Spurs and Guide Banks

E. V. Richardson and D. B. Simons

1.0.0 Introduction

To prevent rivers from eroding their banks and to protect structures constructed on the flood plain (pipelines, highways, buildings, levees, etc.) spurs are often used in place of riprapping the banks. Spurs are embankments projected into the stream from the bank at some angle and for a certain distance. Spurs, by deflecting the current from the bank and causing deposition behind them, may protect the stream bank more effectively and at less cost than riprapping the bank. Also, by moving the location of any scour away from the bank, failure of the riprap on the spur can be repaired before damage is done to structures along the river, whereas failure of riprap on the bank may endanger these structures.

Spurs are also used to protect highway embankments that form the approaches to a bridge crossing. Very often these highway embankments cut off the flood flows across a flood plain causing the flow to run parallel to the embankment to reach the bridge opening. Spurs constructed normal to the highway embankment keep the current away from the highway embankment. The protection provided to the embankment is similar to the protection offered to the banks of a stream.

Spurs are used to narrow or channelize a wide braided river to establish a well defined channel that neither aggrades, degrades nor shifts its location from year to year. In this case, the spurs may be used with long dikes at their outer, stream side to help define the channel. In general, for this problem many miles of a river are controlled by spurs and dikes. An example of this is the work of the Corps of Engineers on the improvement of the channel of the lower Missouri River.

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Finally, spurs are used in short reaches of a river to form a well defined channel in a stream. These spurs may be placed to control the flow into and out of a bend or through a crossing. Their use here is often to increase depth for navigation, to direct the flow to a bridge opening or, as described in the first paragraph, to protect a bank.

Guide banks are a type of spur that is used at waterway openings to straighten the flow through the opening, to increase the discharge through the opening, and to move the location of deep scour away from the abutments of the crossing. Abutments may be either concrete foundations of a bridge or the nose of the embankment that terminates at the waterway opening.

In this report we will restrict ourselves to the discussion of spurs and guide banks. We will define them both more precisely and will give some of the other names that these structures are called. We will review in more detail the functions of spurs and guide banks, and give design considerations for these structures. The literature has many design recommendations for spurs and guide banks which we will present in an abbreviated form. Conflicting statements on the design and use of spurs and guide banks are given in the literature. These very often occur because the designer has designed for a particular condition and from this condition tries to generalize. We will make recommendations for spur and guide bank design based on the literature, our knowledge of fluid and river mechanics, and experience in working with natural streams.

In general, the use of spurs and guide banks has been on sand bed streams with relatively large sediment concentrations. They have not been used as extensively in gravel streams and for streams with aufeis, although they have been used on many streams with ice cover. (aufeis is the accumulation of bottom fast ice in a river channel). For these reasons the recommended design will be conservative and will incorporate the suggestions of Dick Cooper and Vic Galay who have had extensive experience with gravel bed streams. For very important problems in the control of the river, it may be necessary to conduct a model study. In general, the U. S. Army Corps of Engineers with all its experience in river control, relies extensively on model studies for their design.

2.0.0 Definitions

2.1.0 Spurs: Spurs are structures built at an angle to a river bank to deflect flowing water away from critical zones, to prevent erosion of the bank, to establish a more desirable channel for flood control, navigation and erosion control or in general, to establish a predetermined flow alignment.

Some other terms for spurs: spur dikes, dikes - jetties, and groins (groynes).

Adjectives are used with the term spurs to further define them. These adjectives are the material used to construct the spurs (stone spurs, pile spurs, etc.); to describe whether the spurs are impervious or not (pervious spurs, impervious spurs); to describe their shape (round head spurs, L-head spurs, T-head spurs); to describe the elevation of their crown one to another (step up spurs, step down spurs); to describe the angle they make with the bank (attracting spurs, repelling spurs).

2.2.0 Guide banks: Guide banks are embankments built upstream from one or both abutments of a bridge to guide the approaching flow through the waterway opening. Guide banks shift scour holes away from the abutments, increase the volume of flow through the opening, help maintain a constant angle of attack of the flow on piers and abutments, decrease the width of the opening and in general, by establishing smooth parallel stream lines, improve flow conditions in the waterway opening.

3.0.0 Spurs

3.1.0 Design Considerations

The physical quantities to be considered in the design of spurs are: form, angle θ to the bank, length L of spur, spacing S between spurs, materials, spur crest elevation, cross section, and scour. In this section, the design considerations obtained from a literature review will be summarized followed by design recommendations. The numbers in parentheses refer to the bibliography at the end of this report.

3.1.1 Form of Spurs: The form of the spurs are illustrated in Fig. 3.1.1. The straight spur (1, 18, 29, and 31) is set at some angle θ from the bank and normally has a round nose to provide more volume and area for scour protection at the outer end.

The T-head spur (1, 6, 15, 18 and 31) is a straight spur with a rectangular guide vane set at the outer ends. The angle θ at the bank is normally 90°. The angle α varies and is set by the degree of deflection of the current that is desired. However, α that would set the T-head at angles to the flow larger than 10° are not recommended. The length (a) varies, with no particular length recommended in the literature reviewed.

L-head and wing or trail spurs (14, 15, 18, 31 and 40) provide more protection to the bank. The length (a) should close 45-65% of the gap between spurs (14, 40). As with T-heads the angle α should be set so that the L-head has an angle 10° or less to the stream lines of the flow. L-heads are supposed to provide more deposition between spurs and decrease scour around their ends and provide greater protection to the banks. This is rather obvious when their recommended length closes 45 to 65% of the opening between spurs. This small an opening increases their costs but also makes them more effective in channelization for navigation improvement The straight spur is more effective for the amount of money spent.

Hockey shape and inverted hockey shape (14, 18) (also called J shape and inverted L) do not appear to have any advantages over straight or T shape. In fact, their scour holes are more extensive in area than the T-shape (14).

3.1.2 Angle of Spur to the Bank : The angle θ of the spur to the bank (internal angle from downstream bank to spur, Fig. 3.1.2) given in the literature ranges from 30° to 120° (6, 12, 28, 29, 31 and 38). Spurs with angles larger than 90° are called repelling spurs and less than 90° are called attracting spurs (6, 18). Reference 6 cites an example of a downstream pointing spur which caused bank failure. Mamak (28) states the best results (deflecting flow and trapping load) are obtained with spurs inclined upstream (θ from 100 to 110°). Neil (29) makes the same conclusion. The angle for T-head spurs is normally 90° with the deflection of the current obtained by the angle α of the head to the shank. The study by Franco (16) where angles 60°, 90°, and 120° were studied showed that for channelization to improve navigation, the normal or angled downstream spurs performed best. But the downstream angled spur "produced a greater tendency for scouring at their bank end than dikes (spurs) angled upstream."

3.1.3 Length of Spur: The length of a spur depends on its location (straight reach, concave bank of bend, along embankments, etc.), amount of contraction of stream width, and purpose of the spur. The length is also closely related to the spacing because spacing is expressed as some multiplier times the projected length. If the spur is too short then the spacing is close and construction is expensive. If their length is too long they may contract the flow too much or the spacing may be so large that a meander loop may form between two spurs. The length of spurs given in the literature reviewed, ranged from 60 to hundreds of feet (12, 29, 30, 38) with no rules given. The length depends on the spacing, the desired end result, and the economics of construction.

On the Missouri River spurs were used to change a braided shallow stream to a single channel consisting of gentle curves. A fixed width and depth of channel was desired and spurs long enough to establish this width and depth were built. In some cases, spurs 1,000 ft or more long were built. For other streams where only a small shift in channel is required or a bank is to be protected, short spurs (50 ft or less) are built. Neil (states "the length of bank protected by each spur appears to be at least twice its <u>projected</u> length perpendicular to the current ..." Neil states "whether to choose fewer long spurs or a greater number of short ones depends upon their disturbing effects upon the opposite bank and the channel upstream and downstream. For earth work (spurs) the longest spur that will not produce excessive erosion and disturbance should be used, since the major cost of this type is in the slope revetment ... In lieu of a series of short spurs, consideration should be given to .... (riprapping the bank)."

The length of spur depends on the desired result. For bank protection, short lengths (L 50 ft) probably won't be as economical as lengths 100 ft or longer. The maximum permissible length can be established by determining the optimum channel width for the bankful discharge. Optimum channel width is determined by scour, sediment transport, minimum flow disturbance and maximum allowable velocity. For channel control, location, size and shape, the length depends on the desired flow and channel considerations. Spur length varies with location, but usually extends from the river bank to the desired control width of the stream (40).

3.1.4 <u>Spacing between Spurs</u>: Spacing S between spurs is primarily related to the length of the spur, although the velocity of flow, angle of flow streamlines with the spurs, curvature of the bank, and purpose of the spur also affect spur spacing. Spacing distance between spurs in feet or as a function of spur length are given in references 6, 14, 16, 17, 18, 19, 22, 28, 29, 31, 32, 38, 39 and 40. Actual distances range from 200 ft to 4,000 ft. In general, the recommended spacing S is from 1 1/2 to 6 L_i where L_i is the upstream projected spur length into the flow, Fig. 3.1.2. The spacing is, in general, a function of the length of the next upstream spur. Spacing distance S_i equal to 1 1/2 to 2 L is recommended to obtain a well defined deeper channel for navigation and flood control. With these spacings, dredging to keep a deep channel is decreased or eliminated. For bank protection longer spacings are used (S_i = 2 to 6 L_i). With T-head spurs, the recommended S_i is from 3 to 4 L_i for navigation channels.

Reference 38, which is a 1918 report on practice, gives S as a function of channel width. In this case S ranges from .75 to 1 W.

To base spacing on length of spur is logical because flume and wind tunnel studies have shown that the separation zone downstream of a vertical barrier in the flow ranges from 7 to 11 times the barrier height.

3.1.5 Height or Elevation of Spur: Height recommendations depends on the purpose of spur, the amount of contraction of the flow, the magnitude and importance of the overbank flow, and the possible ice problems (19, 22, 38, 39 and 40). Related to the elevation of the spur is whether they are level, slope up from a low point on the streamward end to the bank, or are set at the same elevation or stepped up or stepped down in elevation going from the upstream spur to the downstream spur. In stepped down fields the spurs decrease in elevation in the downstream direction. The sloping crest spur gives a gradually increasing flow area with stage. This type of crest reduces high velocities for the higher stages, helps force the flow into its low water alignment more effectively, and does not hold the flow concentrated at one location over a large change in stage. For these reasons, a sloping crest is often preferred (14).

Laboratory studies by the Corps of Engineers (16) indicate that for navigation purposes a spur system with crest level but successive spurs stepped down was best. However, it states that sloping crest spurs can be designed to be as effective as level crested spurs. For navigation channel control, level-crested spurs should be placed normal to the flow or angled downstream, whereas, sloping-crested spurs should be normal or angled upstream.

The elevation of the crest on the lower Mississippi is from 4 to 15 ft above low water elevation (38, 39, 40). On the Columbia River elevations are from 1 ft below bank level to 1/2 flood stage elevation (22). These elevations appear adequate to maintain a navigable channel in a meandering river system. When spurs are set at elevations where they are overtopped frequently the top and downstream slope of the shank has to be riprapped, increasing their cost.

The L-headed spur may be constructed with the head at a lower elevation than the stem. Fenwick (14) states that "it was found...that little benefit was derived from building the L-head above the water surface." This makes them a little cheaper. He also states "L-heads are expensive so that additional testing and experience are needed to show whether their merits are sufficient to recommend their general use in connection with channel contraction."

