manning's n	n	[R/S]	
	Program will print total head (ft) and conveyance (cfs) at Section 1.		[R/S]*	HEAD= CONV=
11	If new cross section properties are required for upstream section, enter yes (Calculator automatically in alpha mode) and repeat steps 6 through 10. Then go to step 12	yes	[R/S]	A2 ? Z2 ?
	If upstream section properties are identical to those downstream, enter no (Calculator automatically in alpha mode) and go to Step 12	no	[R/S]	Z2 ?
12	Enter bottom elevation of upstream cross section (ft)	Z_2	[R/S]	DX ?
13	Enter distance between cross sections (ft)	Δx	[R/S]	
	Program will calculate and display the following for the upstream cross section:			
	1. water depth (ft)			Y2=
	2. total head (ft)		[R/S]*	HEAD=
	3. conveyance (ft)		[R/S]*	CONV=

* Note: If printer is connected to calculator and turned on, [R/S] is not required here.

Example: Backwater Curve Computation in Natural Channels

This example illustrates the use of Program BWC. Consider the following data:

Q = 10,000 cfs

Conditions and channel properties at cross section 1:

Water depth = 35 ft
Bottom elevation = 100 ft
A = $20\ y^{1}$
P = $20\ y^{0.5}$
n = 0.02

Channel properties at cross section 2:

A = $21\ y^{1.1}$
P = $20\ y^{0.6}$
n = 0.02
Bottom elevation = 105 ft
Δx = 500 ft

Using Program BWC:

```
                XEQ "BWC"
DISCHARGE ?
     10,000.0000    RUN
Y1 ?
         35.0000    RUN
Z1 ?
        100.0000    RUN
A2 ?
         20.0000    RUN
B2 ?
          1.0000    RUN
A4 ?
         20.0000    RUN
B4 ?
           .5000    RUN
MANNING N ?
           .0200    RUN

HEAD=138.1690
CONV=170,128.1594

NEW SECT ?
YES                 RUN
```

```
A2 ?
         21.0000    RUN
B2 ?
          1.1000    RUN
A4 ?
         20.0000    RUN
B4 ?
           .6000    RUN
MANNING N ?
           .0200    RUN
Z2 ?
        105.0000    RUN
DX ?
        500.0000    RUN

Y2=32.7264

HEAD=139.3629
CONV=239,160.4202
```

CALCULATOR PROGRAMS FOR ENGINEERING ANALYSIS OF FLUVIAL SYSTEMS

FLOOD ROUTING PACKAGE

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Flood Routing Package

A simplified version of the DAMBRK flood routing program developed by the National Weather Service is being made available by SLA as part of the Flood Routing package. The simplified dambreak program SMPDBK predicts downstream peak discharges, depth, and travel times with a fraction of the input data and user effort required to implement the DAMBRK program. Furthermore, the accuracy of the simplified version is within ten percent of the results obtained by application of the DAMBRK model in many cases. When applied with an understanding of its limitations, the SMPDBK program is a very useful tool in preparing a dambreak flood forecast.

For routing flood hydrographs in stream channels and through reservoirs, a Muskingum-Cunge channel routing program and level pool reservoir method allows consideration of channel storage when conducting streamflow routing. The level pool reservoir routing program calculates the change in reservoir contents and discharge with time using arbitrary spillway rating and reservoir capacity curves.

National Weather Service (NWS) SMPDBK

Provided in this package is the National Weather Service (NWS) SMPDBK program for developing dambreak flood hydrographs and routing the resulting flood flows through a downstream channel. The complete NWS documentation to this program is included as a separate document.

Muskingum-Cunge Flood Routing

The Muskingum-Cunge flood routing method is based on the assumption of a linear relationship between inflow I, outflow O, and volume of storage V in a channel reach, of the form

$$V = K[XI + (1-X)O] \tag{1}$$

in which K and X are the Muskingum parameters. Equation 1 leads to

$$Q_{j+1}^{n+1} = C_1 Q_j^n + C_2 Q_j^{n+1} + C_3 Q_{j+1}^n \tag{2}$$

from which the outflow discharge at the new time Q_{j+1}^{n+1} can be calculated from a knowledge of the inflow discharge at the present time Q_j^n, the inflow discharge at the new time Q_j^{n+1}, and the outflow discharge at the present time Q_{j+1}^n The coefficients C_1, C_2, and C_3 are the Muskingum coefficients, defined as

$$C_1 = \frac{2X + \frac{\Delta t}{K}}{2(1-X) + \frac{\Delta t}{K}} \tag{3}$$

$$C_2 = \frac{-2X + \frac{\Delta t}{K}}{2(1-X) + \frac{\Delta t}{K}} \tag{4}$$

$$C_3 = \frac{2(1-X) - \frac{\Delta t}{K}}{2(1-X) + \frac{\Delta t}{K}} \tag{5}$$

in which Δt is the time step (also called time interval or routing period).

In the Muskingum-Cunge method, the parameter K is the propagation time of the flood wave, defined as

$$K = \frac{\Delta x}{c} \tag{6}$$

in which Δx is the computational reach length and c is the flood wave celerity. The parameter X is a measure of the amount of numerical diffusion, and can be calculated by

$$X = \frac{1}{2}\left(1 - \frac{q_r}{S_o c \Delta x}\right) \tag{7}$$

in which q_r is the reference discharge per unit width and S_o is the channel bed slope. The parameters K and X are assumed constant.

A method to estimate q_r is expressed as

$$q_r = \frac{Q_r}{T_w} \tag{8}$$

where Q_r is the reference discharge and T is the top width at the reference discharge.

The K and X are calculated based on a reference discharge Q_r. If

$$Q_r = \alpha' A_r^{\beta'} \tag{9}$$

then

$$A_r = \alpha Q_r^{\beta} \tag{10}$$

where $\alpha = (1/\alpha')^{1/\beta'}$ and $\beta = 1/\beta'$.

The celerity of the flood wave c is given by

$$c = \alpha' \beta' A_r^{\beta'-1} \tag{11}$$

Program MCFR Users Instructions.

