SCE COPY

DRAFT COPY

Final Report

Contract No. 41 USC 252(c)(5)

SCOUR DEPTH FROM EMERGENCY SPILLWAY FLOWS
QUEEN CREEK WATERSHED

Submitted to

U.S. Department of Agriculture
Soil Conservation Service
Federal Building
Room 3008
Phoenix, Arizona 85025

Submitted by

Emmett M. Laursen, Professor
Civil Engineering Department

Engineering Experiment Station
College of Engineering
University of Arizona
Tucson, Arizona 85721

January 1978

SUMMARY

The depth of scour at the toe of the emergency spillway of the Queen Creek flood retarding structure will be the sum of the degradation associated with the reach downstream the spillway and the local scour occurring at the base of the spillway. The large magnitude expected flood flows in the reach downstream from the structure will have the capacity and competence to remove the six-foot thick, sand and gravel layer composing the surface material in this wide drainageway. Underlying this sand and gravel layer is a sand, gravel and cobble layer which is over six-feet thick, and the coarsest 20%, with a mean size of four inches, should not be moved even by the highest expected flows. Self-sorting as two feet of this underlying layer is degraded should result in an armour layer of cobble and a limit to the degradation.

A general argument was used to establish the form of the model-prototype relationship for local scour below a spillway such as the baffled-apron. A few data points were then sufficient to draw a tentative curve for the relative depth of local scour d_s/y_c as a function of τ_o'/τ_c which can be approximated as $0.27\ (y_c/d)^{2/3}$ (d_s is the depth of local scour measured from the original stream bed elevation, y_c is the nominal critical depth, d is the diameter of the sediment, and τ_o'/τ_c is the ratio of the boundary shear in a critical flow to the "critical" tractive force.) Especially if during construction, cobbles are placed at the predetermined scour level, the local scour should be no more than four feet.

The best estimate of the possible scour at the emergency spillway of the Queen Creek structure is therefore 6 feet plus 2 feet plus 4 feet or

a total of 12 feet. The reach downstream from the structure for a distance of 1000 feet or more should degrade to form a rectangular channel 1600 feet wide and 8 feet deep.

LOCAL SCOUR BELOW SPILLWAYS

Scour of the stream bed (or banks) occurs when the capacity of the flow to remove sediment from an area is greater then the supply of sediment to that area. This is simply a statement of mass conservation as applied to sediment; although it might be noted that in speaking of the capacity of the flow to remove sediment there is an implication that the flow is competent to move the sediment particles composing the boundary material. This notion of scour can be extended (see "Observations on the Nature of Scour" in Appendix) to demonstrate both the existence of a limit to the size of a scour hole, and the essential difference between scour with and without a supply of sediment (scour by sediment-transporting flow and clear-water scour).

For either case, in a steady, uniform flow there will be no scour. If the flow entering the area under consideration supplies a certain amount of sediment, the flow leaving the area, being the same, can remove exactly the same amount--no more, no less. In the case of clear-water scour a slight non-uniformity of the flow would even be possible; as long as both the entering and leaving flow were unable to transport sediment. Scour occurs then only if the capacity and competence of the flow varies along the flow, and the capacity and competence only vary if the flow itself varies. An analytical solution to a scour problem depends, first, on an ability to describe the flow and, second, on being able to relate the flow characteristics to the capacity and competence of the flow. The use of a hydraulic model as an analog computer does not entirely bypass the need for analysis because the model is always distorted and results must be interpreted and corrected before they will adequately describe prototype behavior.

The only geometry for which an analytical solution of scour has been made is the long contraction--long in the direction of flow--because for this geometry both the flow and the relationship between the flow and the capacity and competence to move sediment can be adequately described (see "Scour at Bridge Crossings" and "Analysis of Relief Bridge Scour" in Appendix). Model studies of scour around bridge piers and abutments can be interpreted in the light of the long contraction solutions, a few key observations of the flow patterns of the models, and a few assumptions. The few prototype measurements of bridge pier scour are less than completely satisfying but seem to verify the predictions based on the model studies.

In the case of scour by sediment-transporting flow, the indication is that the depth of scour can be expanded by the model scale to predict the depth of scour in the prototype and that velocity and sediment size have little or no effect if the transport conditions are well above the critical for movement. This model-prototype relationship should also be true, but would differ in detail, if a spillway is a drop in a canal system transporting bed load.

However, if a spillway is part of a dam and reservoir system, the flow over the spillway would be clear water (possibly muddy with very fine sediment, finer than the sediment that would be scoured out below the spillway), and in this case both velocity and sediment size do make a difference. Because gravity forces dominate, spillway models are run with a model-prototype velocity ratio equal to the square root of the length ratio (or model scale). The major characteristics of the flow are then similar in model and prototype, but certain minor characteristics like the shear and the boundary layer would require that the Reynolds number as well as the Froude number be the same in model and prototype, or that the velocity ratio be the inverse of the length ratio. Velocities and depths can be measured in the model and increased to prototype scale by the model-prototype Froude relationship. Boundary-layer effects must be calculated in model and prototype and compared. Where conditions are very different from the flat plate, such calculations become questionable. The matter of the scour to be expected below spillways is even less well understood, and the prediction of prototype scour from model results if made at all is usually made with many qualifying phrases.

THE HYDRAULICS OF SPILLWAYS AND STILLING BASINS

The standard spillway and stilling basin consists of a dam section modified to an ogee section shaped at the crest to the lower nappe of an overflow weir and with a circular curve at the toe leading into the stilling basin containing baffles and an end sill as shown in Figure 1. The

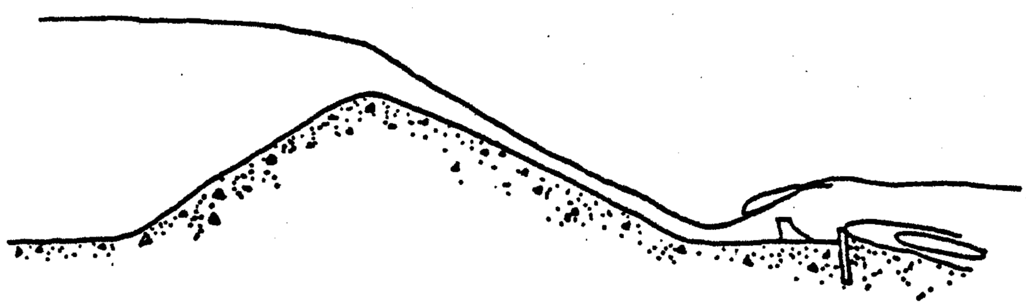
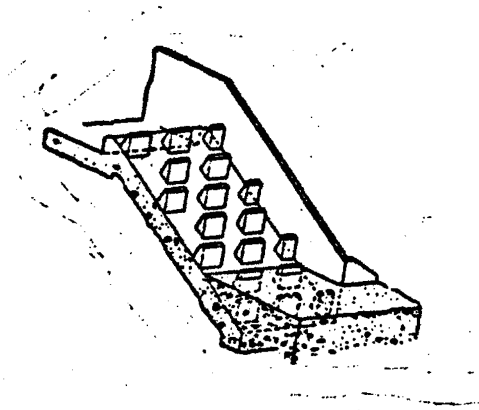


Figure 1. Typical spillway and stilling basin.

crest is a control section with the flow going through critical; the depth and velocity combination having minimum specific energy. However, the crest depth (which is truly critical) is not the nominal critical depth $y_c = \sqrt[3]{q^2/g}$ because the pressure distribution is not hydrostatic and the velocity is neither uniform nor parallel. On the sloping face the flow is super-critical and approaches uniform, normal flow. In the stilling basin a hydraulic jump is formed and the flow out of the basin is again sub-critical. The baffles and end sill contribute to the force system acting to change the momentum flux from the high value of the entering flow to the low value of the flow leaving the basin. It is important to realize, however, that contributing a force to the flow system is not as important as the fact that the force is variable depending upon the position of the jump in the basin. Changes in the tailwater can thereby be compensated with small changes in the jump position. Keeping the jump in the basin with varying tailwater is the primary function of baffles. Although the end sill has this same function, even more importantly it deflects the flow away from the stream bed. A secondary roller is formed between the stream bed and the flow issuing from the basin and stream bed material tends to be dragged back towards the basin. If the slope of the scoured stream bed is much less than the angle of repose, the spillway and stilling basin should be safe even if the prototype scour is larger, relatively, than the model scour. Model scour patterns which do not pose any danger to the structures, and relative scour depths between one geometry and another have usually been considered sufficient interpretation of model results.

THE HYDRAULICS OF THE BAFFLED-APRON SPILLWAY

In the baffled-apron spillway (BuRec Basin IX), the stilling basin is eliminated and the baffle piers are moved to the sloping face of the overflow section. The opening paragraphs of BuRec Engineering Monograph 25 describes the structure and the flow as follows:



Baffled aprons or chutes have been in use on irrigation projects for many years. The fact that many of these structures have been built and have performed satisfactorily indicates that they are practical and that in many cases they are an economical answer to the problem of dissipating energy. Baffled chutes are used to dissipate the energy in the flow at a drop and are most often used on canal wasteways or drops. They require no initial tail water to be effective although channel bed scour is not as deep and is less extensive when the tail water forms a pool into which the flow discharges. The multiple rows of baffle piers on the chute prevent excessive acceleration of the flow and provide a reasonable terminal velocity, regardless of the height of drop. Since flow passes over, between, and around the baffle piers, it is not possible to define the flow conditions in the chute in usual terms. The flow appears to slow down at each baffle pier and accelerate after passing the pier, the degree depending on the discharge and the height of the baffle piers. Lower unit discharges result in lower terminal velocities on the chute.

> *The chute is constructed on an excavated slope, 2:1 or flatter, extending to below the channel bottom. Backfill is placed over one or more rows of baffles to restore the original stream bed elevation. When scour or downstream channel degradation occur, successive rows of baffle piers are exposed to prevent excessive acceleration of the flow entering the channel. If degradation does not occur, the scour creates a stilling pool at the downstream end of the chute, stabilizing the scour pattern. If excessive degradation occurs, it may become necessary to extend the chute.*

The flow would, indeed, be complex because it is around and over roughness elements of the scale of the flow, on a steep slope in a gravitational field. However, after enough rows of baffles so that the flow pattern is repeated around each baffle pier, an approximate momentum analysis can be performed equating the weight component in the direction of flow to the drag force on a single baffle pier. The control volume is taken halfway to the adjacent piers in all directions so that the momentum flux and pressure force on the sections through which the flow enters and leaves are equal. The shear forces resisting movement are small compared to the pressure forces on the pier. For a baffle pier of height h, width 1.5 h, volume 0.675 h^3, 3 h center-to-center in the row and 4 h face-to-face in the column, the weight component down a 1V:2H slope is

$$W \sin \theta = \frac{62.4}{\sqrt{5}} (12 h^2 y - 1.35 h^3)$$

where y is the depth of flow measured normal from the chute (apron) slope. The drag force is

$$F = C_D\ 1.5\ h^2 \frac{\rho}{2} V^2$$

where the average velocity V is based on the clear area at the face of a pier

$$V = \frac{3\ h\ q}{3h\ y - 1.5\ h^2} = \frac{q}{y - 0.5\ h}$$

and

$$q^2 = g\ y_c^{\ 3}$$

Equating the weight component to the drag force results in a relationship between y, h, and y_c or q:

$$(y - 0.112\ h)\ (y - 0.5\ h)^2 = \frac{C_D}{7.15} y_c^{\ 3} = \frac{C_D}{230} q^2$$

These approximate relationships are plotted in Figures 2 and 3 with an assumed value of $C_D = 4$; twice the drag coefficient for a two-dimensional plate to try to account for the separated, high velocity jet flow between a pair of piers striking the pier in the next staggered row. The y vs. q curves fall below roughly averaged nominal depths shown in Monograph 25. The approximate analytical curves could be raised by assuming a higher value of C_D, but the experimental profiles...."are higher than the profiles shown in a photograph of the same test" because "water surface measurements were made with a point gage and a scale, taking the maximum water surface at each measured point."