On braided channels or where side channels are cut off using spurs, their elevations are set at bankful stage. Also, when spurs are used for bank protection their elevation is bankful stage or slightly higher in order to prevent the flow from scouring the bank. In these cases the crest may be sloping to increase flow area, particularly at the large discharges.

With aufeis the elevation of the spurs should be higher than the expected elevation of the ice. Otherwise the ice can build up and cause the stream to flow over the top of the spur. The spurs would no longer confine the flow to the channel and during the spring break up the water could cut a new channel through the ice. If the spurs were lower than the aufeis elevation the new channel could be on the flood plain behind the spurs or on top of them. In the first case the spurs would no longer be effective in maintaining a channel and in the second the spurs would need riprap on the crown and both upstream and downstream sides. With aufeis the spur crest could be level or sloping depending on aufeis elevation and bank height. If the outer or streamward end was above the aufeis then the crest could be level or sloping to the bank.

3.1.6 Construction Materials:

Materials used to construct spurs may be rock (13, 21, 28, 32, 38, 39, 40), timber piles (12, 13, 15, 22, 32, 37, 38, 39, 40), trees (13), sand bags (13), automobile bodies (13), Brownlow weeds (15), brush (21, 28, 29), etc. They may be pervious or impervious (5, 13, 15, 22, 30, 37, 39, 40). Permeability of spurs is a relative term in that impervious spurs, because of cost, are not made water tight.

A study on the Apalachicola River (11) indicated that stone spurs were more effective than pile spurs in river control for navigation. Typical details of the stone and pile dikes (spurs) are given in Figs. 3.1.3 and 3.1.4.

The size of riprap for spurs can be determined by estimating the velocity of flow along, across and around the end of the spur. The appropriate size of stone to resist this velocity may be estimated from Fig. 3.1.5. A more detailed approach would be to use the method developed by Stevens and Simons (36). A short summary of their work is given here, the details and derivations of which can be found in the original paper.

Consider flow along an embankment, as shown in Fig. 3.1.6. The forces on the rock particle are lift force, F_ℓ , drag force F_d , and weight of the particle W_s . Rock particles on side slopes will tend to roll, rather than slide, so it is appropriate to consider stability of rock particles in terms of moments about a contact point 0 about which rotation must take place. The components of forces relative to the plane of motion are shown on (b).

At incipient motion, there will be a balance of moments such that

$$e_2 W_s \cos\theta = e_1 W_s \sin\theta \cos\beta + e_3 F_d \cos\delta + e_4 F_\ell$$

The factor of safety, S.F., of particles against rotation is then determined by the ratio of the moments.

$$\text{S.F.} = \frac{e_2 W_s \cos\theta}{e_1 W_s \sin\theta \cos\beta + e_3 F_d \cos\delta + e_4 F_\ell} \qquad 3.1.1$$

or

$$\text{S.F.} = \frac{\cos\theta \tan\phi}{\eta' \tan\phi + \sin\theta\cos\beta}$$

where

$$\eta' = M + N\cos\delta$$

$$M = e_4 F_\ell / e_2 W_s$$

$$N = e_3 F_d / e_2 W_s$$

η' is called a stability number and ϕ is the angle of repose of the material. If $\delta = 0$ (no angle between the resultant force and the drag vector), we can define η as

$$\eta = M + N$$

η is also a stability number which can also be written in terms of hydraulic variables as

$$\eta = \frac{21\,\tau_s}{(S_s - 1)\,\gamma\,D_{50}} \qquad 3.1.2$$

in which τ_s is the shear stress on the particles with size D_{50}. The unit weight of water is γ and S_s is the specific gravity of the riprap. η' and η are related by

$$\frac{\eta'}{\eta} = \frac{\frac{M}{N} + \cos \delta}{\frac{M}{N} + 1}$$

It is reasonable to assume, in considering incipient motion of riprap particles, that

$$\frac{M}{N} = \frac{e_4 \; F_\ell}{e_3 \; F_d} \approx 1$$

Thus,

$$\frac{\eta'}{\eta} = \frac{1 + \cos \delta}{2} \approx \frac{1 + \sin (\lambda + \beta)}{2} \qquad 3.1.3$$

It can be shown that

$$\tan \;\; = \frac{\cos \lambda}{\frac{2 \sin \theta}{\eta \tan \phi} + \sin \lambda} \qquad 3.1.4$$

where λ is the angle between the horizontal and the velocity vector in the plane of the side slope (see Fig. 3.1.6). By knowing or assuming values of θ , ϕ , λ and S.F., equations 3.1.1, 3.1.3, and 3.1.4 can be solved simultaneously for β , η , and η'. Once the value of η is known, equation 3.1.2 can be solved for D_{50}. A trial and error procedure for solving equations 3.1.1, 3.1.2, 3.1.3 and 3.1.4 is given in the example problem in Appendix B. In many circumstances the flow angularity with the horizontal is small, (i.e. $\lambda \approx 0$), and Eq. 3.1.4 reduces to

$$\tan \beta = \frac{\eta \tan \phi}{2 \sin \theta} \qquad 3.1.5$$

and Eq. 3.1.1 solved for η' becomes

$$\eta' = (\frac{S_m^2 - (S.F.)}{(S.F.)S_m^2} \cos \theta \qquad 3.1.6$$

in which $S_m = \frac{\tan \phi}{\tan \theta}$ is the safety factor of rock particles from rolling down the slope with no flow. Equation 3.1.3 becomes

$$\frac{n'}{n} = \frac{1 + \sin \beta}{2} \qquad 3.1.7$$

Riprap gradation should follow a smooth size distribution curve such as that shown in Fig. 3.1.7. The ratio of maximum size to median size, D_{50} , should be about 2.0 and the ratio between median size and the 20 percent size should also be about 2.0. This means that the largest stones would be about 6.5 times the weight of the median size and small sizes would range down to gravels.

With a distributed size range, the interstices formed by the larger stones are filled with the smaller sizes in an interlocking fashion, prefenting formation of open pockets. Riprap consisting of angular stones are more suitable than rounded stones. Control of the gradation of the riprap is almost always made by visual inspection.

Riprap must be placed on the spur at its outer end to protect it from the high velocity flow and vortex around it. This riprap must be carried around the spur nose in both the upstream and downstream direction until the predicted velocities on these side slopes are less than critical for the base material forming the spur. If the spur is going to be overtopped frequently then the top and downstream slope of shank of the spur must be riprapped.

The determination of the velocity field next to the spur (nose and upstream and downstream face) is extremely difficult. Flow nets can be used and reference (34) gives a method of estimating the velocity.

Riprap should be hard, dense and durable to withstand long exposure to weathering. Visual inspection is most often adequate, to judge quality but laboratory tests may be made to aid the judgment of the field inspector.

Riprap placement is usually by dumping directly from trucks. If riprap is placed during construction of the embankment, rocks can be dumped directly from trucks from the top of the embankment. Rock should never be placed by dropping down the slope in a chute or pushed downhill with a bulldozer. These methods result in segregation of sizes. With dumped riprap there is a minimum of expensive hand work. Poorly graded riprap with slab-like rocks require more work to form a compact protective blanket without large holes or pockets. Draglines with orange peel buckets, back-hoes and other power equipment can also be used advantageously to place the riprap.

Hand placed rock riprap is another method of riprap placement. Stones are layed out in more or less definite patterns, usually resulting in a relatively smooth top surface. This form of placement is used rarely in modern practice because it is usually more expensive than placement with power machinery.

The thickness of riprap should be sufficient to accommodate the largest stones in the riprap. With a well-graded riprap with no voids, a thickness the size of the largest stone should be adequate. If strong wave action is of concern, the thickness should be increased by 50 percent.

Filters should be placed under the stone unless the material forming the core of the structure is coarse gravel or of such a mixture that it forms a natural filter. Two types of filters are commonly used; gravel filters and plastic filter cloths.

With gravel filters, a layer or blanket of well-graded gravel should be placed over the embankment prior to riprap placement. Sizes of gravel in the filter blanket should be from 3/16 in. to an upper limit depending on the gradation of the riprap with maximum sizes of about 3 to 3-1/2 in. Thickness of the filter may vary depending upon the riprap thickness but should not be less than 6 to 9 inches. Filters that are one-half the thickness of the riprap are quite satisfactory. Suggested specifications for gradation are as follows:

$$(1)\quad \frac{D_{50}\ (\text{Filter})}{D_{50}\ (\text{Base})} < 40$$

$$(2)\quad 5 < \frac{D_{15}\ (\text{Filter})}{D_{15}\ (\text{Base})} < 40$$

$$(3)\quad \frac{D_{15}\ (\text{Filter})}{D_{85}\ (\text{Base})} < 5$$

Plastic filter cloths are being used beneath riprap and other revet[illegible] materials such as articulated concrete blocks with considerable success. The cloths are generally in 100 ft long rolls, 12 to 18 ft wide. The edg[illegible] of the plastic sheets are hand sewn in the field with nylon twine. Overla[illegible] of 8 to 12 inches is provided with pins at 2 to 3 ft intervals along the seam to prevent separation in case of settlement of the base material. So[illegible] amount of care must be exercised in placing riprap over the plastic cloth filters to prevent damage. Experiments and results with various cloth filters were reported by Calhoun, Compton and Strohm (4) in which specific manufacturers and brand names are listed. Stones weighing as much as [illegible],0[illegible] lbs have been placed on plastic filter cloths with no apparent damage. Filters can be placed subaqueously by using steel rods as weights fastened along the edges. Additional intermediate weights would assist in sinking the cloth in place. Durability of filter cloths has not yet been establi[illegible] because they have been in use only since about 1967. However, inspections at various installations seem to indicate little or no deterioration had occurred in the few (1 to 4) years that have elapsed for test installati[illegible].

In general, locally available material are used to construct spurs. [illegible] is available then spurs are constructed of it. If large size material is not available for riprap, then gabiens and wire baskets are used to protect against scour.

3.1.7 Cross-section (crest width and slopes):

Typical cross-sections of pile and stone spurs are given in Figs. 3.1.3 and 3.1.4 from Ref. 11. References 6, 11, 13, 15, 16, 17, 21, 22, 28, 37, 38, 39, and 40 give data on cross-section. Crest widths range from 3 - 20 ft and side slopes from 1:1.25 to 1:5. The top width of stone spurs is often controlled by the equipment placing them with a three foot width as a minimum and larger widths being used to facilitate hauling and placing. Winkley (40) states that on the lower Mississippi River, crown width for stone placed by trucks is from 14 - 20 ft and for stone placed by barges a minimum 5 foot crown is used. Side slopes are slightly less than the angle of repose of the material.

3.1.8 Scour:

Scour at contractions occur because the flow area becomes smaller than the normal stream, the average velocity and bed shear stress increase, hence there is increase in stream power locally at the contraction and more bed material is transported through the contracted section than is transported into the section. As the bed level is lowered, the velocity decreases, shear stress decreases and equilibrium is restored when the transport rate of sediment through the contracted section is equal to the incoming rate.

Local scour occurs in the bed of the channel around embankments due to the actions of vortex systems induced by the obstructions to the flow. Local scour occurs in conjunction with or in the absence of general degradation, aggradation, and scour due to contractions. There is need to understand the mechanism of local scour and calculation of potential scour depths.

The basic mechanism causing local scour is the vortex of fluid resulting from the pileup of water on the upstream edge and subsequent acceleration of flow around the nose of the spur. The action of the vortex is to erode bed materials away from the base region. If the transport rate of sediment away from the local region is greater than the transport rate into the region, a scour hole develops. As the depth is increased, the strength of the vortex reduces, the transport rate reduces and equilibrium is reestablished and scouring ceases.