Step	Instructions	Input	Function	Display
1	Load program			
2	Begin execution		[XEQ] MCFR	A1 ?
3	Enter a_1 in $P = a_1 A^{b_1}$	a_1	[R/S]	B1 ?
4	Enter b_1 in $P = a_1 A^{b_1}$	b_1	[R/S]	A2 ?
5	Enter a_2 in $A = a_2 y^{b_2}$	a_2	[R/S]	B2 ?
6	Enter b_2 in $A = a_2 y^{b_2}$	b_2	[R/S]	A3 ?
7	Enter a_3 in $T = a_3 y^{b_3}$	a_3	[R/S]	B3 ?
8	Enter b_3 in $T = a_3 y^{b_3}$	b_3	[R/S]	REF.Q ?
9	Enter reference discharge (cfs)	Q_r	[R/S]	SLOPE ?
10	Enter average channel bed slope	S_o	[R/S]	MANNING N ?
11	Enter Manning's n	n	[R/S]	DELTA X ?
12	Enter computational reach length (ft)	Δx	[R/S]	DELTA T ?
13	Enter time step (min)	Δt	[R/S]	INIT. U/S Q ?
14	Enter initial flow at upstream boundary (cfs)	Q_j^n	[R/S]	INIT. D/S Q ?
15	Enter initial flow at downstream boundary (cfs)	Q_{j+1}^n	[R/S]	INFLOW Q ?
16	Enter upstream inflow for current time step (cfs)	Q_j^{n+1}	[R/S]	
	Program wil compute and display the current time (min) and the downstream discharge (cfs)		[R/S]*	TIME= DISCH.=
	Repeat step 16 for each time step of the simulation			

* Note: If printer is connected to calculator and turned on, executing [R/S] is not required here.

Example: Muskingum-Cunge Flood Routing

This example illustrates the use of Program MCFR. Consider the following data:

$P = 5.8 \; A^{0.5}$

$A = 3.7 \; y^{2.4}$

$T = 11.6 \; y^{1.2}$

$Q_r = 67$ cfs (Reference discharge)

$S_o = 0.03$ (ft/ft)

$N = 0.02$

$\Delta x = 1000$ ft

At the upstream boundary:

Time t_i (min)	Discharge Q_i (cfs)
0	0
10	20
20	50
30	75
40	100
50	80
60	60
70	40
80	20
90	0

Using Program MCFR:

```
              XEQ "MCFR"
A1 ?
          5.8000    RUN
B1 ?

           .5000    RUN
A2 ?
          3.7000    RUN
B2 ?
          2.4000    RUN
A3 ?
         11.6000    RUN
B3 ?
          1.2000    RUN
REF. Q ?
         67.0000    RUN
SLOPE ?
           .0300    RUN
MANING N ?
           .0200    RUN
DELTA X ?
      1,000.0000    RUN
DELTA T ?
         10.0000    RUN
INIT. U/S Q ?
          0.0000    RUN
INIT. D/S Q ?
          0.0000    RUN

INFLOW Q ?
         20.0000    RUN
TIME=10.0000
DISCH.=14.6448

INFLOW Q ?
         50.0000    RUN
TIME=20.0000
DISCH.=45.8713
```

```
INFLOW Q ?
         75.0000    RUN
TIME=30.0000
DISCH.=71.3159

INFLOW Q ?
        100.0000    RUN
TIME=40.0000
DISCH.=95.9918

INFLOW Q ?
         80.0000    RUN
TIME=50.0000
DISCH.=88.2774

INFLOW Q ?
         60.0000    RUN
TIME=60.0000
DISCH.=59.3207

INFLOW Q ?
         40.0000    RUN
TIME=70.0000
DISCH.=45.8505

INFLOW Q ?
         20.0000    RUN
TIME=80.0000
DISCH.=21.0900

INFLOW Q ?
          0.0000    RUN
TIME=90.0000
DISCH.=4.5606
```

Level Pool Routing

Routing of a flood hydrograph through a reservoir is performed using the storage, inflow discharge relationship:

$$\frac{ds}{dt} = I - O \tag{12}$$

where S is the reservoir storage volume, t is time, I instantaneous inflow, and O instantaneous outflow. A finite difference formulation of this equation is programmed and solved simultaneously subject to the reservoir storage-outflow relationship by a Newton-Rapson scheme. The reservoir stage-storage curve and spillway rating curve are utilized in tabular form with the number of data points to describe either curve selected by the user. The program interpolates these data and extrapolates if necessary. The inflow hydrograph is descretized by the user as required to accurately define the flow variation for inflow and for outflow. Varying time steps may be used, with smaller time steps providing higher resolution at times when flows change rapidly.

The starting reservoir conditions for the routing may be at the spillway crest elevation, below or above this elevation. In the case where the routing is started below the spillway elevation, the time step at which the reservoir begins to spill is subdivided by the program, and routing computation performed over the partial time step using an interpolated inflow.

Program ROUTE Users Instructions.

Step	Instructions	Input	Function	Display
1	Allocate storage registers NNN > 26 + 2 (NS+NO) NS = No. of stage-storage pairs NO = No. of stage-discharge pairs		[XEQ] SIZE NNN	
2	Load program			
3	Begin execution		[XEQ] ROUTE	NO. RES PT?
4	Enter number of reservoir stage-storage pairs	NS	[R/S]	STAGE=?
5	Enter reservoir elevation starting with lowest (ft)	Er	[R/S]	STORAGE=?
6	Enter corresponding reservoir storage (acre-feet)	S	[R/S]	
7	Repeat steps 5 and 6 NS times			NO. RATING PT=?
8	Enter number of reservoir stage-discharge pairs	NO	[R/S]	STAGE=?
9	Enter reservoir elevation starting with spillway crest (ft)	Es	[R/S]	DISCHARGE=?
10	Enter spillway discharge starting from zero (cfs)	0	[R/S]	
11	Repeat steps 9 and 10 NO times			INT. INFLOW=?
12	Enter inflow at start of routing (cfs)	I	[R/S]	INT. STAGE=?
13	Enter starting reservoir elevation (ft) and compute initial storage (acre-feet) and discharge (cfs)	Eo	[R/S] [R/S]* [R/S]* [R/S]*	DISCH.= STAGE= STO.= TIME=?
14	Enter time corresponding to following inflow (hr)	T	[R/S]	INFLOW=?
15	Enter inflow (cfs) and compute stage (ft) storage (acre-feet), discharge (cfs) [and fill time (hr) if applicable]	I	[R/S] [R/S]* [R/S]* [R/S]*	FILL TIME= DISCH.= STAGE= STO.=
16	Repeat steps 14 and 15 as desired			
	To rerun with same storage curve but new spillway curve, return to step 8		[GTO]A	
	To restart a routing at any point in time, return to step 12		[GTO]B	
	To correct a time or inflow input error before computation, return to step 14		[GTO]C	

* Note: If printer is connected to calculator and turned on, executing [R/S] is not required here.