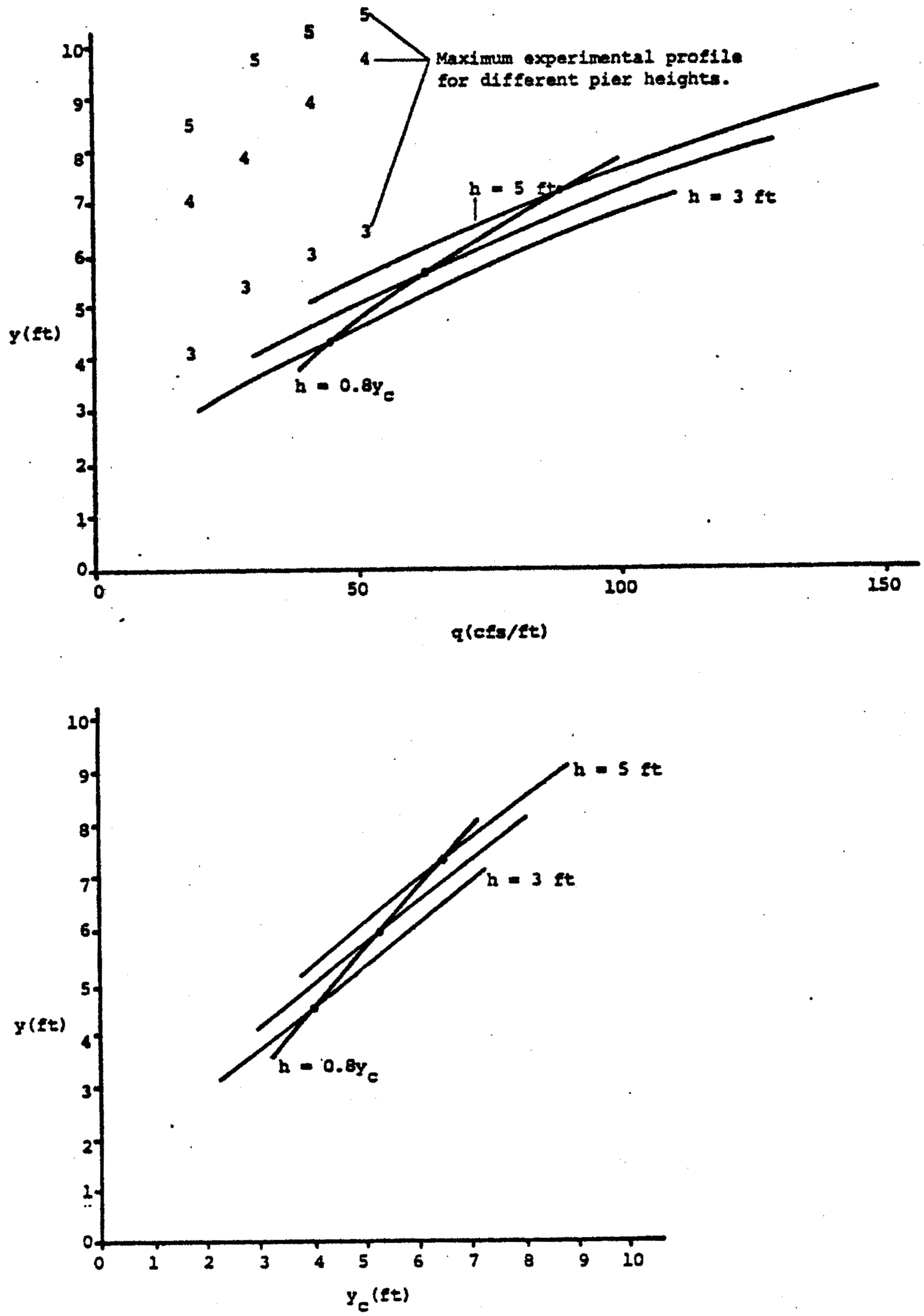


Figure 2. "Theoretical" depth on a baffled apron.

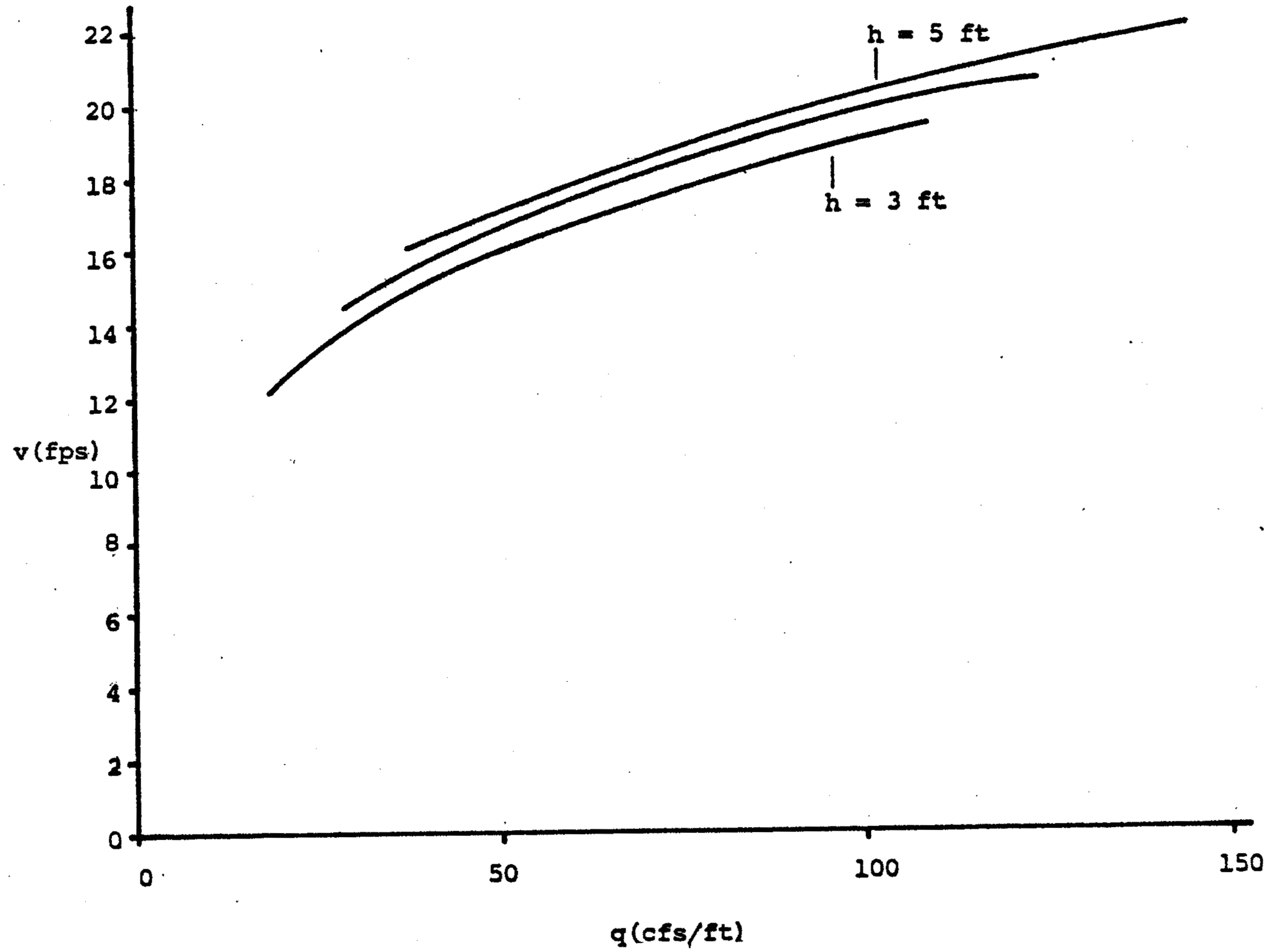


Figure 3. "Theoretical" velocity on a baffled apron.

The plot of the nominal velocity V vs. q shows much higher velocities than the measured velocities cited in Monograph 25, but it is not clear where the experimental velocities were measured. The interesting feature of the approximate analytical curves is that the velocity only increases modestly with a doubling of discharge, and there is even less effect in increasing the baffle pier height from three feet to five feet.

LOCAL SCOUR AT A BAFFLED-APRON SPILLWAY

The flow along and at the tail end of a baffled apron is highly turbulent with an unknown but very non-uniform velocity distribution and a difficult to define thickness. Still and all, it is a jet impinging on the tailwater and eventually on the streambed. In the pool between the tailwater and the streambed it is decelerated and diffused through the turbulence created at the shear face between the jet and the pool as happens with any submerged jet. At the streambed it is turned to the horizontal, and when the scour hole is formed, up. The flow exerts forces on the boundary particles and moves them out of the scour hole until at the limit the pool is big enough to slow the flow sufficiently so that the sediment particles are no longer moved out of the scour hole. There may still be some shifting of the sediment particles by the turbulent flow at this limiting condition and yet no increase in the size of the hole.

An analytical solution is not possible, but even model studies need a modicum of analysis for interpretation. If a model is to be dynamically similar to a prototype it must also be geometrically similar and kinematically similar. Except for the scour hole, geometric similarity is easy to achieve. Kinematic similarity will be very closely approximated for flow in a structure such as this by use of the Froude criterion that the relative velocity ratio, $V_r = V_m/V_p = \sqrt{L_r}$ where the model scale $L_r = L_m/L_p$. The velocity at any point in the model relative to a reference velocity would be the same as the velocity at the comparable point in the prototype relative to a comparable reference velocity in the prototype.

A convenient reference to use is the nominal critical velocity $V_c = \sqrt{g\, y_c} = \sqrt[3]{g\, q}$, and for linear dimensions $y_c = \sqrt[3]{q^2/g}$. Whether this combination of velocity and depth exists anywhere in the model is immaterial; all real velocities and depths will be related to these reference conditions by the flow rate q and the geometry of the structure.

Dynamic similarity for the pressure forces of the baffles on the flow and the consequent effects on the flow can be expected to be similar if the Reynolds number of the model is above 10^3, which ordinarily would be true. Dynamic similarity for the forces on the sediment particles, and the consequent scour, is another matter. Imagine a channel added to the structure upstream of the crest; this channel to at critical slope with the nominal critical depth and critical velocity being characteristic of the flow. This imaginary channel would be composed of fixed sediment particles the same as those which would compose the surface of the limiting scour hole; because of self-sorting this armour layer would be coarser than the sediment to be scoured out.