The flow field and vortex systems around a spur or embankment are illustrated in Fig. 3.1.8. Although the vortex system is known to be the cause of local scour, it has not been possible as yet to calculate the strength of the vortex and relate the velocity field with subsequent scour. Until further research and study makes this possible, average velocity and local depth of flow will be used in the equations to predict local scour depths.

The depth of scour varies with time because the sediment transported into the scour hole from upstream varies, depending upon the presence or absence of dunes. The time required for dune motion is much larger than the time for local scouring action. Thus, even with steady state conditions the depth of scour is likely to fluctuate with time when there are dunes traveling on the channel bed. The larger the dunes, the more variable will be the depth of the scour hole. When the crest of the dune reaches the local scour area, the scour hole will fill and the scour depth will temporarily decrease. When a dune trough approaches, there will be less sediment supply and the scour depth will increase to try to reestablish equilibrium in sediment transport rates. A mean scour depth between these oscillations is referred to as equilibrium scour depth. It is not uncommon (as determined in laboratory tests) to find maximum depths to be 30 percent greater than equilibrium scour depths. The depth that would be reached if no sediment was transported into the scour hole is the "clear-water" scour

Detailed studies of scour around embankments have been made mostly in laboratories. There are very few case studies for scour at field installations. According to the studies of Liu (27) the equilibrium scour depth for local scour is determined by

$$\frac{d_s}{d_1} = 1.1 \left(\frac{L}{d_1}\right)^{0.4} F_1^{0.33} \qquad 3.1.8$$

in which L is the spur length (measured normal to the wall of a flume), d_1 is upstream depth, d_s is depth of scour measured from mean streambed elevation, and F_1 is the upstream Froude number determined as

$$F_1 = \frac{V_1}{\sqrt{gd_1}} \qquad 3.1.9$$

The lateral extent of the scour hole is nearly always determinable from the depth of scour and the natural angle of repose of the bed material.

Field data for scour at embankments for various size rivers are scarce, but data collected at rock spurs on the Mississippi indicate that

$$\frac{d_s}{d_1} = 4\ F_1^{0.33} \qquad 3.1.10$$

determines an equilibrium scour depth. The data are scattered primarily because equilibrium depths were not measured. Dunes as large as 20 to 30 feet high move down the Mississippi and associated time for dune movement is very large in comparison to time required to form local scour holes. Nevertheless it is believed that these data represent the limit in scale for scour depths as compared to laboratory data and enables useful extrapolation of laboratory studies to field installations.

Accordingly, it is recommended that Eq. 3.1.8 be applied for spurs with $0 < L/d_1 < 25$ and Eq. 3.1.10 be used for $L/d_1 > 25$. In applying Eq. 3.1.8 the spur length L is measured from the high water line at the valley bank perpendicularly to the end of the spur. If $L/d_1 > 25$, then scour depth is independent of L/d_1 and depends only on the Froude number and depth of flow. For $L/d_1 < 25$, Eq. 3.1.8 would apply.

It should be recalled that maximum depth of scour is about 30 percent greater than equilibrium scour depth. The lateral extent of scour can be determined from the angle of repose of the material and scour depth.

If the spur is angled downstream the depth of scour will be reduced because of streamlining effect. Spurs that are angled upstream will have deeper scour holes. The calculated scour depth should be adjusted in accordance with the curve of Fig. 3.1.9 which is patterned after Ahmad (1).

Inglis (24) gives an equation that states scour depth D_s is 1.7 times Lacey's regime depth. Lacey's regime depth is given by

$$D_{Lacey} = 0.47\ (Q/f)^{1/3} \qquad 3.1.11$$

where

$$f = 1.76\sqrt{d_m}$$

and d_m is weighted mean diameter of bed material. This equation is similar to Bench's. Gill (12) gave maximum depth of scour as

$$D_s = Kq^{2/3} \qquad 3.1.12$$

where K varies from 1.2 to 1.5 and D_s is measured from the water surface and q is flow intensity in cfs/ft after contraction. Winkley (40) states "Attempts have been made to predict by analytical means the extent and depth of the scour hole caused by a dike (spur), but there have not yet been a sufficient number of correlations to enable design to be based on such forecast with confidence. This hole seems to scour to the optimum depth of the river." These scour equations (3.1.8 - 3.1.12) were derived for sand bedstreams and do not take paving of the scour hole by large particles into account.

In wide braided rivers with many channels the scour depth calculated from the above equations may not be the maximum. The maximum depth of scour at a spur may occur when one of the channels in the braided river impenges on the spur. This may occur at flows much less than bankfull. For this case, depth of scour should be calculated by determining the depth of flow for the largest expected channel in the braided river. This depth is transposed to the tip of the spur. This depth of scour should be compared with the scour depth calculated from the previous equations and the largest scour depth used.

Method of control of scour is to place a stone blanket around the toe at the outer edge. This blanket must have sufficient rock to armor plate the scour hole after it forms. For scour that will occur if the shank is overtopped, excess stone is put at the downstream toe of the shank to armor plate any scour hole that forms.

It should be noted that most scour prediction equations are for sand bed streams. That with gravel bed streams there will be armoring of the scour hole by selective transport of the material forming the bed. Thus, the blanket to control scour on spurs in gravel bed streams need not be as extensive in thickness or area as for sand bed streams.

3.2.0 RECOMMENDATIONS ON SPUR DESIGN

3.2.1 Spur Form:

In general, the straight spur with a round nose should be used for most bank protection of straight streams and to protect embankments across streams. To protect concave banks at bends, short (30 - 50 ft) straight spurs are effective if the bank between is armored or resists erosion.

To channelize and guide the flow, T-head spurs with the head set at a small α to the flow is recommended. They should be less expensive if the head of the T is made relatively long as the spacing can be increased decreasing the number of shanks. L-head spurs may also be used.

3.2.2 Number of Spurs:

Spurs to protect stream banks or to contract the stream should number no less than three. For protection of embankments across the stream one or two spurs may be sufficient.

3.2.3 Spur Spacing:

The distance between spurs S to protect the banks of straight reaches long radius bends or braided channels from erosion may be from 3 to 4 times the upstream spur length. To obtain a well defined channel for navigation, the spacings should be 1-1/2 to 2 times the upstream spur length. However, the spacing should not be longer than 0.5 times the meander wave length of the stream.

If spurs are placed on the concave side of bends then spacing may be 4 to 6 spur lengths. Their use here is to move the high velocity flow away from the bank. The spurs must be short (20 - 30 ft) in order to be effective and not disrupt the flow around the bend. In addition the bank may need riprap.

Spurs placed on embankments across streams may be 6 to 10 times spur length or greater if the velocity along the embankments is low. If the velocity is large then the spacing should be from 4 to 6 L.

3.2.4 Spur Length:

For bank protection of straight reaches, long radius bends and braided channels the minimum length is 50 ft and the maximum length should be less than 10 to 15% of bankful channel width W. Maximum length can be larger ... 15% W but only after analysing the affect of this larger constriction on the flow and the channel. The 50 ft length appears the most economical minimum length. With spurs shorter than 50 ft, it is probably cheaper to riprap the bank. With more information on costs of spurs and cost of riprapping the bank a more realistic minimum width of spur can be determined.

The maximum length is not only limited by stream contraction and economics but also by spacing. If spur spacing is limited by the meander wave length then it is not economical for bank protection to establish spur lengths longer than 1/6 to 1/2 S .

For channelization and flow contraction, spur length is determined by the width of the desired channel.

For bank protection along embankments across the river spur length may be as short as 20 to 30 ft. But if there is a channel along the embankment, the spur should extend across this channel.

3.2.5 <u>Angle with Bank</u>:

In general, a spur at 90^{o} to the bank is recommended. This is the most economical length for bank protection.

When the purpose of the spur is to channel or guide the flow then angles of 100 to 110^{o} (spur angled upstream) may be more effective in deflecting the flow.

3.2.6 <u>Spur Elevation</u>:

The elevation of the crest of the spur should be 1 ft above the aufeis elevation or bankful elevation whichever is larger. The 1 ft should provide an adequate safety factor. Spurs that are higher than pipe line design flood stage may constrict the flow too much causing erosion problems. The crest may be level or slope up to the bank.

In a series of spurs, the elevation at the stream end may step down. The elevation of the first spur in a series being at design elevation and the outer ends of the next one in a series stepping down. If stepped down elevations are used the shank should be constructed so as to resist erosion by water flowing over it.

3.2.7 Cross-section:

Cross section and shape similar to Fig. 3.1.10 is recommended. Top width may be as narrow as 3 ft and sides slope should be 2:1.

3.2.8 Material:

Stone or local bed and soil material protected by stone may be used.

3.2.9 Scour:

The scour hole dimension should be estimated using Eq. 3.1.8 for $L/d_1 < 25$ and Eq. 3.1.9 for $L/d_1 > 25$. The regime equations should be used as a check. These equations are conservative because they do not consider the affect of large size material in the bed armoring the hole. Sufficient rock material should be placed in a blanket around the outer edge of the spur to assure armoring the scour hole.

If the shank is set lower than bankful so that overtopping will be frequent then its crest and downstream toe must be riprapped. If the shank is constructed of gravel it need not be riprapped to protect it from overtopping by pipeline design flood.

Riprap should be placed on the upstream side of the shank if it is made of erodible soil and it is anticipated that flow will occur along it.

Riprap should be designed using the Corps of Engineers present practice or the method of Stevens and Simons. However, quantities of riprap should be sufficient to allow for material removal by ice where pertinent.

4.0.0 Guide Banks

4.1.0 DESIGN CONSIDERATIONS

Guide banks have been used in many parts of the world to guide the flow of water through a bridge opening and to move the scour away from the abutments. They have been used on sand bed and gravel streams. References to guide banks used in this report are 2, 6, 20, 24, 25, 29, and 32. References 6, 25 and 29 give design procedure.

Principal factors in the design of guide banks are their convergence or parallelism to the opening, plan shape, upstream and downstream length, cross-section, crest elevation, scour and riprap. These are defined in Fig. 4.1.1 and discussed in the following sections.

4.1.1 Convergent or parallel:

American practice is to give the guide banks an elliptical form convergent to the opening whereas in Pakistan and India the banks are straight and parallel to the opening with a curved section at the upstream and downstream ends. The form of the elliptical guide bank is given in Fig. 4.1.2 and the design dimensions as determined by Karaki (25) are given in Fig. 4.1.3. The design layout for straight guide banks is given in Fig. 4.1.4 from Ref. 6. Mahmood (personal communication) stated that parallel guide banks straighten the flow more effectively than convergent ones. Straight guide banks probably do a better job of straightening the flow which could be important if piers are placed in the opening and of reducing the attack on the abutments. Elliptical guide banks move the scour hole further upstream and downstream of the bridge opening.

4.1.2 Plan Shape:

The plan shape of the guide banks depends on the type of channel (meander or braided), direction of streamlines of the flow approaching the opening and location of the crossing. Neil (29) fairly well summarized the plan shape for guide banks for bridge openings. These are reproduced in Fig. 4.1.5. In general, the designer should pick the shape that best fits the streamlines of the flow in the channel. If the streamlines are curving then a straight guide bank on concave side and curved on convex side may be best (Fig. 4.1.5 c, d). For short guide banks the ellipse of Karaki (25) can be used.

The curved portion of curved guide banks should be a quarter ellipse with major axis 2.5 times minor axis (25, 29, 32).

The radius of curvature for the curved portion at the upstream end of straight guide banks is given in Table 1 from Ref. 6.