Example: Level Pool Routing

This example illustrates the use of Program ROUTE. Consider the following data:

Stage (ft)	Storage (ac-ft)
49.0	0.10
53.0	0.33
56.0	0.64
59.0	0.99
62.0	1.50
64.0	1.82
66.0	2.31

Outflow rating curve:

Stage (ft)	Discharge (cfs)
56.5	0.0
65.0	0.7
65.2	3.5
65.4	8.5
65.6	15.1
65.8	22.8
66.0	36.0
66.2	55.0

Inflow hydrograph:

Time (hr)	Discharge (cfs)
0	0
0.05	15.0
0.10	27.0
0.15	60.0
0.20	89.0
0.225	96.0
0.25	100.0
0.275	97.0
0.30	93.0
0.35	75.0
0.40	57.0
0.45	42.0
0.50	27.0
0.55	15.0
0.60	7.0

Inital Stage = 49.2 feet

Using Program ROUTE:

```
          XEQ "ROUTE"
NO. RES PT?
          7.0000    RUN
STAGE=?
         49.0000    RUN
STORAGE=?
           .1000    RUN
STAGE=?
         53.0000    RUN
STORAGE=?
           .3300    RUN
STAGE=?
         56.0000    RUN
STORAGE=?
           .6400    RUN
STAGE=?
         59.0000    RUN
STORAGE=?
           .9900    RUN
STAGE=?
         62.0000    RUN
STORAGE=?
          1.5000    RUN
STAGE=?
         64.0000    RUN
STORAGE=?
          1.8200    RUN
STAGE=?
         66.0000    RUN
STORAGE=?
          2.3100    RUN

NO. RATING PT=?
          8.0000    RUN
STAGE=?
         56.5000    RUN
DISCHARGE=?
          0.0000    RUN
STAGE=?
         65.0000    RUN
DISCHARGE=?
           .7000    RUN
STAGE=?
         65.2000    RUN
DISCHARGE=?
          3.5000    RUN
STAGE=?
         65.4000    RUN
DISCHARGE=?
          8.5000    RUN
STAGE=?
         65.6000    RUN
DISCHARGE=?
         15.1000    RUN
STAGE=?
         65.8000    RUN
DISCHARGE=?
         22.8000    RUN
STAGE=?
         66.0000    RUN
DISCHARGE=?
         36.0000    RUN
STAGE=?
         66.2000    RUN
DISCHARGE=?
         55.0000    RUN

INT. INFLOW=?
          0.0000    RUN
INT. STAGE=?
         49.2000    RUN

INITIAL COND.

DISCH.=0.0000
STAGE=49.2000
STO.=0.1115

TIME=?
           .0500    RUN
INFLOW=?
         15.0000    RUN

DISCH.=0.0000
STAGE=49.7390
STO.=0.1425

TIME=?
           .1000    RUN
INFLOW=?
         27.0000    RUN

DISCH.=0.0000
STAGE=51.2481
STO.=0.2293

TIME=?
           .1500    RUN
INFLOW=?
         60.0000    RUN

DISCH.=0.0000
STAGE=53.7647
STO.=0.4090

TIME=?
           .2000    RUN
INFLOW=?
         89.0000    RUN

FILL TIME=0.1539

DISCH.=0.2158
STAGE=59.1202
STO.=1.0104

TIME=?
           .2250    RUN
INFLOW=?
         96.0000    RUN

DISCH.=0.3081
STAGE=60.2413
STO.=1.2010

TIME=?
           .2500    RUN
INFLOW=?
        100.0000    RUN

DISCH.=0.4058
STAGE=61.4280
STO.=1.4028
```

```
TIME=?
          .2750    RUN
INFLOW=?
        97.0000    RUN

DISCH.=0.5072
STAGE=62.6583
STO.=1.6053

TIME=?
          .3000    RUN
INFLOW=?
        93.0000    RUN

DISCH.=0.6076
STAGE=63.8778
STO.=1.8005

TIME=?
          .3500    RUN
INFLOW=?
        75.0000    RUN

DISCH.=5.6131
STAGE=65.2845
STO.=2.1347

TIME=?
          .4000    RUN
INFLOW=?
        57.0000    RUN

DISCH.=38.4668
STAGE=66.0260
STO.=2.3164
```

```
TIME=?
          .4500    RUN
INFLOW=?
        42.0000    RUN

DISCH.=48.2819
STAGE=66.1293
STO.=2.3417

TIME=?
          .5000    RUN
INFLOW=?
        27.0000    RUN

DISCH.=36.0173
STAGE=66.0003
STO.=2.3101

TIME=?
          .5500    RUN
INFLOW=?
        15.0000    RUN

DISCH.=25.2779
STAGE=65.8375
STO.=2.2702

TIME=?
          .6000    RUN
INFLOW=?
         7.0000    RUN

DISCH.=17.4982
STAGE=65.6623
STO.=2.2273
```

CALCULATOR PROGRAMS FOR ENGINEERING ANALYSIS OF FLUVIAL SYSTEMS

SEDIMENTATION PACKAGE

By

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Sedimentation Package

The Sedimentation package consists of programs to calculate sediment transport based on different transport relationships and a utility program for evaluating critical and effective shear stresses and particle fall velocity. Particle fall velocities may be calculated using Stoke's Law for small sizes and Rubey's expression for larger sizes. The effective boundary shear stress acting on a given size may be calculated as a function of channel flow conditions. The shear stress required to initiate sediment motion for a given grain size may be calculated using the Shield's criteria. Sediment transport relationships which are programmed in the package are the Meyer-Peter, Muller bed load equation, the Einstein suspended load approximation, and the transport relationships developed by Yang and by Shen and Hung.