If the relative roughness and the Reynolds number of the flow are large enough the boundary shear can be written as

$$\tau_0' = V^2\, d^{1/3} \,/\, 30\, y^{1/3}$$

where d is the mean size of the sediment in feet. The expression comes from using $n = 0.0355\, d^{1/6}$ in Manning's equation. Now if this boundary shear is compared to the critical tractive force required to move the sediment particles an expression of scour tendency results

$$\tau_0'/\tau_c = V^2/120\, y^{1/3}\, d^{2/3}$$

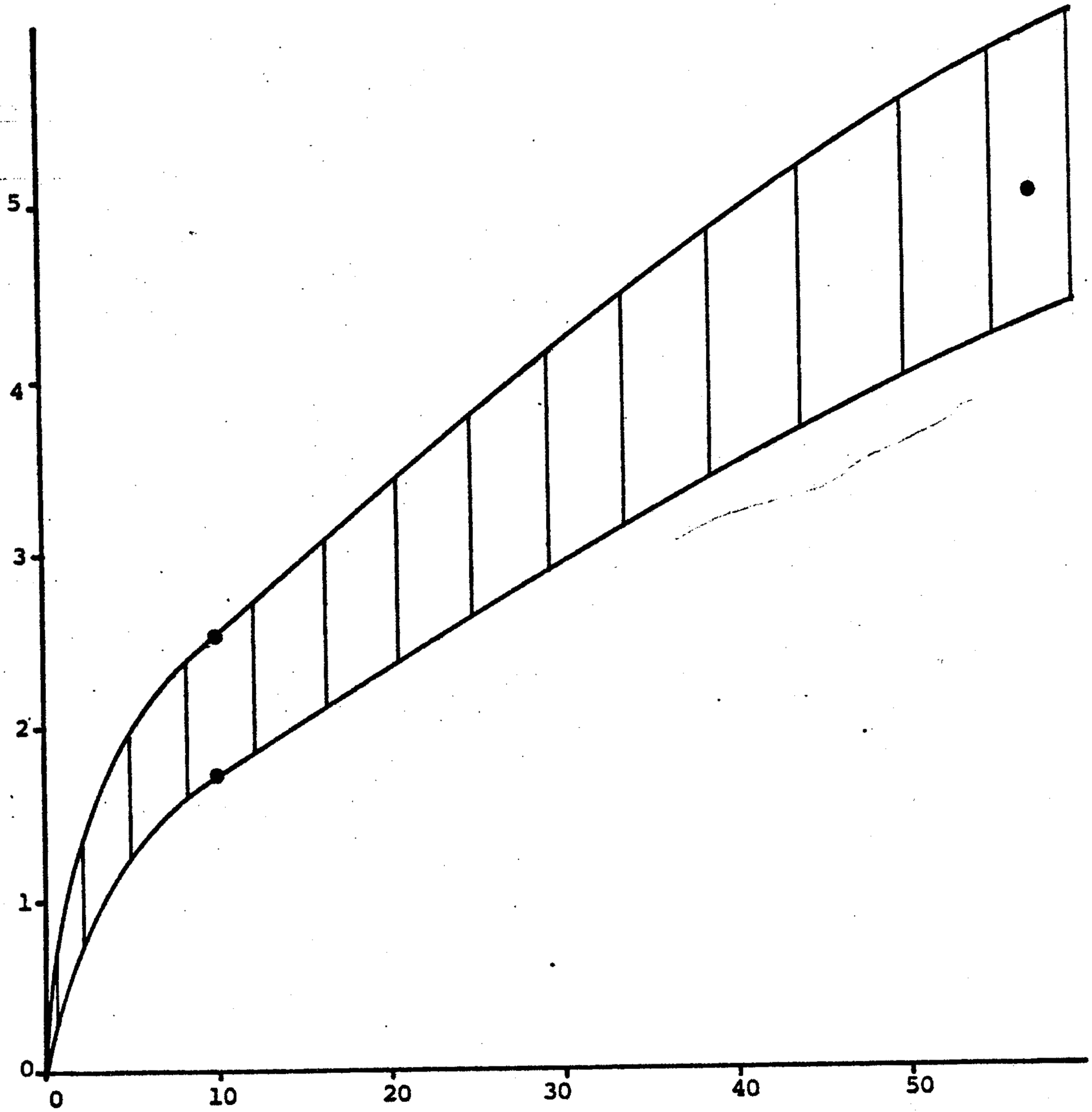
if the critical tractive force $\tau_c = 4$ d (note that this meaning of "critical" is for the beginning of sediment movement and has nothing to do with the other sense of critical as being the minimum specific energy level.) If the sediment in model and prototype are both sand or gravel and the size of the sand is larger than the thickness of any possible laminar sublayer in a comparable flow on a smooth surface, the value of τ_0'/τ_c should be about the same in model and prototype if the size of the sediment in model and prototype are related by the model scale. The tendency for movement would certainly be the same in the imaginary channel upstream of the crest. At the surface of the limiting scour hole $\tau_0'/\tau_c = 1$ in both model and prototype, and although it is difficult to say what velocity and length should be used in evaluating τ_0' and just how they would be related to the boundary shear, if the scour holes in model and prototype were in proportion to the model scale, it is reasonable to suppose that the condition of $\tau_0'/\tau_c = 1$ would be satisfied in both model and prototype. Or, to reverse the statement, if the sediment size is scaled up, the depth of scour and the shape of the scour should be similar in model and prototype and in the ratio of the length scale.

For a given geometry of structure, one might expect the relative scour depth d_s/y_c to be function of τ_0'/τ_c; the larger the boundary shear/critical tractive force ratio, the larger the d_s/y_c ratio. A somewhat different but roughly parallel function could be expected for every different spillway geometry or change in y_c. According to Monograph 25 for a prototype discharge rate of 60 cfs/ft and presumably using a 0.5 mm sand, a 1:16 model gave prototype scour depths of 8 feet to 12 feet (prototype scale) with baffles of various size. At lesser discharges, the scour depths were somewhat less.

A comparable scour hole is found at the toe of rapids in the Colorado River; the geometry of the rapids is random rather than regular and the rocks (baffles) range in size with most smaller than 0.8 y_c but some larger. However, there is a jet similar to that of the baffled-apron spillway penetrating the tailwater and scouring the bed. The flow flucuates on a daily and weekly basis for power generation at Glen Canyon dam and for longer time periods with storage requirements in Lake Powell. The sediment being transported has a mean size of about 0.3 mm but the scour hole may be armoured. The unit discharge at the typical rapids is about 66 cfs/ft and the scour depth is about 25 feet. Thus three data points of more-or-less precision are available. Since V_c and y_c are related to q, the boundary shear/critical tractive force ratio can be written as

$$\frac{\tau_0'}{\tau_c} = \frac{1}{3.7} \left(\frac{y_c}{d}\right)^{2/3}$$

The graphical scour depth relationship shown in Figure 4 is a guess, but an informed guess. For some size of sediment the scour would be zero; this size in reality might be so big that the geometry is changed. As the sediment size decreases the scour increases, but the function should be approximated by a power less than unity. A family of curves within the band shown would be expected for different geometry. The scale of the structure is given by y_c, all other dimensions would be fractions or



$$\tau_o'/\tau_c = \frac{1}{3.7}\left(\frac{y_c}{d}\right)^{2/3}$$

Figure 4. Possible scour at a baffled-apron spillway.

multiples of y_c. The sediment must be granular and cohesionless and, if $\tau_c \neq 4\ d$, a proper correction should be made.

One very important caveat should be noted. The Bureau of Reclamation tests were for one-half hour and the limiting scour might not have been attained. If the limiting scour is greater than cited, the lower part of the curve, the part to be used, would be raised some unknown amount.

DEGRADATION BELOW SPILLWAYS

Another kind of scour which must also be considered is the degradation which occurs in the reach below the spillway. The reservoir created by the dam traps the sediment supplied from upstream. Although if the reservoir is shallow some very fine sediment (clays) might not be trapped, the sands and gravels, the type of material to be scoured out downstream will be. The reach immediately downstream from the spillway will be scoured out to the limit where the flow can no longer move the boundary materials. At the toe of the spillway this action is aside from but will be intermingled with the local scour. The sediment scoured out supplies a load temporarily to the next area downstream, but when the first area is scoured out no more material is supplied and the next area is scoured out. As a first approximation each finite area is scoured out to be followed by the next. The lowered velocity in the degraded portion of the stream has a lesser velocity and, therefore, a lesser slope to the energy gradient and to the water surface. Thus the water surface drops and more scour takes place. The degraded reach becomes longer and longer and the stream bed elevation just below the spillway becomes lower and lower until some other factor stops the degradation. This other factor can be a lake or reservoir which limits the drop in water surface, or a layer of coarse sediment which the flow cannot move and therefore limits the drop in stream bed elevation. This layer of coarse sediment can be created during the degradation of the reach if enough coarse material to armour is left behind from the material being scoured out of the bed.

PREDICTION OF THE PROBABLE SCOUR AT THE QUEEN CREEK EMERGENCY SPILLWAY

There are two parts to the prediction of the probable scour at the Queen Creek emergency spillway. The first is the prediction of the local scour at the base of the spillway; the second is the prediction of the degradation in the reach below the spillway for this is the base from which the local scour is measured. Both predictions must be qualified, but should serve for preliminary design. Further studies will be needed for final design.

Predicted Local Scour

The basis for the prediction of the scour was laid in a previous section of this report as d_s/y_c as a function of y_c/d; a log-log plot of the desired relationship is shown in Figure 5.

Assuming the height of the baffle piers $h = 0.8\ y_c$ and the other dimensions as tested by the Bureau of Reclamation and recommended by them, determinations were made of the probable depth of the scour for unit rates of discharge of 60, 80 and 100 cfs/ft, and mean sediment sizes of 1/2, 1, 2, 4 and 8 inches with the results shown in Figure 6.