4.1.3 Upstream and Downstream Length:

The upstream and downstream length for straight guide banks is as follows:

	Reference
GU = 3/4 to 1 W'	2
GD = 1/4 W'	6, 29
GU + GD $\leq$ 150'	32
GU + GD = W'	24
GU = 1 to 1.1 W'	6
GD = 1/10 to 1/5 W'	6
GU = 1.25 to 1.5 W'	6
GU = 3/4 W'	29

In general, the lengths are given as a function of W' , the opening width. This width would be established by determining desired opening for the design flow taking into account scour. In the opening width determination, local scour resulting from low flow meandering in too large an openi[illegible] must be considered.

For the elliptical guide bank, Fig. 4.1.3 can be used to design and select the length.

In determining the length it is not necessary that both guide banks [illegible] the upstream side be the same length. For some flow conditions a short curved guide bank on one side and a long straight bank on the other may be the best solution, see Fig. 4.1.5.

Table 1

Sand Classification	Probable maximum abnormal scour below bed level in ft.	Fall per mile of river in inches (These are average values, slopes may be much steeper locally)				
		3	6	9	12	18
		Radius of Upstream Curved End of Guide Banks				
1	2	3	4	5	6	7
Very Coarse . .	Under 20	200	250	300	350	400
	Over 20	250	310	375	440	500
Coarse . .	Under 30	300	360	425	490	550
	Over 30	350	430	510	590	670
Medium . .	Under 40	400	425	550	625	700
	Over 40	450	550	650	750	850
Fine . .	Under 50	500	590	675	760	850
	Over 50	600	725	825	925	1020
Very Fine . .	Under 60	600	700	800	900	1000
	Over 60	800	900	1000	1100	1200

Some considerations of the effect of selecting too long or too short a guide bank is given in Fig. 4.1.6 from Ref. 29.

4.1.4 Cross-Section:

The design considerations for the cross-section of the guide banks should be similar to that given for spurs. That is, at angle of repose of the material, see Fig. 4.1.7.

4.1.5 Crest Elevation:

The crest elevation should be 1 ft higher than the elevation of the pipeline design flood taking into consideration the affect of the contraction of the flow. The reason for this is that flow should not overtop the guide bank.

4.1.6 Scour:

For elliptical guide banks, the depth of scour is given in the design procedure given in Fig. 4.1.3. For the straight guide banks the design considerations are the same as for spurs.

4.1.7 Riprap:

Design considerations for riprap is the same as for spurs.

4.2.0 RECOMMENDATIONS

4.2.1 Guide Bank Form:

For the gravel streams the convergent guide bank should be satisfacto.. and more economical. The ellipse form with major axis 2.5 minor axis sho: be satisfactory. A heel of the same general shape should be placed on the downstream side.

4.2.2 Upstream and Downstream Length:

The upstream length GU should be 3/4 opening width W' and the downstream length GD should be 1/4 W'. However, the sum of GU and GD should be equal to or less than 150 ft.

4.2.3 Cross-Section:

The same design considerations may be used as for spurs. Top width may be equal to or greater than 3 ft. Side slopes of 2:1, or slightly flatter than the angle of repose of the material, may be used.

4.2.4 Crest Elevation:

The crest elevation may be one ft higher than the elevation of the pipeline design flood.

4.2.5 Scour:

The design scour for the elliptical guide banks can be determined using Fig. 4.1.3.

4.2.6 Riprap:

The riprap should be designed using the procedure for spurs.

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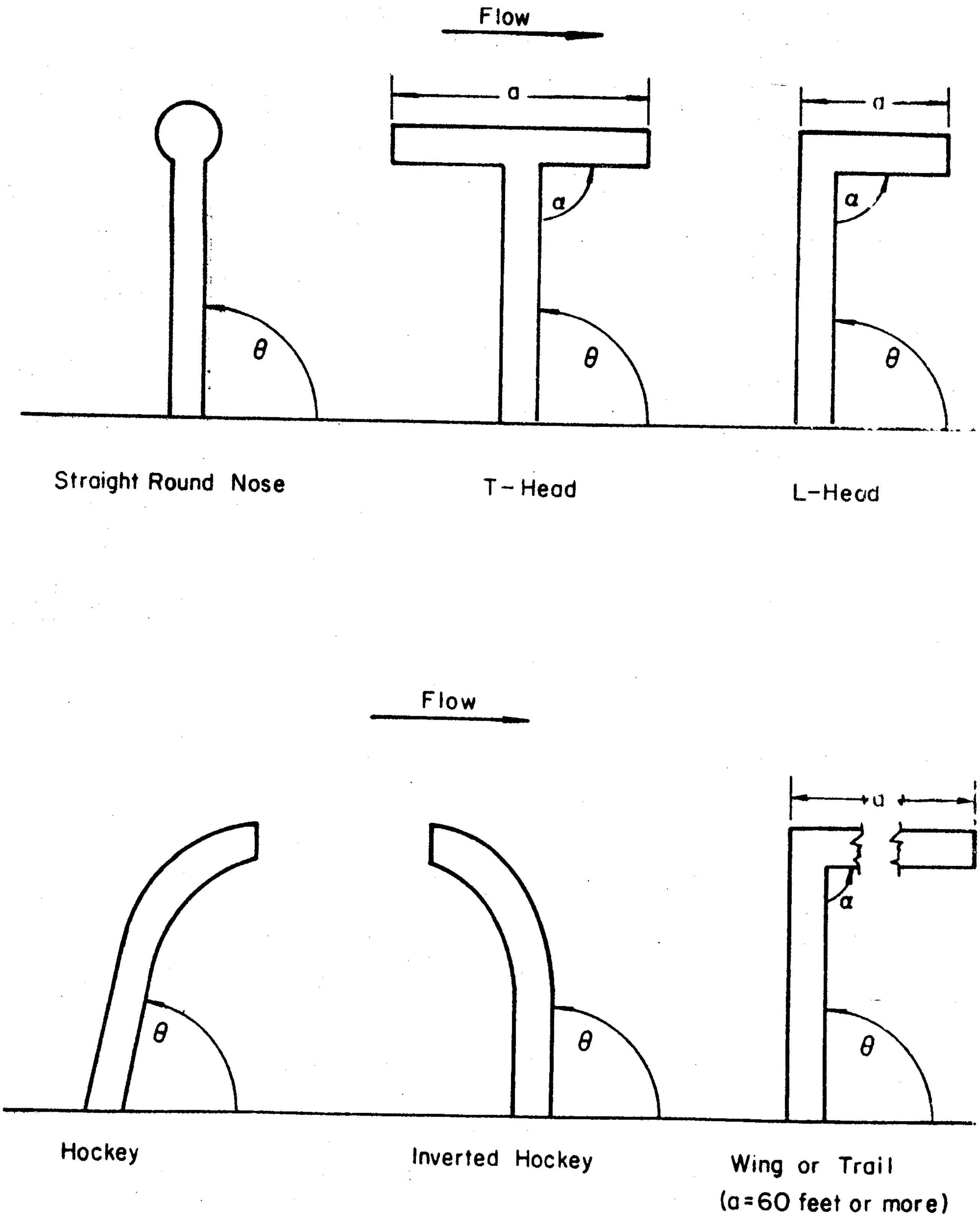
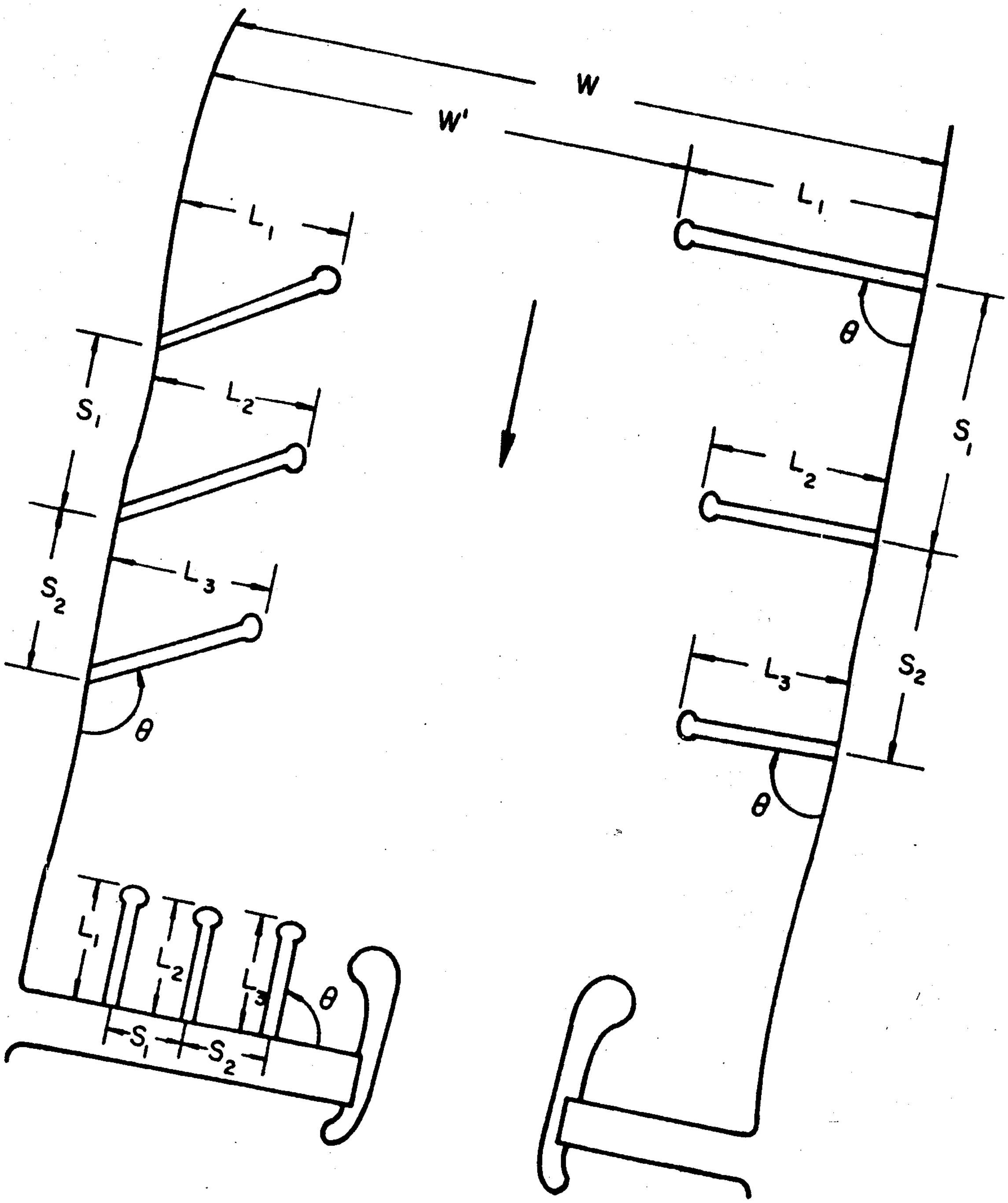


Fig. 3.1.1 Forms of spurs.



W = Stream Width
W' = Contracted Stream Width
S_i = Distance Between Spurs i and i+1
L_i = Projected Length of Spur i
θ = Angle of Spur from Downstream Bank

Fig. 3.1.2 Definition sketch for spurs.