Particle Fall Velocity

Particle fall velocities are a function of particle size and water properties that are represented by the kinematic viscosity. The fall velocity for sediment diameters less than 0.0002 feet may be calculated from Stoke's Law. For larger sizes, Rubey's (ASCE, 1975) expression is used, or

$$w = \frac{2.9517\ d_s^2}{\nu}\ ;\ \text{for } d_s < 0.0002 \text{ ft} \tag{1}$$

and

$$w = \frac{[36.064\ d_s^3 + 36\ \nu^2]^{1/2} - 6\nu}{d_s}\ ;\ \text{for } d_s \geq 0.0002 \text{ ft} \tag{2}$$

Effective Shear Stress

The effective boundary shear stress, τ_e, acting on the grain can be expressed as

$$\tau_e = \frac{1}{8}\ \rho\ f\ V^2 \tag{3}$$

where ρ is the density of water, f is the Darcy-Weisback friction factor and V is the average flow velocity.

Critical Shear Stress

The shear stress necessary to initiate sediment motion is known as the critical shear stress. The Shield's diagram (Figure 1) is most often used to compute critical shear stress. The procedure is to determine either the boundary Reynolds number, $R_* = U_* d_s/\nu$ or the parameter $d_s/\nu \sqrt{0.1\ (\gamma_s/\gamma - 1)\ gd_\nu}$ and finding the point of intersection with the dimensionless shear stress curve. The corresponding ordinate, F_*, is the set equal to the Shield's parameter, that is

$$F_* = \frac{\tau_c}{(\gamma_s - \gamma) d_s} \tag{4}$$

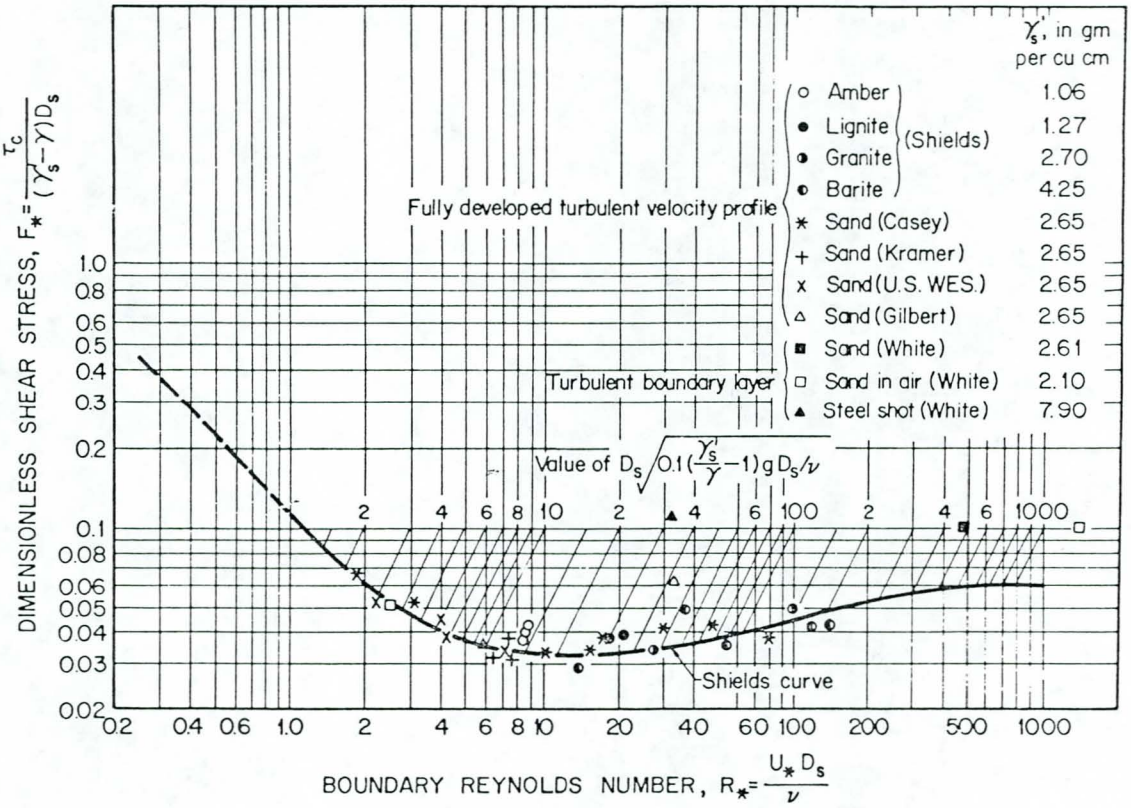

Figure 1. Shield's diagram: dimensionless critical shear stress.

For values of the boundary Reynolds number greater than 500, $F_\star = 0.060$. However, Gessler (1965) suggested reducing $F_\star = 0.047$ for Reynolds numbers greater than 500. The program allows the user to enter $F_\star$ or use a default value of $F_\star = 0.047$.

Program FALL Users Instructions.

Step	Instructions	Input	Function	Display
1	Load program			
2	Set calculator to USER mode		[USER]	
3	Begin execution		[XEQ] FALL	OPTION ?
4	To determine particle fall velocity, execute local label A		A	VISCOSITY ?
5	Enter kinematic viscosity (ft/sec^2)	ν	[R/S]	PART. SIZE ?
6	Enter mean particle diameter (mm)	d_s	[R/S]	
	Program will compute and display fall velocity (ft/s)			FALL VEL=
4	To determine effective shear stress, execute local label B		B	FRICTION F ?
5	Enter friction factor f	f	[R/S]	VELOCITY ?
6	Enter velocity (ft/s)	V	[R/S]	
	Program will compute and display effective shear stress (lb/ft^2)			EFF. STRESS=
4	To determine critical shear stress using default value of F* = 0.047, execute local label C		C	PART. SIZE ?
5	Enter mean particle diameter (mm)	d_s	[R/S]	
	Program will computer and display critical shear stress (lb/ft^2)			CR. STRESS=
4	To determine critical shear stress using an input value of F*, execute local label D		D	PART. SIZE ?
5	Enter mean particle diamter (mm)	d_s	[R/S]	F* ?
6	Enter F*	F*	[R/S]	
	Program will compute and display critical shear stress			CR. STRESS=

Example: Particle Fall Velocity, Effective Shear Stress, and Critical Shear Stress

This example illustrates the use of Program FALL. Consider the following data:

$\nu = 1.16 \times 10^{-5}$

$d_s = 0.2$ mm

$f = 0.03$

$V = 2.0$ ft/s

$F_* = 0.06$

Using program FALL to determine particle fall velocity, effective shear stress, critical shear stress ($F_* = 0.047$) and critical shear stress ($F_* = 0.06$):

```
                XEQ "FALL"
OPTION ?
                     XEQ A
VISCOSITY ?
          1.16-05    RUN
PART. SIZE ?
             .2000   RUN

FALL VEL=7.965E-2

                     XEQ B
FRICTION F ?
               .03   RUN
VELOCITY ?
                2    RUN

EFF. STRESS=2.907E-2

                     XEQ C
PART. SIZE ?
               .2    RUN
CR. STRESS=5.089E-5

                     XEQ D
PART. SIZE ?
               .2    RUN
F* ?
              .06    RUN

CR. STRESS=6.496E-5
```

Meyer-Peter, Muller Bed Load and Einstein Suspended Load Equations

The bed load transport capacity for a given size material is given by the Meyer-Peter, Muller equation.

$$q_{bi} = P_{\ell i} \frac{12.85}{\gamma_s \rho} (\tau_o - \tau_{ci})^{1.5} \tag{5}$$

in which q_{bi} is the bed load transport rate for that size fraction in volume per unit width, τ_o is the boundary shear acting on a grain, τ_{ci} is the critical tractive shear for that grain size, ρ is the density of water, γ_s is the unit weight of sediment, and $P_{\ell i}$ is the fraction of material for the given size present in the loose soil layer. The equation for suspended load discharge for each fraction of sediment is demonstrated by the Einstein method as:

$$q_{si} = q_{bi} (P_E I_1 + I_2) \tag{6}$$

where P_E is a hydraulically defined parameter

$$I_1 = 0.216 \frac{E^{z-1}}{(1-E)^z} \int_E^1 \left(\frac{1-y^2}{y}\right) dy \tag{7}$$

$$I_2 = 0.216 \frac{E^{z-1}}{(1-E)^z} \int_E^1 \left(\frac{1-y^2}{y}\right) \ln y \, dy \tag{8}$$

and

$$E = \frac{a}{D} \tag{9}$$

$$a = \frac{w}{0.4U_*} \tag{10}$$

where

- a is the level of reference above the bed.
- U_* is the shear velocity due to grain roughness.
- w is the fall velocity of suspended sediment
- D is the hydraulic depth of flow.

Equation 6 relates the bed load transport to suspended load transport for all size fractions for which the bed load function exists. The transport capacity for a given size material may be expressed as

$$q_{ti} = q_{bi} + q_{si} \tag{11}$$

The shear stress is determined using the average flow velocity as

$$\tau_o = \frac{1}{8} \rho f u^2 \tag{12}$$

where f is the Darcy-Weisbach friction factor.

The critical tractive force τ_{ci} is computed from the equation

$$\tau_{ci} = \delta_s (\gamma_s - \gamma) d_{si} \tag{13}$$

where d_{si} is the sediment size is δ_s is the Shield criteria for incipient motion parameter dependent on flow conditions. For most flow conditions including those described within, δ_s is 0.047.

The total transport capacity is then determined by summing the transporting capacity for all sizes:

$$q_t = \sum_i (q_{si} + q_{bi}) \tag{14}$$

Program MPM is provided in the package using this theory. The first part of the program determines bed load only; the second part combines both bed load and Einstein's suspended load approximation. The program may be used for only representative sediment size at a time; however, best results will be achieved by routing by size functions.

Program MPM Users Instructions.

Step	Instructions	Input	Function	Display
1	Load program (USER mode)		[USER]	
2	Begin execution		[XEQ] MPM	SHIELDS F* ?
3	Enter Shields incipient motion parameter	δ_s	[R/S]	FRICTION F ?
4	Enter Darcy-Weisback friction factor	f	[R/S]	KIN. VISC ?
5	Enter Kinematic Viscosity (ft/sec^2)	ν	[R/S]	TOP WIDTH ?
6	Enter channel top width (ft)	W	[R/S]	VELOCITY ?
7	Enter velocity (ft/sec)	U	[R/S]	SLOPE ?
8	Enter channel bed slope	S	[R/S]	DEPTH ?
9	Enter flow depth (ft)	D	[R/S]	SED. SIZE ?
10	Enter geometric mean size of sediment (mm) in size fraction being routed	d_{si}	[R/S]	FRACTION ?
11	Enter fraction of total bed material being routed in current size fraction	fract.	[R/S]	
	Calculation program will compute and develop the following results for the Meyer-Peter, Muller Bed Load:			
	1) sediment size (mm)			SED SIZE=
	2) bed load yield (cfs)		[R/S]*	SED Q CFS=
	3) bed load yield (lb/sec)		[R/S]*	SED Q LB/S=
12	To calculate total sediment yield considering both bed load and Einstein suspended load, execute local label A		A	
	Program will compute and display following results for total sediment yield:			
	1) total sediment (cfs)			SED Q CFS=
	2) total sediment yield (lb/sec)		[R/S]*	SED Q LB/S=

* Note: If printer is connected to calculator and turned on, executing [R/S] is not required here.

Example: Sediment Transport using the Meyer-Peter, Muller Bed Load and Einstein Suspended Load Equations

This example illustrates the use of Program MPM. Consider the following conditions:

δ_s = 0.047

f = 0.025

ν = 1.1×10^{-5} ft/sec^2

w = 8.0 ft

u = 2.9 ft/sec

s = 0.002

D = 0.52 ft

d_s = 0.20 mm

fract = 1.0

Using Program MPM to determine bed load:

```
                XEQ "MPM"
SHIELDS F* ?
            .0470    RUN
FRICTION F ?
            .0250    RUN
KIN. VISC. ?
           1.1-05    RUN
TOP WIDTH ?
           8.0000    RUN
VELOCITY ?
           2.9000    RUN
SLOPE ?
            .0020    RUN
DEPTH ?
            .5200    RUN
SED. SIZE ?
            .2000    RUN
FRACTION ?
           1.0000    RUN

SED SIZE=0.2000
SED Q CFS=4.6599E-3
SED Q LB/S=0.7706
```