The samples of top layer of alluvium, about 5 to 6 feet deep, have a mean size of between 1/2 and 3 mm and maximum sized gravel of about 3 inches. This layer would probably completely vanish in a large flood because there is very little material to provide an armour layer. The next layer of sediment is much coarser and goes down to over 12 feet below the land surface. The mean size of this mixture of sand, gravel and cobble is

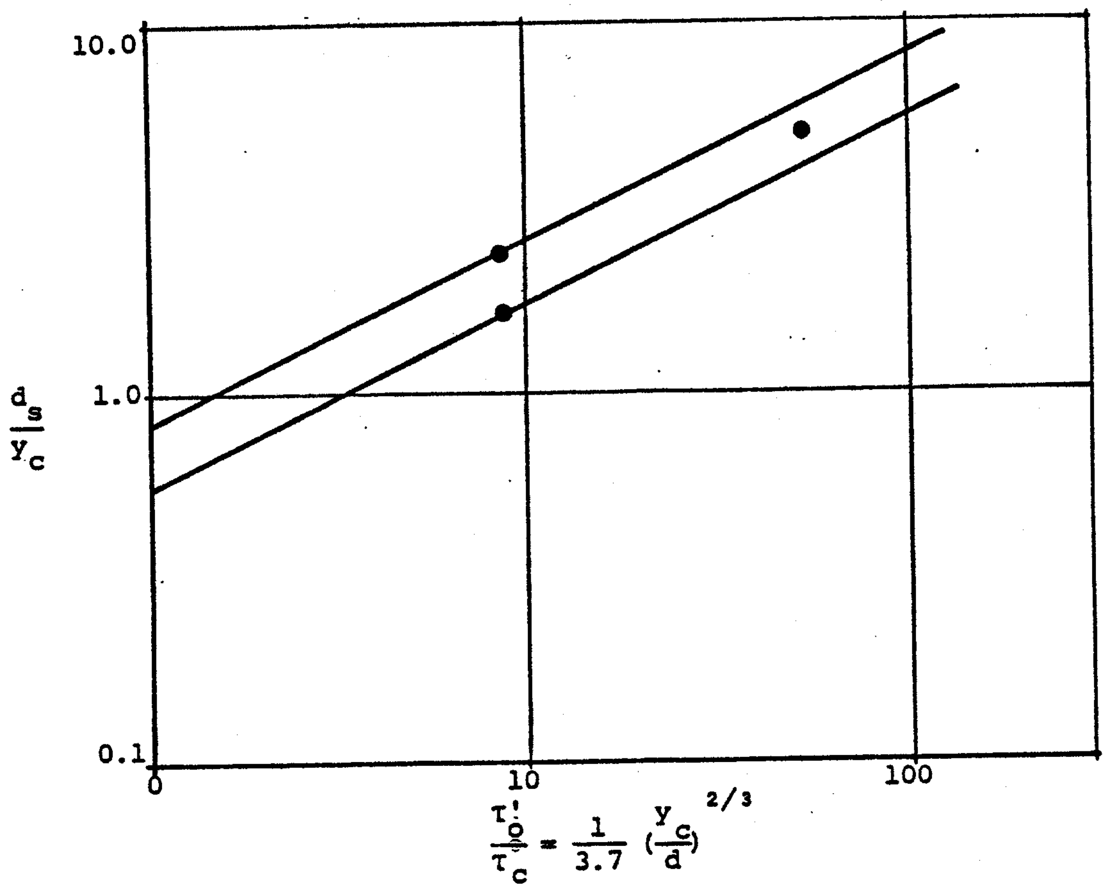


Figure 5. Assumed local scour relationship for Queen Creek emergency spillway.

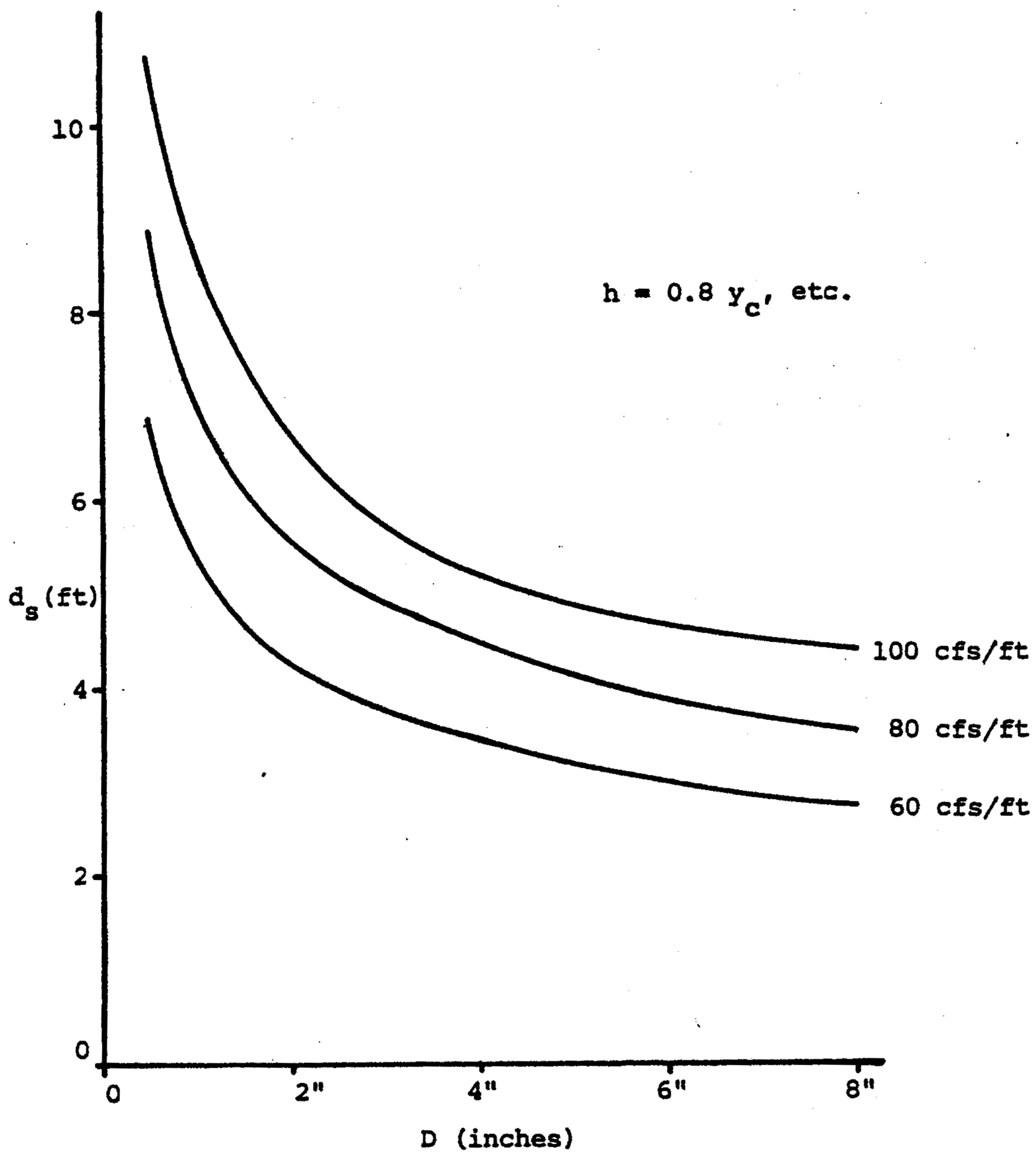


Figure 6. Estimated local scour at Queen Creek emergency spillway.

between 3/8 and 2 inches in the different samples and the maximum size cobble is between 6 and 12 inches. The 90% size in the samples varied between 2 and 8 inches with the average about 5 inches. There probably would be enough coarse gravel and cobble size particles to armour the scour hole and limit the local scour to 4 feet. Since part of the layer has to be removed to form and place the concrete it would be more certain if the material was roughly sieved and the coarsest 20% laid out from the base of the structure in the shape of a natural scour hole as shown schematically in Figure 7. Details of the shape and length of the scour hole should be available from past Bureau of Reclamation model studies, or from the final design tests planned for the Queen Creek structure. The armour (riprap) strip should be 12 to 16 feet wide, but need not extend to the downstream end of the scour hole--only far enough to turn the jet up.

Depending on the final decision as to the unit rate of discharge, the dimensions chosen for the baffle piers, and the availability of riprap material to armour the shaped scour hole, it should be possible to limit the depth of local scour to four feet--but four feet down from where, is a question.

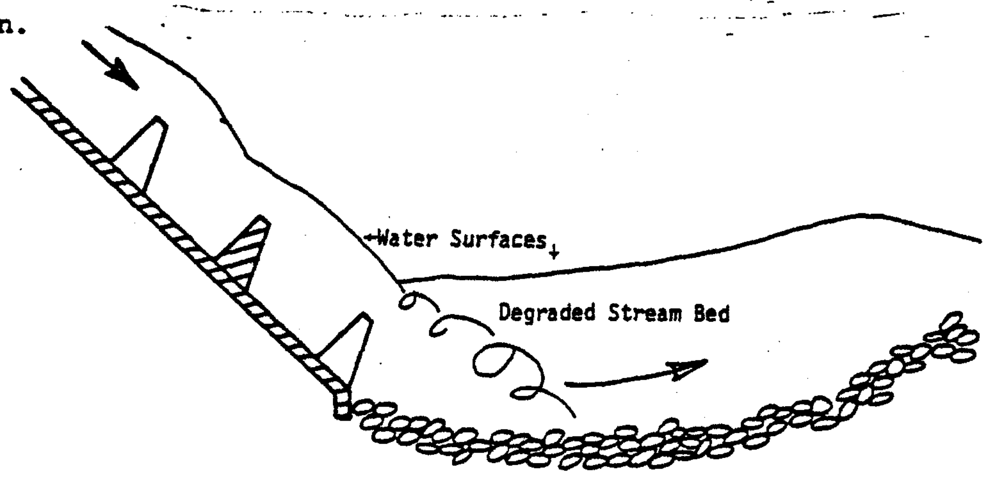


Figure 7. Artificially riprapped local scour hole.

Predicted Degradation

The depth of local scour must be measured from the degraded stream bed--not the original stream bed. Except for possibly some very fine sediment particles, the reservoir behind the dam serves as an excellent trap. As a result the flow away from the spillway has an ability to transport sediment, and it will do just that. The sediment it transports will come from the streambed which lowers and lowers and lowers as time goes on. Trying to follow this process through time and space is: (1) difficult, (2) depends on the probability of occurrence of various series of extreme events, and (3) requires a good understanding of the self-sorting action whereby the composition of the bed material gradually changes. If a limit to the degradation will not be reached during the life of the dam and reservoir, such a solution must be sought even if the confidence that can be placed in such a solution is less than desirable. A more certain answer can be obtained if a limit sensibly exists and the details of just how it is achieved can be bypassed.

The first step is to describe the normal, sediment-transporting flow. This has been done in Table I assuming a rectangular cross-section so that the discharge is categorized by the unit rate of discharge q. Mannings formula with a n = 0.30 has been used to find depth y and velocity V, and Strickler's n to find the particle shear $\tau_o' = V^2 d_m^{1/3}/30\ y^{1/3}$.

A sand and gravel layer about six feet thick overlies a sand, gravel and cobble layer at least six feet in thickness. The percent finer curves for these two sediments are shown in Figure 8. In averaging the samples a weight was given to the thickness of the layer from which individual samples were obtained.