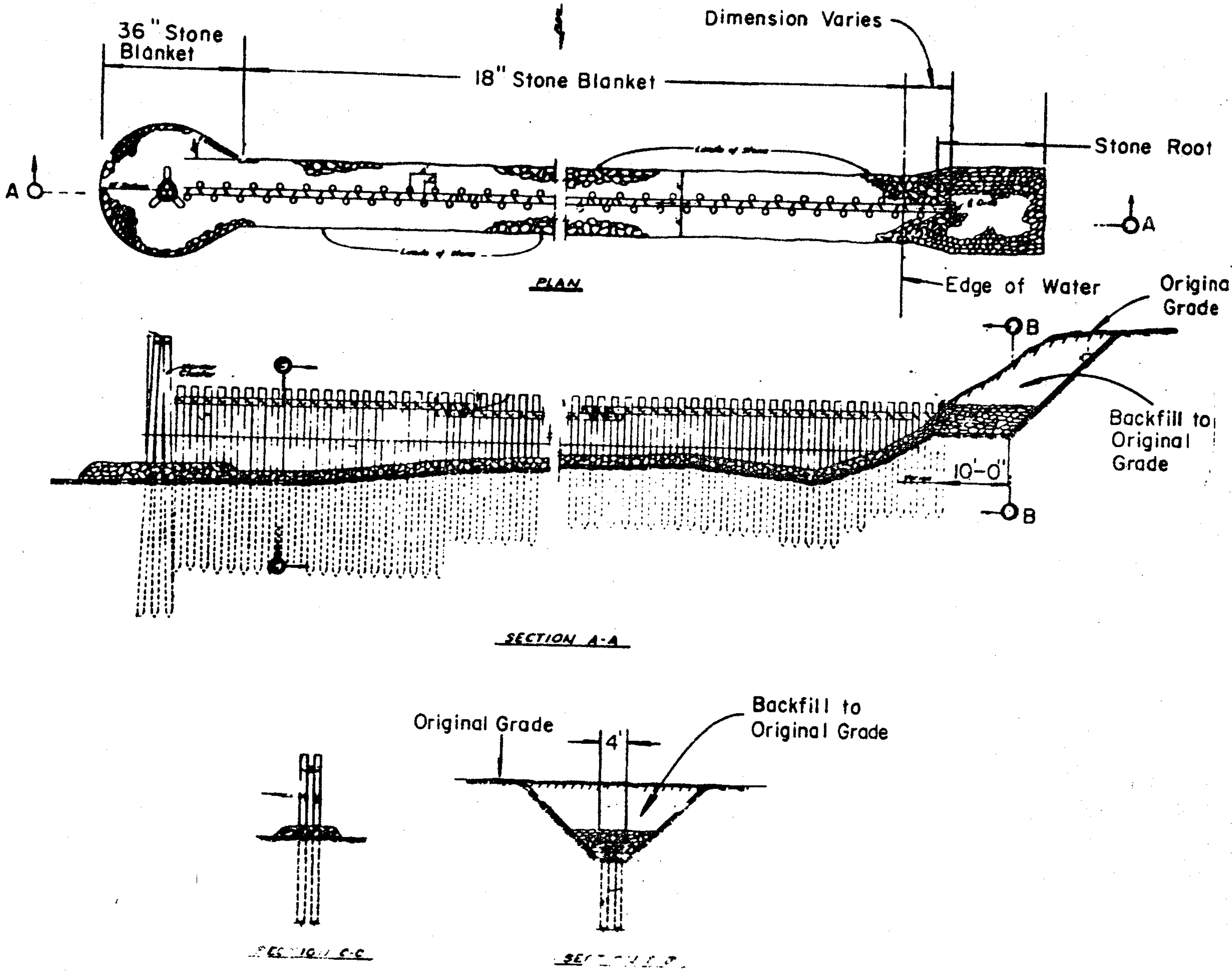


Fig. 3.1.3 Typical pile dike (from Ref. 11).

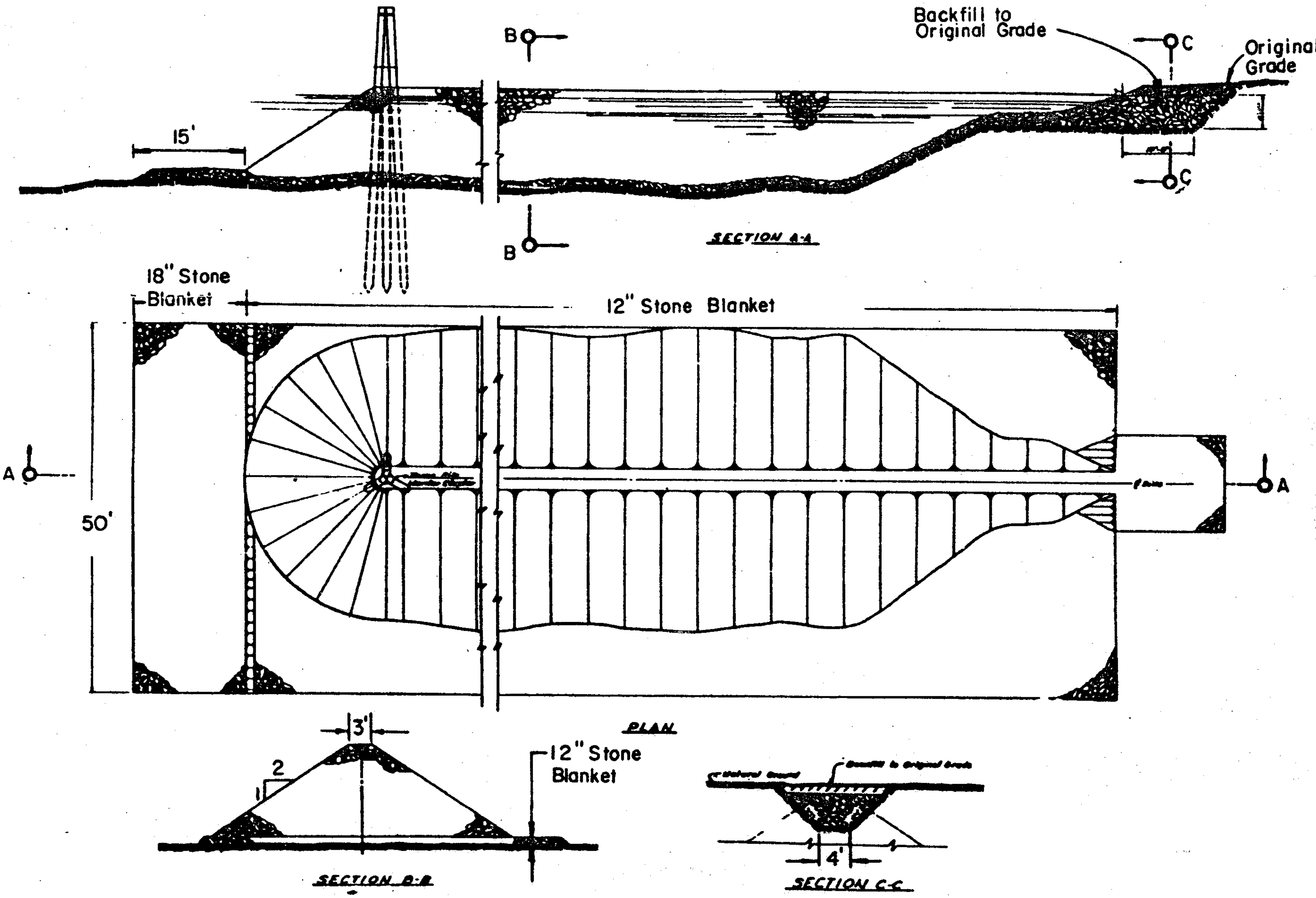


Fig. 3.1.4 Typical stone dike (from Ref. 11).

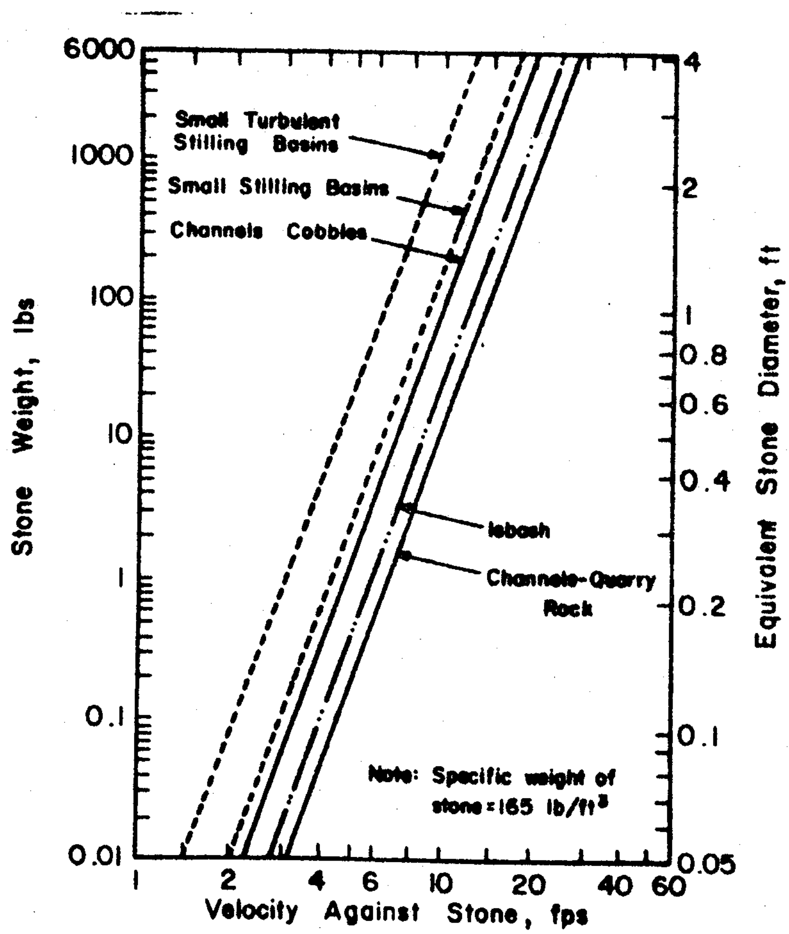


Fig. 3.1.5 Critical velocity as a function of stone size (from Ref. 32).