To determine total yield considering both bed load and suspended load:

```
                XEQ A
SED Q CFS=1.6672E0
SED Q LB/S=275.6912
```

Yang's Sediment Transport Equation

According to Yang (1976) the following stream power equation can be utilized for calculation and prediction of total bed material concentration transported in sand bed flumes and rivers:

$$\log C_t = 5.435 - 0.286 \log \frac{wD}{\nu} - 0.457 \log \frac{U_*}{w}$$

$$+ \left(1.799 - 0.409 \log \frac{wD}{\nu} \; 0.314 \log \frac{U_*}{w}\right) \log \left(\frac{US}{w} - \frac{U_{cr}S}{w}\right. \qquad (15)$$

where

C_t = total sediment concentration in parts per million by weight
D = median sieve diameter
S = water surface slope or energy slope
U_* = shear velocity
U = average water velocity
U_{cr} = critical average water velocity at incipient motion
ν = kinematic viscosity
w = terminal fall velocity

The term U_{cr}/w can be calculated as

$$\frac{U_{cr}}{w} = \frac{2.5}{\log \left(\frac{U_*D}{\nu}\right) - 0.06} + 0.66 \text{ when } 1.2 < \frac{U_*D}{\nu} < 70 \qquad (16)$$

and

$$U_{cr} \quad .05 \text{ when } 70 \leq \frac{U_*D}{\nu} \qquad (17)$$

In this program, the terminal fall velocity is calculated from the median grain size and the kinematic viscosity by the method presented in the fall velocity program.

Transport Relationship of Shen and Hung

Shen and Hung (1971) began with the assumption that sediment transport is such a complex phenomenon that no single Reynolds number, Froude number, or combination of them can be found to describe sediment motion under all conditions. They recommend a regression method be used to develop a formula based on all available data for immediate engineering purposes. They selected the sediment concentration (bed material load) as the dependent variable and the fall velocity of the median sediment particle of the bed sample as well as the flow velocity and flow depth as the independent variables. The equation is

$$\log C = a_0 + a_1 X + a_2 X^2 + a_3 X^3 \tag{18}$$

where

$$X = V^{a_4} S^{a_5} W^{a_6} D^{a_7} \tag{19}$$

a_0 through a_7 are the parameters to be determined by regression technique, C is the sediment bed material load concentration by weight (wash load is arbitrarily taken to less than 0.0625 mm), V is the average flow velocity, S is the energy slope, W is the fall velocity of the median size sediment of the bed sample, and D is the flow depth. The sediment particle fall velocity was corrected to that of the actual measured water temperature and no attempt was made to include the effect of heavy sediment concentration (if existent).

With a total of 587 data points, the final results were a_0 = -107404.45938164, a_1 = 324214.74734085, a_2 = -326309.58908739, a_3 = 109503.87232539, a_4 = 0.00750189, a_5 = 0.00428802, a_6 = -0.00239974, and a_7 was very small as compared to a_4, a_5, and a_6 and subsequently was dropped for the final analysis.

Program SEDTR Users Instructions.

Step	Instructions	Input	Function	Display
1	Load program (USER mode)		[USER]	
2	Begin execution		XEQ [SEDTR]	OPTION ?
3	To run Yang's method, execute local label A		A	DISCHARGE ?
4	Enter discharge (cfs)	Q	[R/S]	VELOCITY ?
5	Enter velocity (ft/s)	U	[R/S]	SLOPE ?
6	Enter channel bed slope	S	[R/S]	DEPTH ?
7	Enter flow depth (ft)	d	[R/S]	KIN. VISC ?
8	Enter kinematic viscosity (ft/sec^2)	ν	[R/S]	SIZE D50 ?
9	Enter mean sediment diameter (mm)	D_{50}	[R/S]	
	Program will compute and display the following:			
	1) sediment concentration (ppm)			SED CONC=
	2) sediment yield (cfs)		[R/S]*	SED Q CFS=
	3) sediment yield (lb/sec)		[R/S]*	SED Q LB/S=
3	To run Shen & Hung method, execute local label B		B	DISCHARGE ?
4	Enter discharge (cfs)	Q	[R/S]	KIN. VISC ?
5	Enter kinematic viscosity (ft/sec^2)	ν	[R/S]	VELOCITY ?
6	Enter channel bed slope	S	[R/S]	SIZE D50 ?
7	Enter mean sediment diameter (mm)	D_{50}	[R/S]	
	Program will compute and display the following:			
	1) sediment concentration (ppm)			SED CONC=
	2) sediment yield (cfs)		[R/S]*	SED Q CFS=
	3) sediment yield (lb/sec)		[R/S]*	SED Q LB/S=

* Note: If printer is connected and turned on, executing [R/S] is not required here.

Example: Sediment Transport Using Yang's Method and Relationship of Shen and Hung

This example illustrates the use of Program SEDTR. Consider the following data:

Q = 23.2 cfs

U = 2.9 ft/s

S = .002

Depth = 0.52 ft

δ = 1.1×10^{-5} ft/sec^2

D_{50} = 0.20 mm

Using Program SEDTR and Yang's Method:

```
                XEQ "SEDTR"
OPTION ?
                      XEQ A
DISCHARGE ?
          23.2000    RUN
VELOCITY ?
           2.9000    RUN
SLOPE ?
            .0020    RUN
DEPTH ?
            .5200    RUN
KIN. VISC. ?
           1.1-05    RUN
SIZE D50 ?
            .2000    RUN

SED CONC=2,471.3126
SED Q CFS=0.0216
SED Q LB/S=3.5777
```

Now using Program for the relationship of Shen and Hung:

```
                      XEQ B
DISCHARGE ?
          23.2000    RUN
KIN. VISC. ?
           1.1-05    RUN
VELOCITY ?
           2.9000    RUN
SLOPE ?
            .0020    RUN
SIZE D50 ?
            .2000    RUN

SED CONC=2,277.1938
SED Q CFS=0.0199
SED Q LB/S=3.2966
```

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CALCULATOR PROGRAMS FOR ENGINEERING ANALYSIS OF FLUVIAL SYSTEMS

RIVER MECHANICS PACKAGE

By

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River Mechanics Package

The River Mechanics package provides the capacity to estimate the aggradation or degradation of a river reach over time, and includes programs to estimate equilibrium local scour at an encroachment in the river or at a bridge pier and for riprap design.