Table 1. Normal Stream Flow Characteristics

q (cfs/ft)	20	40	60	80	100
y (ft)	2.82	4.26	5.43	6.45	7.41
V (fps)	7.10	9.40	11.05	12.40	13.50
F $(V/\sqrt{gy})$	0.74	0.80	0.84	0.86	0.88
$\sqrt{gyS}$ (fps)	0.69	0.85	0.96	1.05	1.12
$\tau_o'(d_m=2.1\text{mm})$	0.23	0.35	0.44	0.52	0.59
$\tau_o'(d_m=17.2\text{mm})$	0.45	0.69	0.88	1.04	1.18

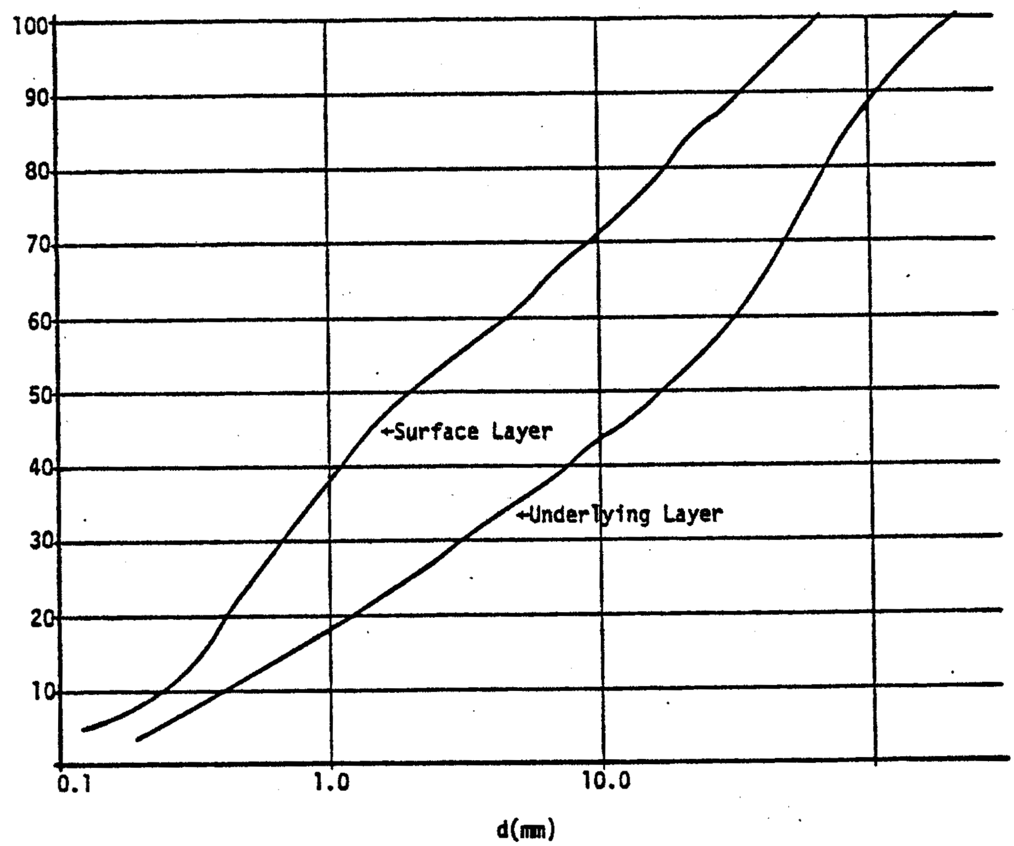


Figure 8. Size distribution of surface and underlying material at Queen Creek.

In order to evaluate the sediment-transporting characteristics of the flows, each sediment was divided into five equal fractions with the characteristics shown in Table II. The critical tractive force was evaluated as $\tau_c = 4\ d$, and the fall velocity w as if the particle was a quartz sphere falling by itself in 10°C water.

Table II. Bed Material Characteristics

Surface Layer

	d_{90}	d_{70}	d_{50}	d_{30}	d_{10}
d (mm)	33	9.2	2.1	0.68	0.25
d (ft)	0.11	0.030	0.0069	0.0022	0.00082
τ_c (psf)	0.43	0.12	0.027	0.0089	0.0033
w (fps)	4.44	2.36	0.92	0.32	0.092

Underlying Layer

	d_{90}	d_{70}	d_{50}	d_{30}	d_{10}
d (mm)	110	48	17.2	3.2	0.42
d (ft)	0.36	0.16	0.057	0.011	0.0014
τ_c (psf)	1.44	0.63	0.23	0.042	0.0055
w(fps)	8.13	5.38	3.21	1.18	0.148

The sediment-transporting characteristics of the flows can then be evaluated by the use of the equation

$$\bar{c} = \sum p\left(\frac{d}{y}\right)^{7/6} \left(\frac{\tau_o'}{\tau_c} - 1\right) f\left(\frac{\sqrt{gyS}}{w}\right)$$

from which both the concentration and composition of the total sediment load can be obtain ($\bar{c}$ is percent by weight). (For a full explanation of the relationship see "The Total Sediment Load of Streams" in the Appendix.) The computations are summarized in Tables III and IV.

What do these computations imply? First, it is obvious that most of the sediment load is derived from the finest fraction of the bed material. This finding is not unusual--rather it is typical--and the 1.5% by weight concentration for a unit discharge q = 80 cfs/ft is also to be expected for a stream such as this. If the flow could get at this finest fraction and maintain a concentration of 1.5% by weight, it could sweep out the finest 20% of the six-foot thick surface layer for a distance of 1000 feet in 16 minutes. However, it cannot get at all of the finest fraction because the fraction does not move out at the same rate; it would take over five days to move the coarsest 20% if the conditions (depth, velocity, sediment, etc.) stayed as calculated. The bed material would coärsen, the sediment load would reduce by a factor of 10, but at rates of flow of 60, 80, 100 cfs/ft all or almost all the surface layer would be removed for a considerable distance downstream from the spillway. It is possible that the very coarsest of the coarsest 20% would remain, but probably not enough to armour the stream bed.

As the streambed in the first short reach below the spillway is degraded (lowered) the depth of flow is increased and the velocity is decreased for the same unit discharge. This decreases the capacity of the flow to transport sediment and decreases the rate of degradation, but this tendency does not last. As the degraded reach lengthens, the water surface

Table III. Computation of Total Sediment Load for Surface Layer

d(mm)	p	$(d/y)^{7/6}$	$(\tau_o'/\tau_c - 1)$	$f(\sqrt{gyS}/w)$	$\bar{c}$(% by weight)	% of total load
				$q = 20$ cfs/ft, $d_m = 2.1$ mm		
33	0.2	0.0224	0	6.5	0	0
9.2	0.2	0.00493	0.86	7.9	0.007	1.1
2.1	0.2	0.000895	7.21	13	0.017	2.7
0.68	0.2	0.000239	24.3	40	0.046	7.4
0.25	0.2	0.0000746	67.6	550	0.555	88.8
					0.625	
				$q = 40$ cfs/ft, $d_m = 2.1$ mm		
33	0.2	0.0138	0	7.0	0	0
9.2	0.2	0.00304	1.86	8.2	0.009	1.0
2.1	0.2	0.000551	11.6	14	0.018	1.9
0.68	0.2	0.000147	37.9	42	0.047	5.0
0.25	0.2	0.000046	104.2	900	0.863	92.1
					0.937	
				$q = 60$ cfs/ft, $d_m = 2.1$ mm		
33	0.2	0.0105	0.18	7.3	0.003	0.2
9.2	0.2	0.00229	2.65	8.8	0.011	0.8
2.1	0.2	0.000417	15.1	15.5	0.020	1.6
0.68	0.2	0.000111	48.5	62	0.067	5.5
0.25	0.2	0.0000347	133	1200	1.108	91.7
					1.209	
				$q = 80$ cfs/ft, $d_m = 2.1$ mm		
33	0.2	0.00853	0.235	7.5	0.003	0.2
9.2	0.2	0.00187	3.34	9.0	0.011	0.7
2.1	0.2	0.000341	18.1	18	0.022	1.5
0.68	0.2	0.000091	58	80	0.084	5.7
0.25	0.2	0.0000284	158	1500	1.346	91.7
					1.466	
				$q = 100$ cfs/ft, $d_m = 2.1$ mm		
33	0.2	0.0072	0.37	7.9	0.004	0.2
9.2	0.2	0.00159	3.91	10.0	0.012	0.8
2.1	0.2	0.000288	20.6	19	0.023	1.4
0.68	0.2	0.000077	65.6	90	0.091	5.6
0.25	0.2	0.000024	179	1750	1.504	92.0
					1.634	

OBSERVATIONS

E.M. Laursen

Observations on the Nature of Scour

by

EMMETT M. LAURSEN

Iowa Institute of Hydraulic Research

STATE UNIVERSITY OF IOWA

Iowa City, Iowa

STATE UNIVERSITY OF IOWA
REPRINTS IN ENGINEERING
REPRINT NO. 123

Reprinted from

PROCEEDINGS OF THE FIFTH HYDRAULICS CONFERENCE

Bulletin 34, State University of Iowa Studies in Engineering

1952

OBSERVATIONS ON THE NATURE OF SCOUR

By

EMMETT M. LAURSEN

Iowa Institute of Hydraulic Research, Iowa City, Iowa

FUNDAMENTAL CHARACTERISTICS OF SCOUR

Within the framework of our knowledge of physical phenomena, certain general characteristics of the process of scour can be formulated. Since in many areas the framework exists only in broad outline, our insight into the more detailed features of scour must depend on experimentation. These details, however, must be consistent with the general characteristics; and the formulation of general characteristics, therefore, establishes an outline for the interpretation of experimental observations.

Scour can be defined as the enlargement of a flow section by the removal of material composing the boundary through the action of the fluid in motion. Implicit in this definition is the fact that the moving fluid exerts forces on the particles composing the boundary, causing their movement. The amount of material which the fluid can move or transport, in unit time, is termed the capacity of the flow.

If the principle of conservation of matter is accepted, the local rate of scour is equal to the difference between the rate of removal and the rate of supply. Moreover, since no distinction is made between the material supplied and the material scoured, the rate of removal must equal the local capacity of the flow. Application of this concept to the more detailed characteristics of the scour process is possible if certain assumptions regarding the flow conditions at the boundary are made.

Even if the bulk flow is constant with time, the change in the boundary configuration because of scour results in unsteady flow conditions along the boundary. In general, the enlargement of the flow section will result in a reduction of velocity along the boundary and, therefore, a reduction in capacity for transport. The rate of scour must then decrease as the difference between the capacity and the rate of supply decreases. Implicit in the foregoing statement is the notion of a limiting extent of scour. The rate of scour will equal zero when the capacity is exactly equal to the supply. That a limit exists, for which the rate of scour is equal to zero, can be deduced with the aid of two further assumptions.

The premise that the velocity decreases as the flow section enlarges can be expanded to require that the velocity becomes zero when the boundary extends to infinity. If the rate of flow is finite this assumption is assuredly acceptable, and is sufficient to prove that a limit exists for the case in which material is supplied to the scoured area. If the capacity decreases with the velocity then there must be some finite boundary position for which the capacity equals the supply. This position will be the limit to the extent of scour. For the case in which there is no supply to the scour hole an additional assumption is needed — that below some critical velocity the capacity is zero. There must then be some finite boundary position for which the velocity decreases to this critical value and the rate of scour becomes zero. For the case of no supply, this position satisfies the notion of limit.