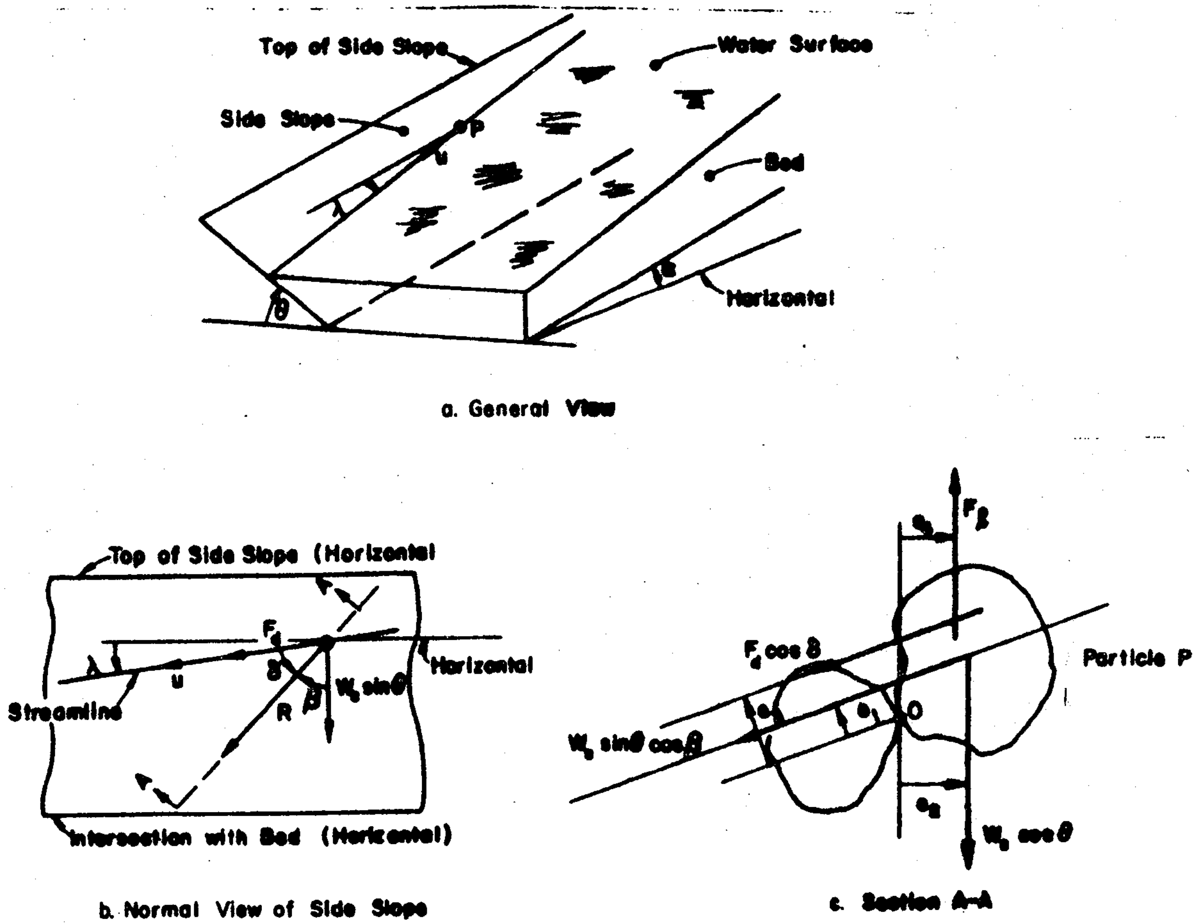


Fig. 3.1.6 Diagrams for riprap stability analysis (from Ref. 32).

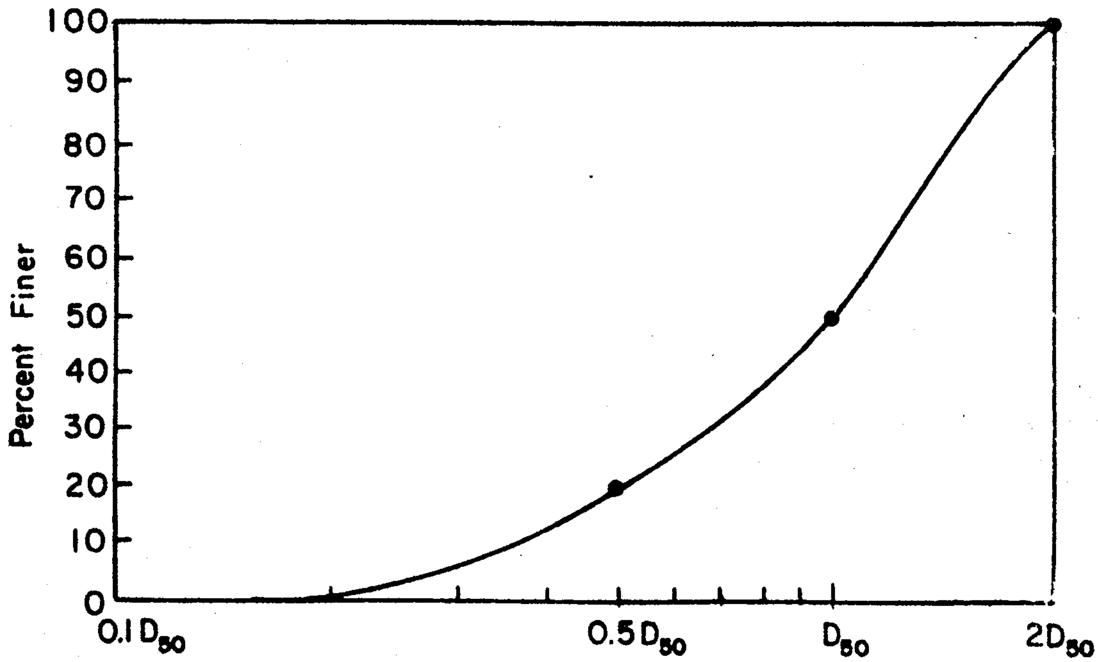


Fig. 3.1.7 Suggested gradation for riprap (from Ref. 32).

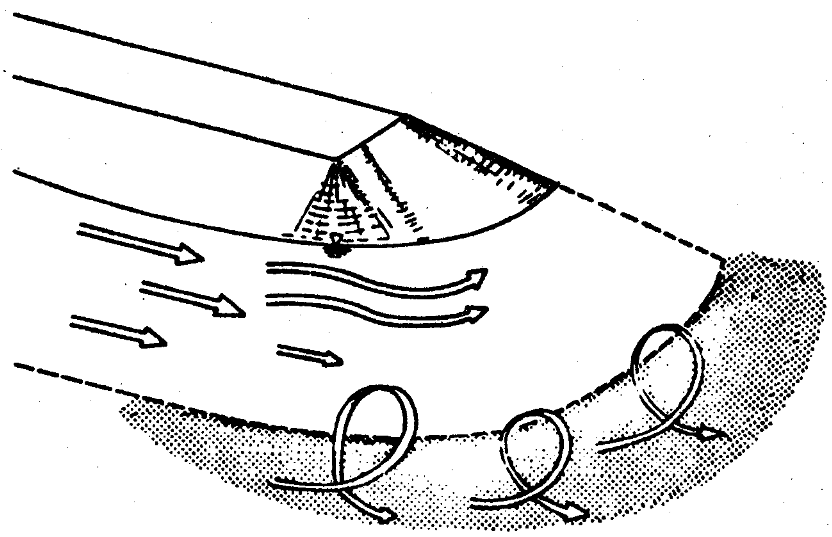


Fig. 3.1.8 Schematic representation of scour at an embankment (from Ref. 32).

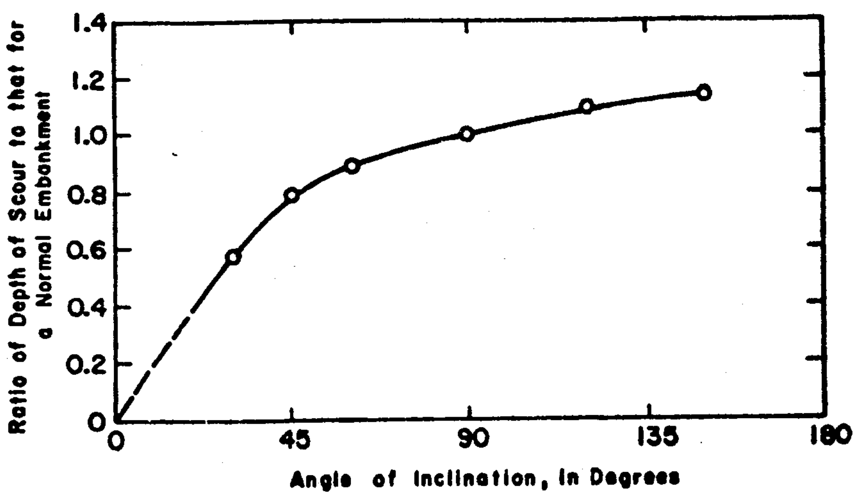


Fig. 3.1.9 Scour reduction due to embankment inclination (from Ref. 32).

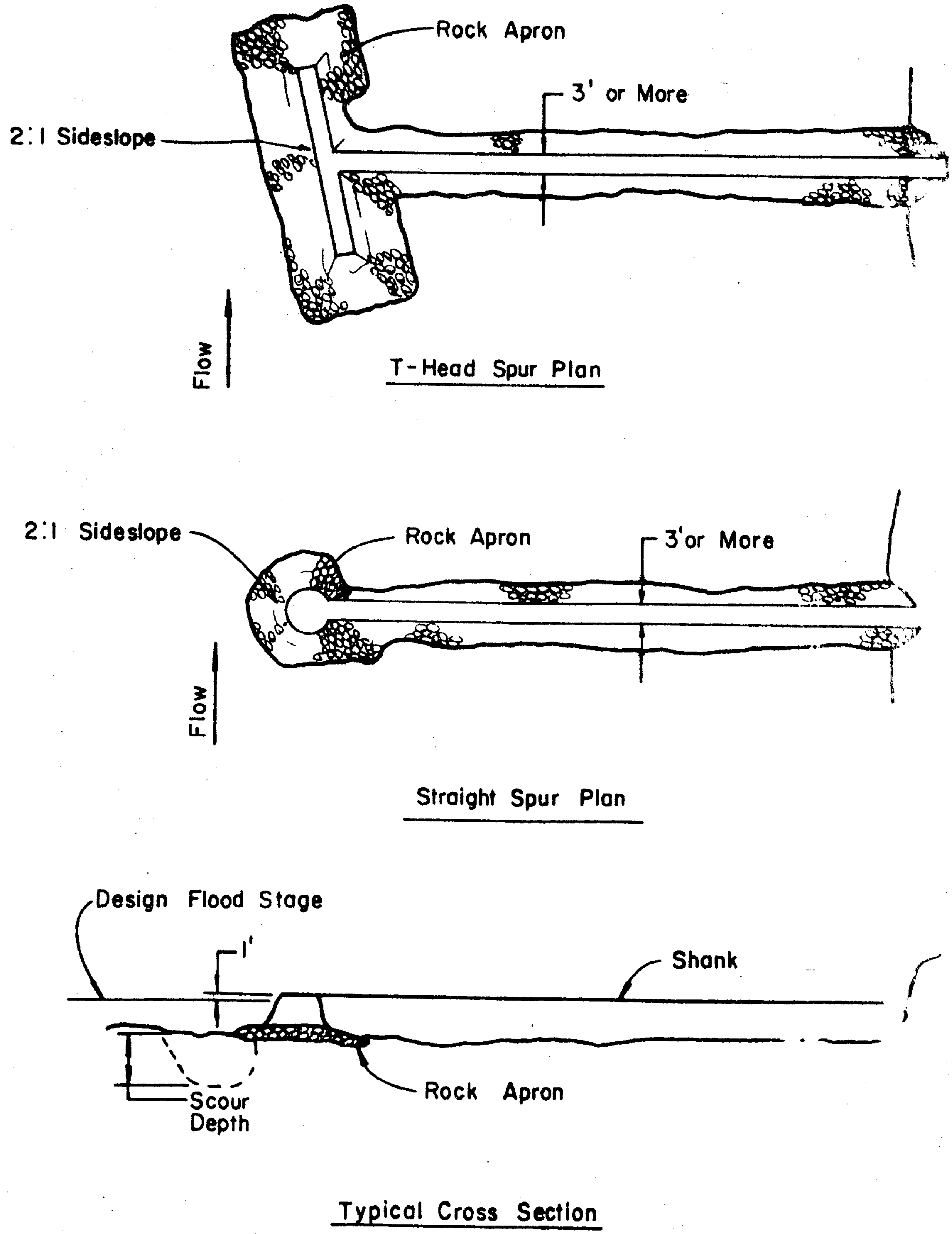


Fig. 3.1.10 Typical cross section and shape.