The change in bed elevation with time for a river reach is calculated based upon a sediment continuity relationship for the reach and the channel geometry. When combined with a transport relationship, the dynamic change in bed elevation and slope can be approximated for a discharge hydrograph.

The safety factor of rock riprap for the shear stress of a given flow condition and channel configuration may be calculated using the riprap design program.

Degradation/Aggradation Analysis

A simple calculation of degradation or aggradation may be obtained using a method based on the sediment continuity equation. Assuming no lateral inflow, the sediment continuity equation may be written as

$$\frac{\partial Q_s}{\partial x} + (1 - \lambda) \frac{\partial A_s}{\partial t} = 0 \tag{1}$$

where Q_s is the volume sediment discharge, λ is the bed material porosity, A_s is the change in cross-sectional bed area, t is time and x is distance. In simple finite difference formulations

$$\frac{\Delta Q_s}{\Delta x} + (1 - \lambda) \frac{\Delta A_s}{\Delta t} = 0 \tag{2}$$

Therefore, the change in area may be calculated as

$$\Delta A_s = - \frac{\Delta Q_s \Delta t}{\Delta x (1 - \lambda)} \tag{3}$$

The change in Q_s may be expressed as

$$q_{s_2} B_2 - q_{s_1} B_1 = \Delta Q_s \tag{4}$$

where B_1 is the channel width at Station 1 and q_{s_i} is the sediment volume discharge per unit width at Station 1. In addition, ΔA_s may be expressed as

$$\Delta A_s = \frac{B_1 + B_2}{2} \Delta z \tag{5}$$

where Δz is the average change in bed elevation. Equating Equations 3 and 5 yields

$$\Delta z = - \frac{2 \Delta Q_s \Delta t}{\Delta x (1 - \lambda)(B_1 + B_2)} \tag{6}$$

or using Equation 4

$$\delta z = - \frac{2 \Delta t (B_2 q_{s_2} - B_1 q_{s_1})}{\Delta x (1 - \lambda)(B_1 + B_2)} \tag{7}$$

This change in bed elevation is an average change over the entire Δx. If there is a control at the downstream end, the average change in bed eleva-

tion should be redistributed over Δx giving no change at the downstream end and $2\Delta z$ at the upstream end. Equation 7 may be used in conjunction with a transport equation to provide for dynamic slope. That is, if the water hydrograph is subdivided into several Δts. The degradation/aggradation equation may be run for the first Δt and the slope updated by $2\Delta z$. Then the new slope may be used in the transport equation to calculate a new value of q_s. This q_s may then serve as input to the aggradation/degradation equation, and so forth.

Program SEDCON Users Instructions.

Step	Instructions	Input	Function	Display
1	Load program			
2	Begin execution		XEQ [SEDCON]	QS1 ?
3	Enter upstream unit sediment discharge (cfs/ft)	q_{s_1}	[R/S]	QS2 ?
4	Enter downstream unit sediment discharge (cfs/ft)	q_{s_2}	[R/S]	B1 ?
5	Enter upstream channel width (ft)	B_1	[R/S]	B2 ?
6	Enter downstream channel width (ft)	B_2	[R/S]	DELTA T ?
7	Enter time step (min)	Δt	[R/S]	DELTA X ?
8	Enter distance step (ft)	Δx	[R/S]	POROSITY ?
9	Enter porosity	λ	[R/S]	
	Program will compute and display depth (in feet) of aggradation (+) or degradation (-)			DZ=

Example: Degradation/Aggradation Analysis

This example illustrates the use of Program SEDCON. Consider the following data:

q_{s_1} = 300 cfs/ft
q_{s_2} = 310 cfs/ft
B_1 = 200 ft
B_2 = 200 ft
Δt = 1.0 min
Δx = 200 ft
λ = 0.40

Using Program SEDCON:

```
                XEQ "SEDCON"
QS1 ?
          300.0000    RUN
QS2 ?
          310.0000    RUN
B1 ?
          200.0000    RUN
B2 ?
          200.0000    RUN
DELTA T ?
            1.0000    RUN
DELTA X ?
          200.0000    RUN
POROSITY ?
             .4000    RUN

DZ=-0.5000
```

Local Scour Around Embankments

Detailed studies of scour around embankments have been made, mostly in laboratories. According to the studies of Liu et al., (1961), the equilibrium local scour depth may be determined by

$$S = 1.1 \quad y(\frac{a}{y}) \; F_r^{0.33} \tag{8}$$

where a is the embankment length (measured normal to the wall of a flume), y is upstream depth and F_r is the upstream Froude number

$$F_{r_1} = \frac{v}{U_g y} \tag{9}$$

where v is the upstream velocity.

Local Scour Around Bridge Piers

A number of available formulas for predicting local scour around bridge piers are given in "Scour at Bridge Watewasys" (Highway Research Board, 1970). Two of these formulas have been found to be particularly successful based on previous SLA experience. These two involve relationships developed by Shen and Neil. Neil's relationship is

$$d_s = 2.2 \; y \; (\frac{b}{y})^{0.65} Fr^{0.43} \tag{10}$$

where d_s is the depth of scour, b is the pier width, y is the flow depth, and Fr (pier Froude number) is $V/(gb)_{1/2}$.

Shen's equation study is

$$d_{sc} = 0.00073 \; R^{0.619} \tag{11}$$

where d_{sc} = equilibrium depth of scour measured from mean bed elevation, R = pier Reynolds number = $V\,b/\nu$, V = mean velocity of the undisturbed flow, b = width of pier projected on a plane normal to the undisturbed flow, and ν = kinematic viscosity. The relationships are based on model study data and are supposedly the maximum scour that can occur at any velocity.

Program SCOUR Users Instructions.