Establishing the existence of a limit to the extent of scour gives no indication as to the time necessary to attain the limit. That the limit must be approached asymptotically can be shown. If the limit were to be reached in finite time, the scour must continue beyond the limit, or deposition (negative scour) must occur after the limit is reached, or the scour process must be described by two functions, one before and one after the limit. None of these possibilities is admissible. Deposition would require a two-valued relation between the capacity and the difference between the actual and the limiting boundary. Continued scour is not compatible with the concept of limit. Unless some new force is added there is no reason why the scour function should change at the limit. If the limit is approached asymptotically, however, no matter how small the difference between the actual and limiting boundary, there is always a small rate of scour so that the limiting position is approached more closely. Not until the limit is reached at infinite time does the rate of scour become zero. This process is orderly and continuous.

To recapitulate, the following general characteristics which should be basic to any detailed analysis of local scour have been deduced:

1. The rate of scour will equal the difference between the capacity for transport out of the scoured area and the rate of supply of material to that area.

2. The rate of scour will decrease as the flow section is enlarged.
3. There will be a limiting extent of scour.
4. This limit will be approached asymptotically.

The premises necessary to form these characteristics are a definition of scour, the principle of conservation of matter, and two restrictive assumptions that describe the kind of process the scour phenomenon is expected to be. The two assumptions, which are not overly restrictive but rather credible in the light of general knowledge of fluid mechanics and sediment transportation, are:

1. The movement of the fluid along the boundary ceases when the boundary extends laterally to infinity.

2. The capacity of the flow decreases in a single-valued continuous relation as the flow section is enlarged, and decreases to zero before the movement of the fluid ceases.

Nothing has been said or need be said, at this stage, about the mechanism of transport or the method of supply. It should be kept in mind, however, that only conditions at the boundary are under consideration.

APPLICATION OF FUNDAMENTAL PRINCIPLES

By using symbolic terms the first general characteristic can be written as an equation of scour,

$$\frac{d}{dt}[f(B)] = g(B) - S \tag{1}$$

where B is a mathematical description of the boundary, so that

$\frac{d}{dt}[f(B)]$ is the rate of scour,

$g(B)$ is the capacity of the flow as a function of the boundary position, and

S is the rate of supply.

To apply this equation to a specific situation, the rate of supply and the capacity of the flow (as it varies with the boundary position) must be known. If the relation between capacity and the velocity distribution near a boundary were known exactly and if methods were available to specify the flow pattern for any boundary condition, the equation of scour could be used to solve any scour problem. For most instances such a relation and such methods are not available and recourse must be had to approximations and experimentation.

How the solution for a specific instance of scour could be obtained within this general framework can be seen by an examination of a paper by Straub [1]. The specific situation was the equilibrium depth obtaining in long channel contractions. The procedure was equivalent to setting $\frac{d}{dj}[f(B)] = 0$ and solving for the limiting condition of scour. At the limit the capacity for transport in the contracted, scoured section must equal the rate of supply — which is equal to the capacity for transport in the uncontracted section. The contraction was sufficiently long that essentially uniform flow was established.

The flow was described by Manning's formula with the same value of n in the contracted and uncontracted sections,

$$Q = \frac{1.49}{n} W_1 d_1^{2/3} S_1^{1/2} = \frac{1.49}{n} W_2 d_2^{2/3} S_2^{1/2} \qquad (2)$$

and the capacity was described by a sediment transport formula of the DuBoys type with coefficients experimentally determined,

$$g(B) = W_1 \psi \gamma d_1 S_1 (\gamma d_1 S_1 - \tau_e) = W_2 \psi \gamma d_2 S_2 (\gamma d_2 S_2 - \tau_e) = S \quad (3)$$

Upon combining Eqs. (2) and (3), an equation for depth of flow in the contracted portion is formed,

$$d_2 = \left(\frac{d_1}{(1-\beta)^{3/7}}\right) \left\{ \frac{\frac{-\tau_e}{\gamma S_1} + \left[\left(\frac{\tau_e}{\gamma S_1}\right)^2 + \frac{4d_1\left(d_1 - \frac{\tau_e}{\gamma S_1}\right)}{1-\beta}\right]^{1/2}}{2\left(d_1 - \frac{\tau_e}{\gamma S_1}\right)} \right\}^{3/7} \quad (4)$$

In these and the following equations,

Q and Q_s are the rates of flow of water and sediment

ψ and τ_e are constants

S_1 and S_2 are the slopes in uncontracted and contracted sections,

β is the contraction so that $W_2 = (1-\beta) W_1$, and

d_1 and d_2 are the depths in uncontracted and contracted sections.

Experiments in the laboratory confirmed this relationship very closely. Such confirmation could be expected since the coefficients in the transport equation were determined under similar conditions. Whether Eq. (4) will apply to field conditions depends not on Eq. (1), which must be valid, but on Eq. (2) and especially Eq. (3), which are approximate. Equation (4), with τ_e equal to zero, is very

nearly Griffith's equation [2], which is based on field observations,

$$d_2 = d_1 \left(\frac{W_1}{W_2}\right)^{0.857} \qquad (5)$$

This agreement between analysis, laboratory, and field indicates that even approximate knowledge of transport and flow conditions can lead to useful results.

Scour by a Submerged Jet

Under the sponsorship of the Office of Naval Research, the Iowa Institute is conducting an investigation of the effect of sediment characteristics on the scour process. A submerged jet is being used as the active scouring agent, and the flow as well as the sediment characteristics is being varied. For uniform sands it has been found possible to analyze this scour process by means of Eq. (1) and the necessary empirically determined relationships.

The experimental boundary conditions are shown schematically in Fig. 1. It was found necessary to restrict the pendulation of the jet by the lip shown at the upper edge of the slot. The sand profile was obtained by photographing with back lighting, and the time of observation was recorded by the including of a clock in the photograph. A typical series of profiles is superposed on Fig. 1. Sands having the characteristic curves designated as M, A, and B in Fig. 2 were used.

Similarity of scour profiles was established by plotting the dimensionless coordinates of the profiles; the horizontal distance from the slot to the crest of the dune was used as the repeating variable (Fig.

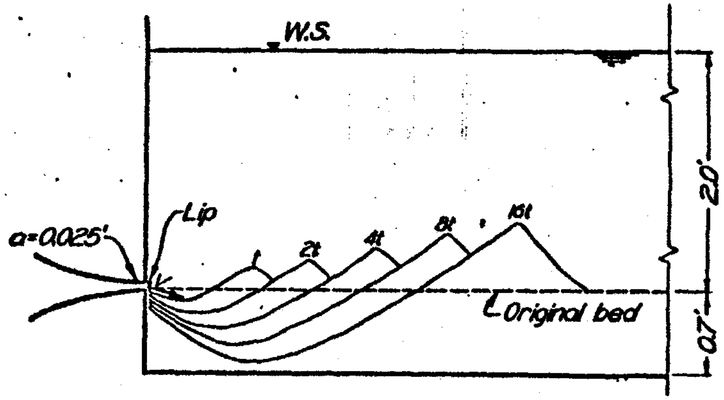


Fig. 1. Schematic of Experimental Equipment for Scour by Submerged Jet.

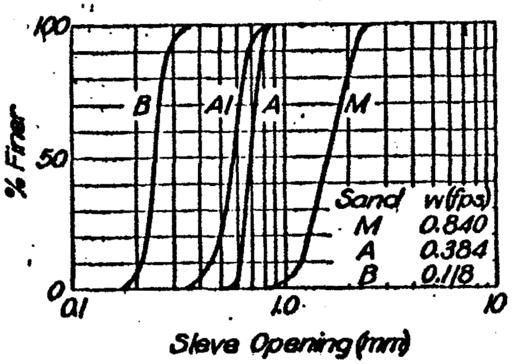


FIG. 2. SAND CHARACTERISTICS.

3), Except for the profiles representative of an initial transitory stage of scour, all the profiles of a run superpose if plotted in this manner. Moreover, the profile forms for all runs of any one sand were almost identical and the forms for the various sands differed only slightly.

At the beginning of each run the sand was moved as bed load. During this transitory stage, the vertical dimensions of scour hole and dune increased faster than the horizontal dimensions. When the upstream face of the dune reached the natural angle of repose, the mechanism of transport changed to suspension. The sand was then entrained by the flow, largely at the point of impingement, and was lifted in suspension by the upward currents of the flow. Some of the sediment was returned to the scour hole by the large counter-clockwise eddy shown in Fig. 4. The greater part of the sediment was deposited on the upstream face of the dune and slumped back into the scour hole. The sand removed from the scour hole was con-

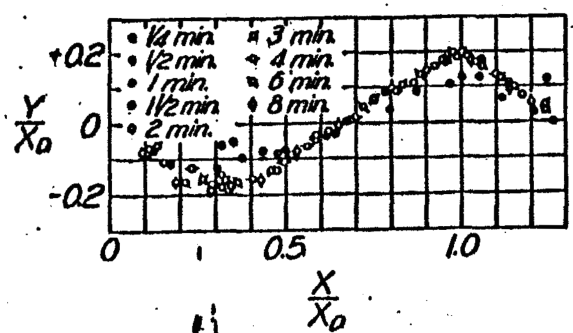


FIG. 3. SIMILARITY OF SCOUR PROFILES.

fined to that portion which deposited downstream from the crest of the dune.

Having established similarity of profiles it was sufficient to examine one typical dimension in the investigation of the variation of the extent of scour with time. The variation of the extent of scour with time is plotted in Fig. 5, the distance x_D (see Fig. 4)

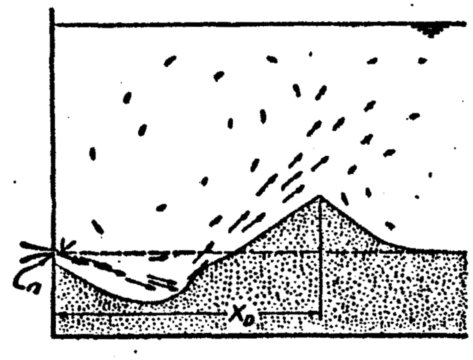


FIG. 4. FLOW PATTERN OF SUBMERGED JET.

having been chosen as a typical length. After the transitory stage it can be seen that the plotted points scatter around the straight lines with the logarithm of a time parameter as abscissa. This would seem to indicate that the extent of scour would become infinite as time became infinite — in contradiction to characteristic No. 3 of the first section, that there must be a limit to the extent of scour.

By reducing the velocity of flow after a scour hole had developed, conditions could be imposed so that the sand particles rarely moved at the point of impingement, and movement of any particle over the crest was hardly conceivable. An increase in velocity would increase the amount of movement. The limiting velocity for any given size of scour hole was arbitrarily defined as the velocity which appeared to carry particles to, but not over, the crest, during a period of observation of several minutes. The points in Fig. 6 were thus obtained. To provide a distinguishing notation, x_D at the limit has been called x_L. A limit such as indicated in Fig. 6 and a relationship between extent of scour and time as indicated in Fig. 5 can both exist only if the true function is approximately logarithmic over a considerable range and yet approaches a finite limit. That such a function can describe the phenomenon will be shown.