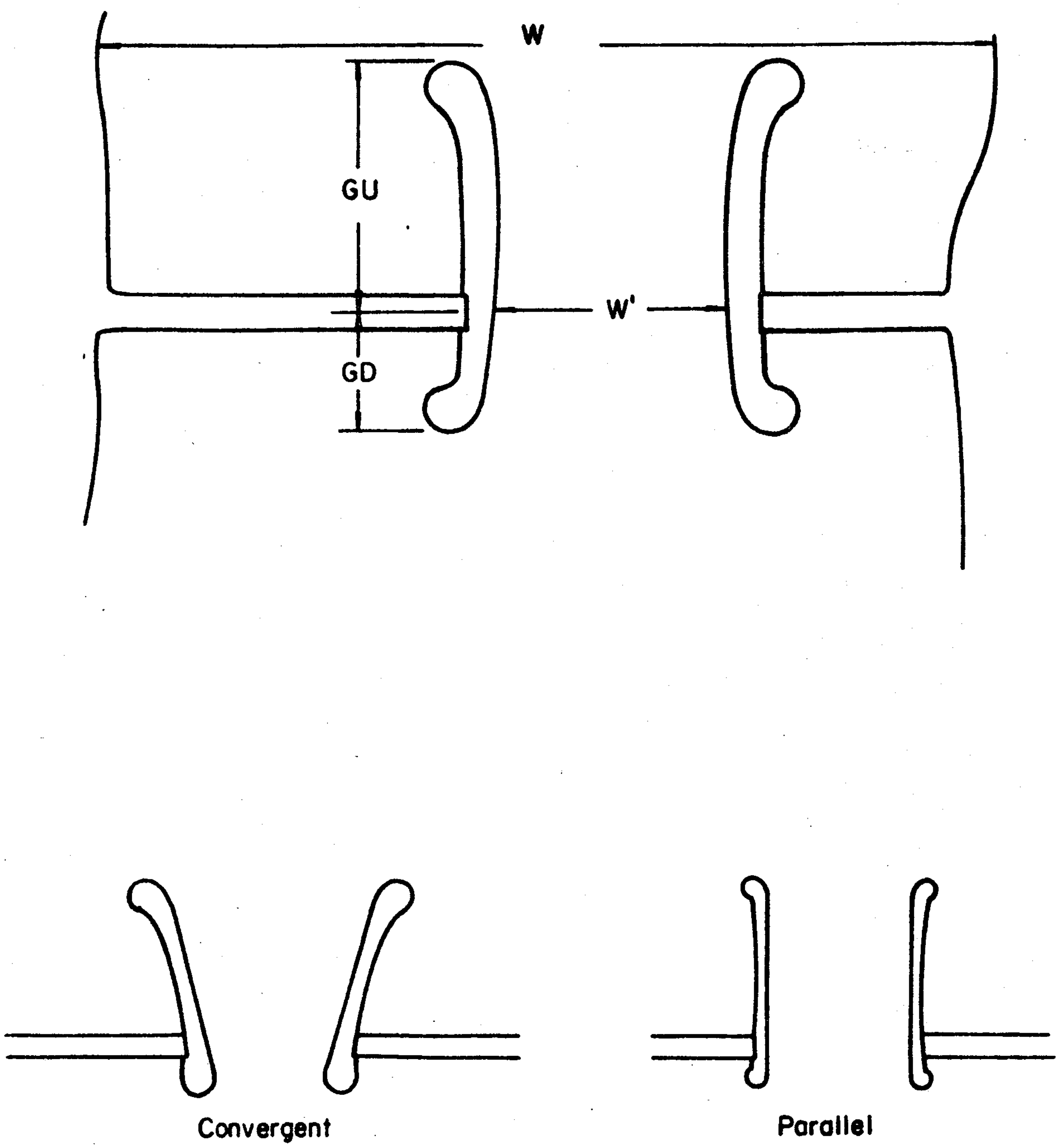


Fig. 4.1.1 Definition sketch for guide banks.

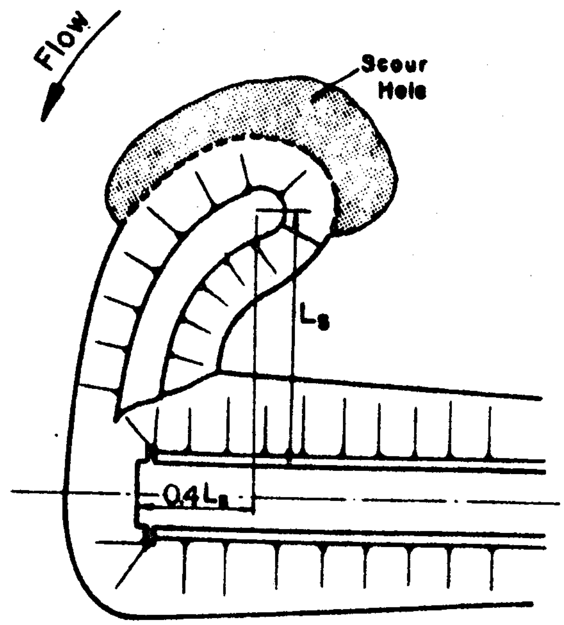


Fig. 4.1.2 Typical elliptical guide bank (from Ref. 32).

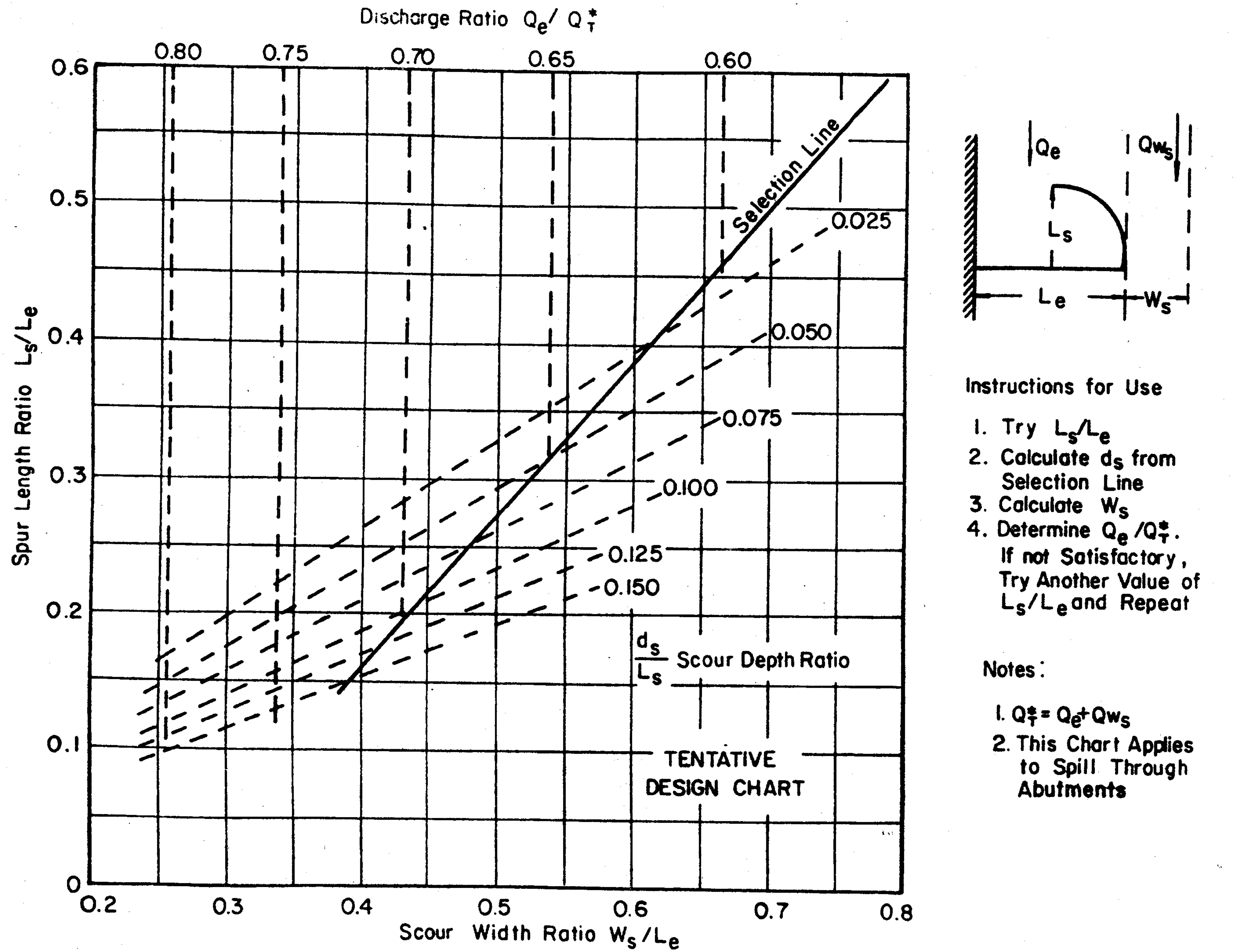


Fig. 4.1.3 Guide bank design procedure (from Ref. 25).

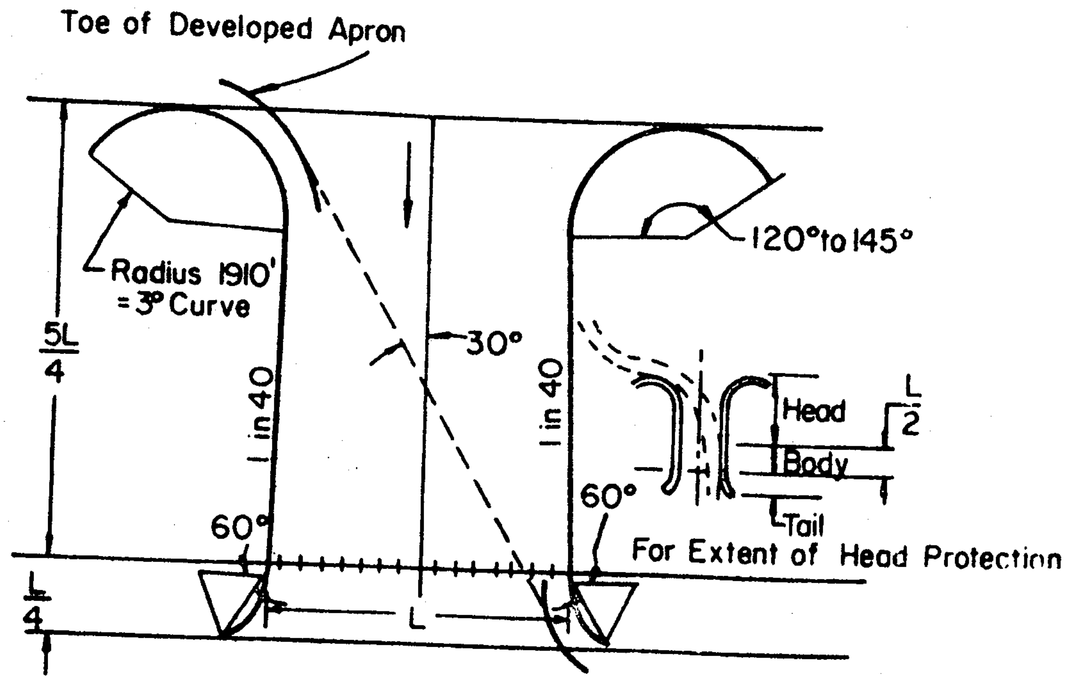
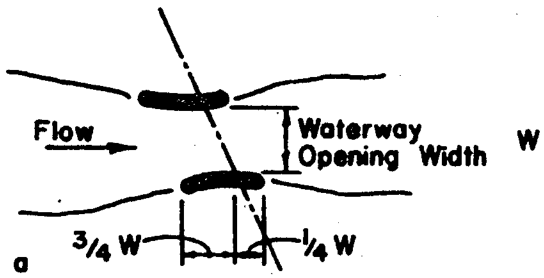
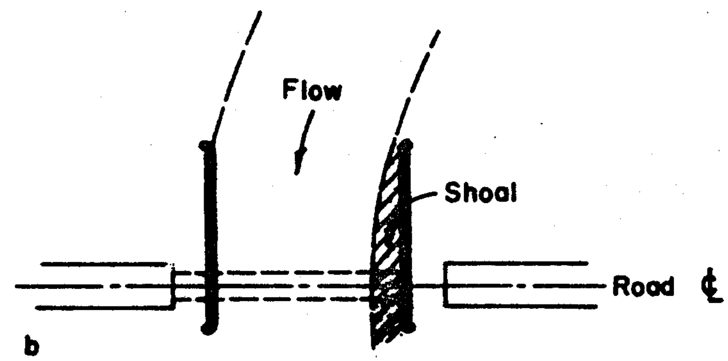


Fig. 4.1.4 Straight guide bank design (from Ref. 6).