Step	Instructions	Input	Function	Display
1	Load program (USER mode)		[USER]	
2	Begin execution		[XEQ] SCOUR	OPTION ?
3	To compute local scour around embankments execute local label A		A	DEPTH ?
4	Enter upstream flow depth (ft)	y	[R/S]	VELOCITY ?
5	Enter upstream flow velocity (ft/s)	V	[R/S]	LENGTH ?
6	Enter embankment length (ft) This is the length of encrouchment into the flow from a vertical bank	a	[R/S]	
	Program will calculate and display local scour depth (ft)			S=
3	To compute local scour around bridge piers execute local label B		B	DEPTH ?
4	Enter upstream flow depth (ft)	y	[R/S]	VELOCITY ?
5	Enter upstream flow velocity (ft/sec)	v	[R/S]	WIDTH ?
6	Enter pier width (ft)	b	[R/S]	KIN VISC ?
7	Enter kinematic viscosity (ft/sec^2)	ν	[R/S]	
	Program will calculate and display local scour depth (ft) using both Neil's (d_s) and Shen's (d_{sc}) relationships			DS= DSC=

Example: Local Scour

This example illustrates the use of Program SCOUR. Consider first the potential local scour around the end of an embankment that encroaches into the flow ten feet from the bank:

y = 5 ft

V = 10 ft/sec

a = 10 ft (length embankment encroaches into flow)

Using Program SCOUR:

```
                XEQ "SCOUR"
OPTION ?
                      XEQ A
DEPTH ?
          5.0000    RUN
VELOCITY ?
         10.0000    RUN
LENGTH ?
         10.0000    RUN

S=6.7089
```

Next consider local scour around one-foot bridge piers:

y = 12 ft

v = 6 ft/sec

b = 1 ft (pier width)

ν = 1×10^{-5} ft/sec^2

Using Program SCOUR to compute the potential scour considering both Neil's (d_s) and Shen's (d_{sc}) relationships:

```
                XEQ "SCOUR"
OPTION ?
                      XEQ B
DEPTH ?
         12.0000    RUN
VELOCITY ?
          6.0000    RUN
WIDTH ?
          1.0000    RUN
KIN VISC ?
             1-05   RUN

DS=5.3771

DSC=2.7542
```

Design of Riprap

The common acceptalbe methods for sizing the riprap are discussed in Simons and Senturk (1977). The method developed by Steven and Simons (1971) is utilized in this analysis. The details are given in Simons and Senturk (1977).

The safety factor for rock riprap on a channel side slope is

$$SF = \frac{\cos\theta\tan\phi}{\eta'\tan\phi + \sin\theta\cos\beta} \tag{12}$$

where SF is the safety factor, θ is the side slope angle and ϕ is the angle of repose and β and η' are given below

$$\beta = \tan^{-1}\left(\frac{\eta\tan\theta}{\sin\theta}\right) \tag{13}$$

and

$$\eta' = \eta\left(\frac{1 + \sin\beta}{2}\right) \tag{14}$$

The parameter of η is given by

$$\eta = \frac{21\tau_s}{(\gamma_s - \gamma)d_{50}} \tag{15}$$

Where γ_s is the specific weight of the rock, γ is the specific weight of water, d_{50} is the median diameter of the riprap and τ_s is given below.

$$\tau_s = r\,\tau_o \tag{16}$$

where

$$\tau_o = \gamma y_o S \tag{17}$$

where y_o is the normal flow depth and S is the friction slope. The ratio r is obtained from Figure 1. The angle of repose, θ may be obtained from Figure 2.

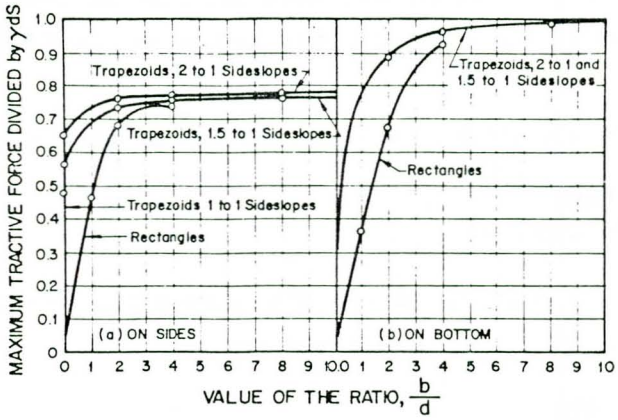

Figure 1. Variation of τ in a traditional cross section.

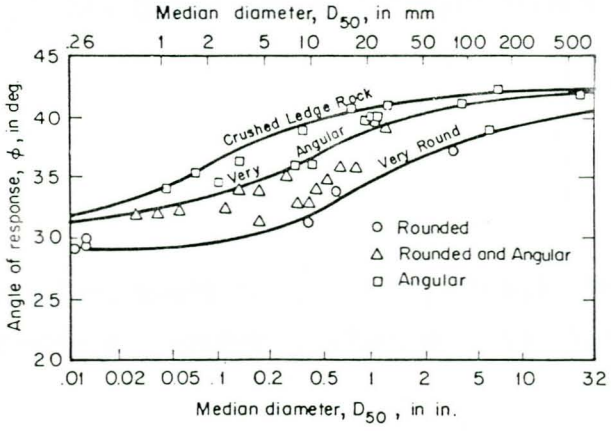

Figure 2. Angle or repose for dumped riprap (after Simons, 1957).

Program RIPRAP Users Instructions.

Step	Instructions	Input	Function	Display
1	Load program			
2	Begin execution		[XEQ] RIPRAP	SIZE D50 ?
3	Enter mean riprap diameter (ft)	D_{50}	[R/S]	∡ REPOSE ?
4	Enter riprap angle of repose (deg)	ϕ	[R/S]	SIDE SLOPE ?
5	Enter channel side slope (H:V)	S.S.	[R/S]	BND SHEAR ?
6	Enter boundary shear stress (lb/ft^2)	τ_o	[R/S]	RATIO ?
7	Enter ratio of side slope shear stress to boundary shear stress	r	[R/S]	
	Program will calculate and display factor of safety			

Example: Riprap Design

This example illustrates the use of Program RIPRAP. Consider the following data:

Side Slope = 2.5

D_{50} = 2.0 ft

τ_o = 4.66 lb/ft^2

From Figure 1:

r = 0.77

From Figure 2:

θ = 41° (very angular rock)

Using Program RIPRAP:

```
          XEQ "RIPRAP"
SIZE D50 ?
          2.0000    RUN
∠ REPOSE ?
         41.0000    RUN
SIDE SLOPE ?
          2.5000    RUN
BND SHEAR ?
          4.6600    RUN
RATIO ?
           .7700    RUN

F.S.=1.4334
```

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