In order to obtain an independent measure of the capacity func-

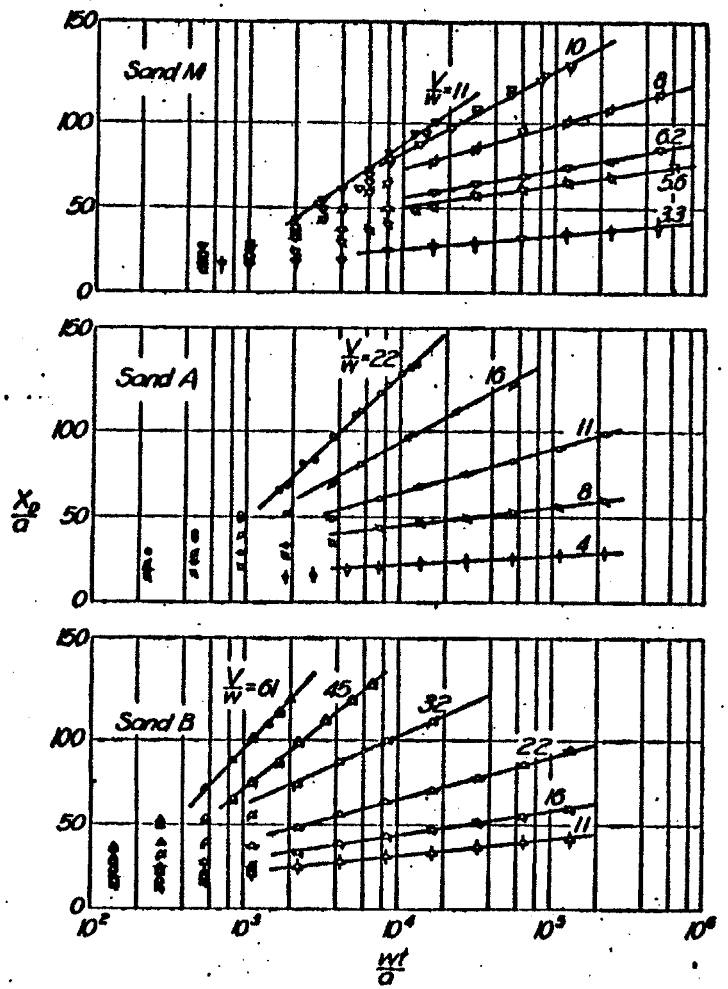


FIG. 5. VARIATION OF EXTENT OF SCOUR WITH TIME.

tion, a sand hopper was added to the experimental equipment. The same experimental procedure was used as before, except that, above the slot, sand was supplied at a measured rate. A scour hole and dune formed as before and the pattern was essentially the same as without the sand feed. The dune advanced downstream until the rate of removal of sand from the scour hole equaled the sediment supply. The position of the upstream face of the dune then moved upstream. The effect of the boundary configuration on the capacity of the flow was determined by relating each rate of sediment sup-

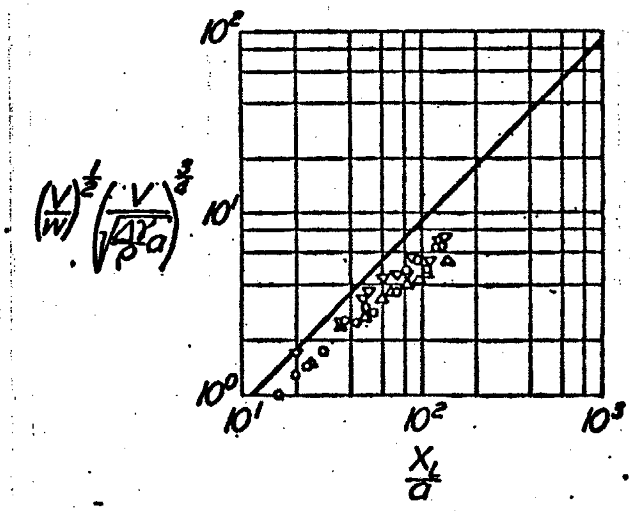


FIG. 6. LIMITING EXTENT OF SCOUR.

ply to the position of the crest of the dune when the upstream face was at its maximum distance from the slot. By varying the velocity and by using a second sand the composite plot of Fig. 7 was obtained, thereby determining a capacity function,

$$\frac{Q_s}{b} = \frac{K_s V^{6.5}}{w^2 x_D{}^4} \tag{6}$$

in which b is the width of the channel, K_s is a dimensional constant, and w is the fall velocity of the sediment.

Since the similarity of shape had been established, the rate of

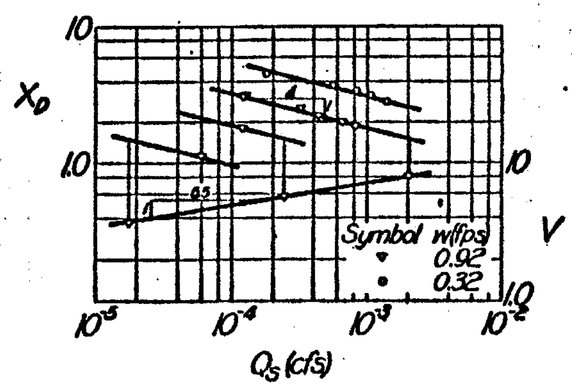


FIG. 7. CAPACITY OF SUBMERGED JET.

scour per unit width could be written as:

$$\frac{d}{dt}[f(B)] = \frac{dA}{dt} = K_s\frac{d(x_D^2)}{dt} \qquad (7)$$

in which the coefficient K_s for the different sands is 0.14 for sands M and A, and 0.15 for sand B. These coefficients were determined from similarity plots such as Fig. 3. By modifying the capacity function to include a limit x_L for which capacity would be zero, and inserting Eqs. (6) and (7) in Eq. (1) a differential equation of scour was obtained,

$$K_s\frac{d(x_D^2)}{dt} = \frac{K_t V^{6.5}}{w^2}\left(\frac{1}{x_D^4}-\frac{1}{x_L^4}\right) \qquad (8)$$

This equation can be integrated to give,

$$f\left(\frac{x_D}{x_L}\right) = -\left(\frac{x_D}{x_L}\right)^2 + \frac{1}{2}\ln\left[\frac{1+(x_D/x_L)^2}{1-(x_D/x_L)^2}\right] = \frac{K_t V^{6.5}}{K_s w^2 x_L^6}t \qquad (9)$$

As is seen from Fig. 8, the function $f(x_D/x_L)$ has the desired characteristics of approximating, over a considerable range, a straight line with semi-logarithmic plotting and also being asymptotic to a limit.

Agreement between the above theory and experiment was realized when values of x_L from the straight line on Fig. 6 were used in Eq. (9). That the x_L values to be used in Eq. (9) are smaller than those determined by experiment might be expected, because Eq. (6) is a simple approximation of the capacity function. The effect of turbulence is not fully included therein — especially at the limit for

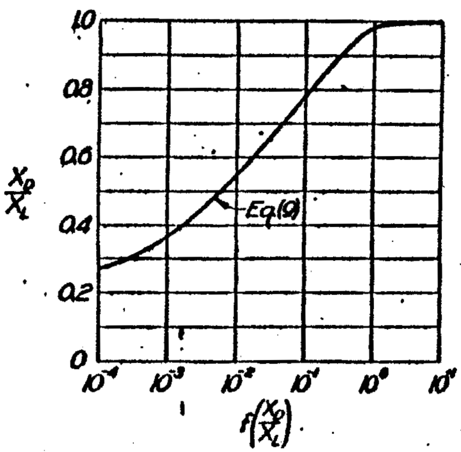


FIG. 8. GRAPHICAL REPRESENTATION OF EQ. (9).

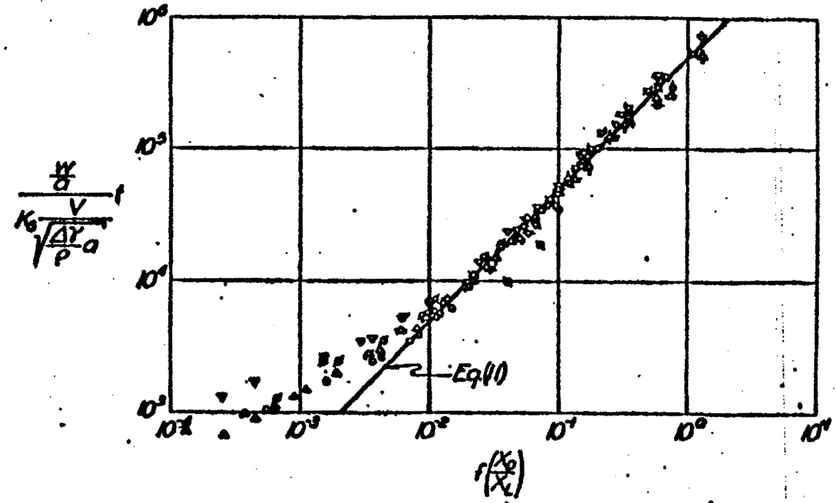


FIG. 9. VERIFICATION OF EQ. (11).

which occasional turbulent velocities, but not the mean velocity, can be sufficient to enlarge the scour hole. If the equivalent expression for x_L,

$$x_L = c_1 a\left(\frac{V}{w}\right)^{1/3}\left(\frac{V}{\sqrt{\frac{\Delta\gamma}{\rho}a}}\right)^{3/4} \qquad (10)$$

is substituted in Eq. (9), a new set of parameters for the coefficient of t is obtained, and Eq. (9) takes the form:

$$f\left(\frac{x_D}{x_L}\right) = \frac{K_s}{c_1^6}\,\frac{(\Delta\gamma/\rho)^{7/4}}{a^{13/4}}\;\frac{w/a}{K_s V/\sqrt{a\Delta\gamma/\rho}}\,t \qquad (11)$$

$$= c_2\frac{w/a}{K_s V/\sqrt{a\Delta\gamma/\rho}}\,t$$

This equation is a mean line through the points in Fig. 9. The points off the lower end of the line are from high-velocity runs of short duration. Subtraction of a time for flow establishment would bring these points closer to the theoretical curve. A combination of Figs. 8 and 9 results in Fig. 10, wherein the extent of scour, instead of a function of the extent, is plotted against time.