Length and plan shape of guide banks: (a) suggested length of guide banks in shifting alluvial rivers; (b) straight, parallel guide banks tending to cause formation of a shoal on one side (an elliptical shape is preferable on the inner bank here); (c) combination of straight and curved banks on a channel bend.

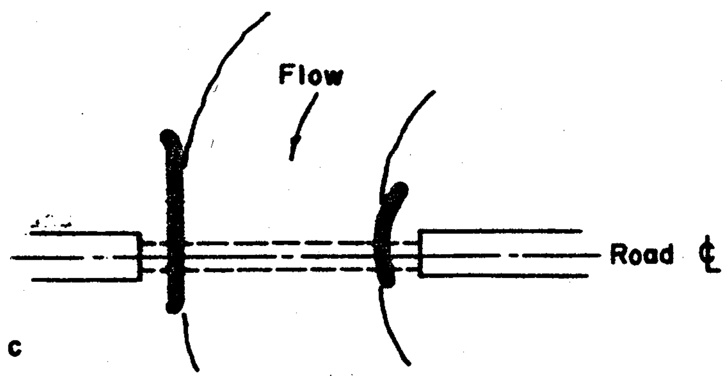
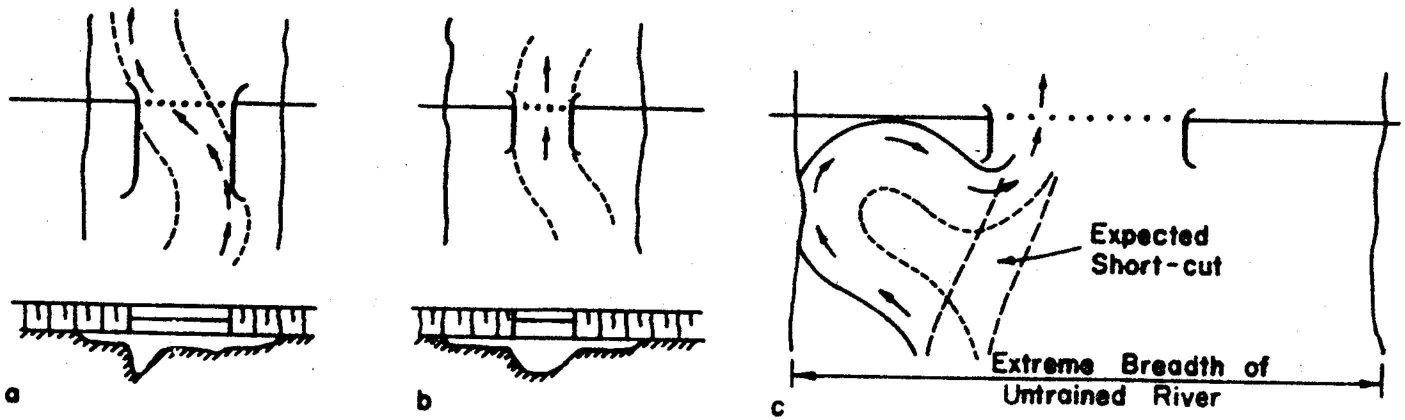


Fig. 4.1.5 Guide bank plan (from Ref. 29).



(a) bridge and guide banks unnecessarily long (note the skewed flow and irregular cross-section); (b) shorter length of bridge, permitting a more efficient cross-section and shorter guide banks; (c) excessively short guide bank, permitting the breaching of the embankment; (d) longer guide bank, preventing situation (c) and providing dead water protection to the embankment.

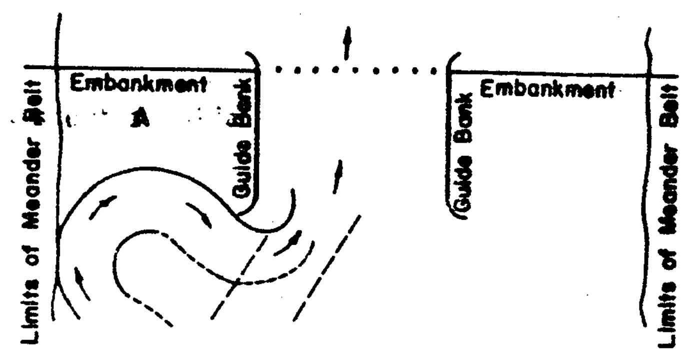


Fig. 4.1.6 Guide bank length selection (from Ref. 29)

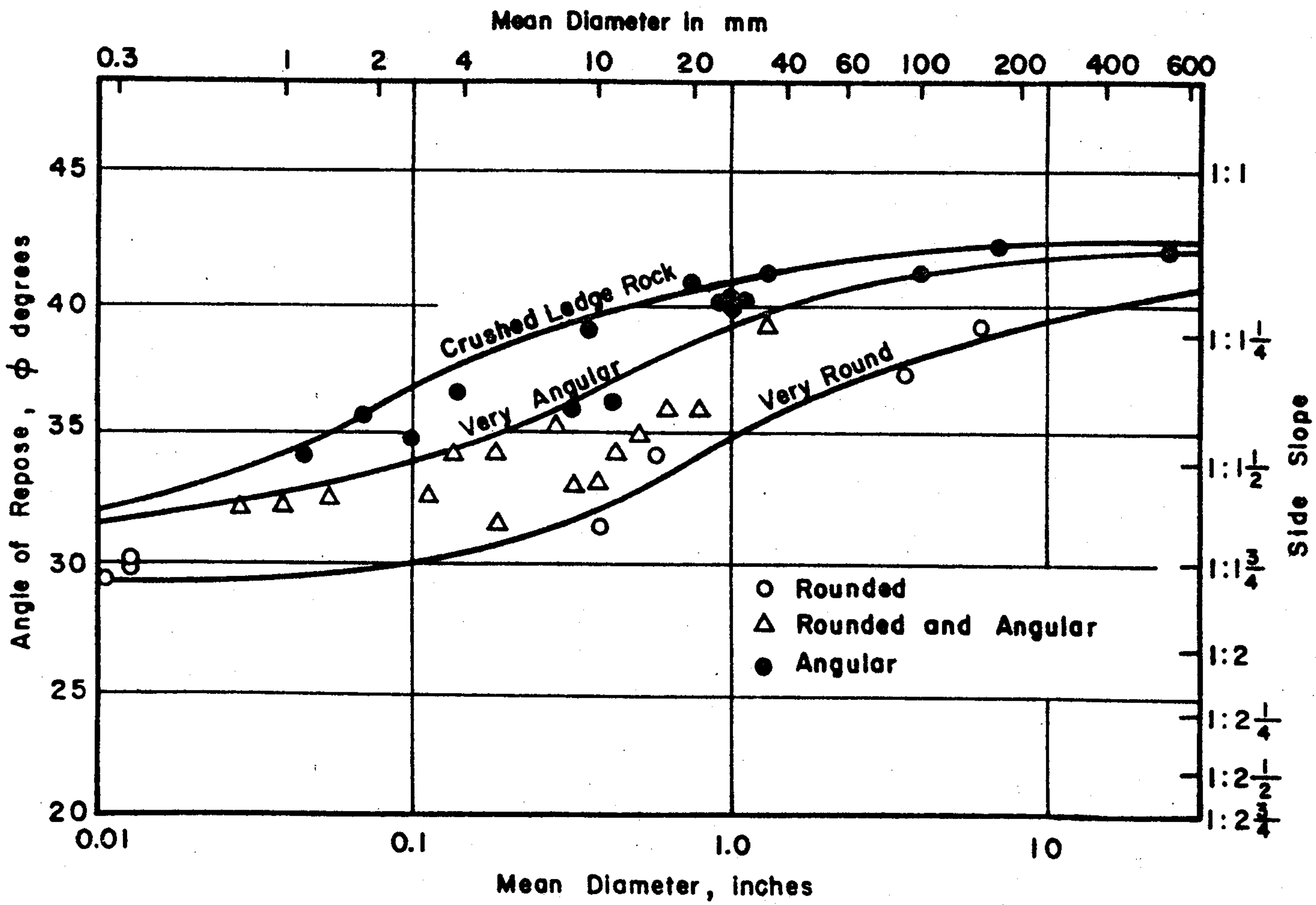


Fig. 4.1.7 Angle of Repose (from Ref. 32).

Example Riprap Problem

Embankment Protection with Riprap

Size the riprap for the end of an embankment for which the critical flow conditions are

$$\tau_s = 2.0 \text{ psf}$$

$$\lambda = 22^o \quad \text{(downslope)}$$

$$\theta = 18.4$$

The riprap should be sized so that it will not move; yet oversizing the materials may be costly. The safety factor is the ratio of moments tendin to remove the particle from its resting place to moments holding the particle at rest. A suitable safety factor for design is

$$S.F. = 1.5$$

Solution

Instead of solving simultaneously Eqs. 3.1.1, 3.1.2, 3.1.3 and 3.1.4 determine the required rock size, D_{50} by trial and error method.

Select $D_{50} = 0.3$ ft normally rounded

$$S_s = 2.65$$

and $\phi \simeq 39^o$

$$\eta = \frac{21\tau_s}{(S_s - 1)\gamma D_{50}} = \frac{21 \times 2.0}{1.65 \times 62.4 \times 0.3} = 1.4$$

$$\tan = \frac{\cos \lambda}{\frac{2 \sin \theta}{\eta \tan \phi} + \sin \lambda} = \frac{0.927}{\frac{0.631}{1.10} + .375} = 0.98$$

$$\beta = 44^o$$

$$\eta' = \left(\frac{1 + \sin (\lambda + \beta)}{2}\right) \eta = \left(\frac{1 + .914}{2}\right) 1.4 = 1.3$$

$$S.F. = \frac{\cos \theta \tan \phi}{\eta' \tan \phi + \sin \theta \cos \beta} = \frac{.949 \times .81}{1.3 \times .81 + .316 \times .719} = .60$$

Thus a rock diameter of 0.3 ft is unsatisfactory. Select other rock sizes greater than 0.3 ft in diameter and compute the corresponding safety factors. The computations are shown in tabular form.

SAFETY FACTORS

D_{50} ft	ϕ deg.	tan ϕ	η	tan β	β deg.	η'	S.F.	Remarks
0.3	39	.81	1.4	.98	44	1.3	.60	Riprap will fail
0.6	40	.84	.68	.63	32	.62	1.01	
0.9	41	.87	.45	[illegible]	25	.39	1.32	Rock too small
1.0	41	.87	.41	.43	23	.35	1.39	Rock too small
1.5	41	.87	.27	.30	17	.22	1.67	Rock slightly large
1.2	41	.87	.34	.37	20	.28	1.52	Use this

The computations indicate that a riprap with $D_{50} = 1.2$ ft will provide a 1.5 safety factor for a shear stress of 2.0 psf directed downward 22^{o} from the horizontal on a 3:1 side slope.