SCOUR AROUND BRIDGE PIERS AND ABUTMENTS

Under the sponsorship of the Iowa State Highway Commission and the Bureau of Public Roads, the Iowa Institute is conducting

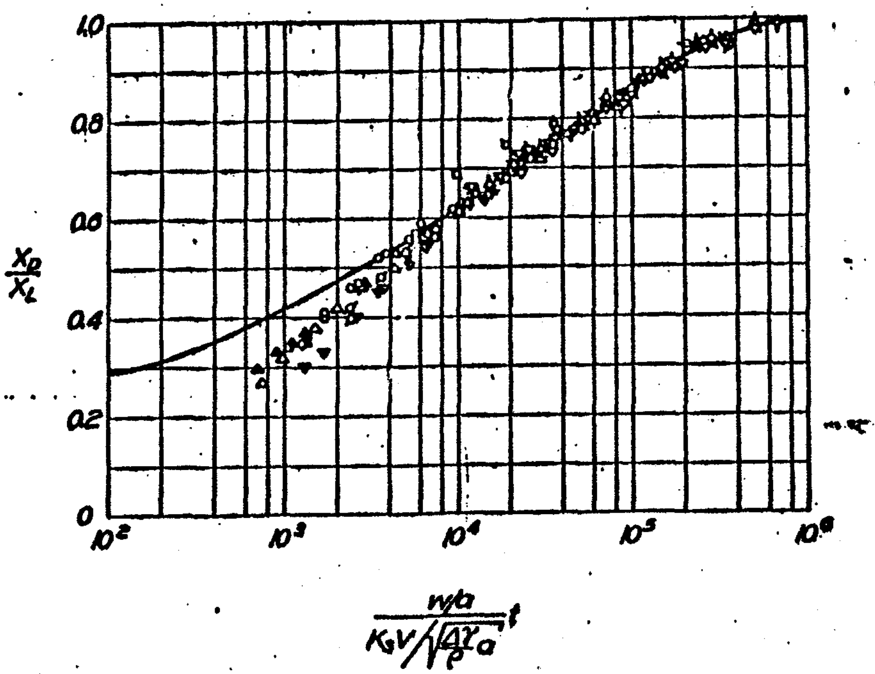


Fig. 10. Extent of Scour Relative to Limiting Extent as a Function of Time.

a comprehensive study of the bridge-pier scour problem. The results of the first two phases of this study have been reported in detail elsewhere [3, 4] and only those results pertinent to the general scour problem will be stressed here.

If an obstruction, such as a bridge pier, is placed in a natural sediment-bearing stream, the flow pattern in the immediate vicinity of the obstruction is greatly modified. Since the capacity for sediment transport is dependent largely upon the velocity at the level of the particles in motion on the bed, the transport capacity at points near the obstruction will also be modified. As a result of the variation in capacity in the vicinity of the pier, scour will occur where capacity exceeds supply. The enlargement of the boundary caused by the scour will further modify the flow pattern — and in turn the scouring action — continually approaching a limiting, or equilibrium, flow and boundary condition.

For the study of the effects of velocity, depth of flow, and sediment size, a transport flume, with a trap for determining the rate

of sediment transport and an elevator for adding sand at that rate, was used. A typical pier with a web set at an angle of 30° to the flow has been used in all these experiments to date. The range of tests has included depths of flow from 0.2 to 0.9 foot and velocities from 1.0 to 2.25 fps. The lower limit on the velocity was the requirement of appreciable bed-load movement; the upper limit was imposed either by an approach to critical flow in the flume or by sand jumping the trap. The variation of velocity and depth resulted in a fifty-fold variation in the rate of sediment transport. Sands A-1 and M of Fig. 2 have been tested.

The relation between equilibrium depth of scour and the velocity, depth of flow, and sediment size is indicated in Fig. 11. The effect, if any, of velocity and sediment size is so small as to be within the precision of the measuring instrument. The effect of depth of flow is considerable, although the relationship is not one of direct proportionality.

These experimental results can be rationalized in the light of the general characteristics of the scour process. For the velocity to have no effect on the equilibrium depth of scour, the capacity for transport of the spiral roller at that scour depth must always equal the rate of sediment supply as furnished by the bed-load movement in the flume. This can be true only if the capacity of the roller and the capacity in the flume bear the same relation to the mean velocity of flow. In the scour hole, the velocity at the grain level is a function

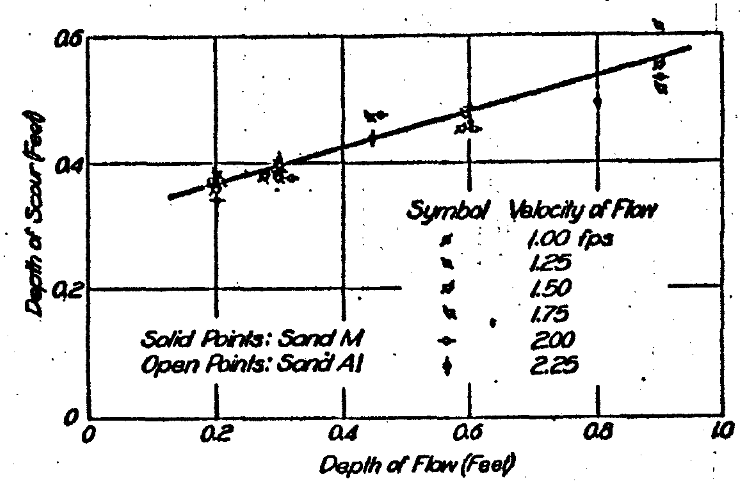


Fig. 11. Transport Balance at Equilibrium Depth of Scour.

of the velocity of the roller; in the flume, it is a function of the mean velocity. The velocity of the roller, as a first approximation, could be expected to bear a constant ratio to the mean velocity of flow. Therefore, the velocity at the level of the moving grains in the scour hole and in the flume should result in balanced capacities no matter what the absolute magnitude of the mean velocity. Essentially the same argument will explain the lack of change of scour depth with a change in the size of the uniform sand.

A similar analysis will show that the depth of scour should increase with the depth of flow. As a result of the vertical velocity distribution, the velocity at the level of the moving sand grains in the flume will decrease as the depth of flow increases. Since this velocity is the active agent of transport, the rate of sediment transport in the flume will also decrease. For an equilibrium depth of scour to obtain, the capacity of the roller must similarly decrease. The vertical velocity distribution has only a secondary influence on the roller velocity. The roller must, therefore, increase in size to be reduced sufficiently in velocity, and thereby in capacity — resulting in a greater depth of scour for greater depths of flow.

The rationalization in respect to the effect of velocity on the equilibrium depth of scour was confirmed by a series of tests on rate of scour. For this series the sand bed of the flume was replaced by bricks except in the vicinity of the pier. Transport into the hole was thus minimized. The rate of scour as a function of the depth of scour is shown in Fig. 12. The scour rate at small depths of scour should be disregarded because of the unsteadiness of flow during establishment, and at depths approaching the equilibrium because transport into the hole becomes significant. If the middle portion of each rate-of-scour curve is extrapolated, the capacity of the roller at the equilibrium depth is found to be equal to the transport into the hole under normal transport conditions.

Two qualifications, implicit heretofore, limit this analysis of bridge-pier scour. The lower limit is expressed by the requirement for general bed-load movement. As flow conditions approach the critical for sediment movement, the turbulence structure assumes greater importance. The upper limit is expressed by the requirement for sub-critical flow ($F<1$). The flow patterns for Froude numbers greater and less than unity will be markedly different.

Large scale experiments are needed to explore fully the effect of depth of flow on the equilibrium depth of scour. The depth of flow

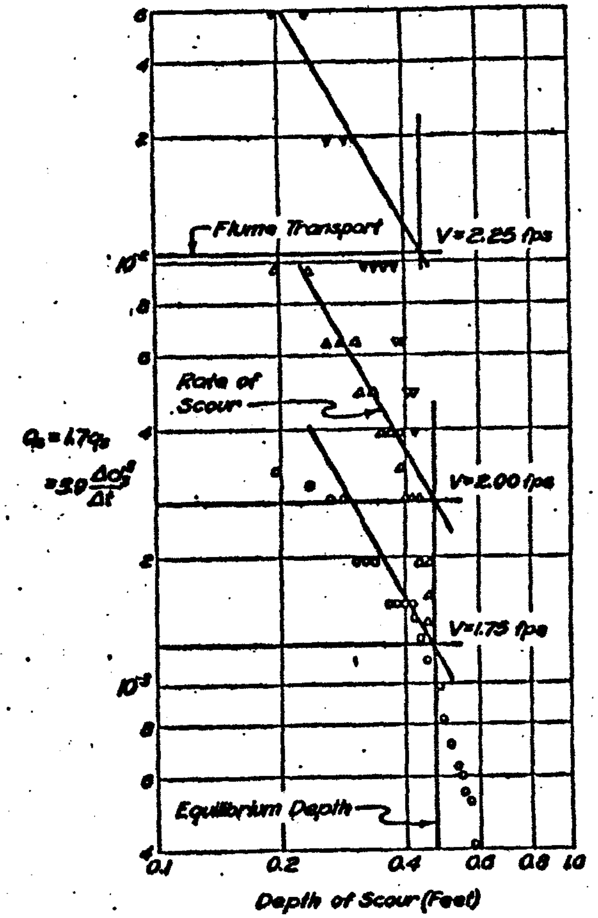


FIG. 12. DEPTH OF SCOUR AS A FUNCTION OF VELOCITY, DEPTH OF FLOW AND SEDIMENT SIZE.

should not be considered merely a geometric length variable. Indeed, its influence does not appear to be direct, but indirect through its influence on the relationship between the transport capacity in the unobstructed stream and in the scour hole. The dynamics of the flow and the state of the bed may, therefore, have secondary influences implicit in the effect of the depth of flow. However, if the qualifications mentioned above are met, the velocity of flow and sediment size, insofar as they determine the absolute rate of transport, should not affect the equilibrium depth of scour no matter what the scale. It is this simplification which gives promise that a correlation between field and laboratory may be possible.

CONCLUSIONS

By reasoning from a few premises, several general characteristics of scour were demonstrated — that the rate of scour is equal to the difference between the capacity of flow and the rate of supply, that the rate of scour decreases as the extent of scour increases, and that there is a finite limit to the extent of scour which is approached asymptotically. The first of these characteristics must be true of all scour phenomena. The others are true if the flow pattern complies with the restrictive assumptions made. For most cases of scour, the assumptions are credible. Other assumptions, of course, might be adopted to describe the flow pattern in some special circumstances. The deduction of similar characteristics should then follow.

The re-examination of Straub's solution for the depth of flow in a long contraction showed the assumed characteristics to be in accord with a particular case of scour for which laboratory and field measurements were already available.

The effectiveness of these principles in an original analysis was then demonstrated for the case of scour by a submerged jet. By using an approximate experimental capacity function the differential equation of scour was integrated to give a relationship between the extent of scour and time. Experimental data confirmed the analysis.

The usefulness of the general concepts in interpreting experimental results was illustrated for the case of scour around bridge piers and abutments. The experimental observations that velocity of flow and sediment size had no measurable effect on the equilibrium depth of scour were rationalized from the premise that the transport capacities in the unobstructed flume and in the scour hole have the same relationship to the mean velocity of flow. The equilibrium then does not depend on the absolute rate of transport, but only on a balance between the two capacities. The same reasoning was used in explaining why the depth of flow had an influence on the equilibrium depth of scour. Although this influence cannot even be approximately expressed from the limited range of the experminents, large-scale experimentation should provide data for such an expression. The simplification of the problem that has been obtained indicates that correlation between field and laboratory should be possible since the absolute rate of transport does not need to be scaled.

In essence, the formulation of the general characteristics transforms the scour problem into the problems of the determination of flow patterns and of the relation between the flow pattern and the transport capacity of the flow. Although these two problems usually depend upon experimental determination, the general concepts provide a guide for interpretation and analysis.

DISCUSSION

Mr. Parsons said that Mr. Laursen has employed straightforward reasoning in the analysis of the scour problem. A logical concept of the overall process of scour has been used as the basis for the testing program and for orderly determination of the associations of variables that enter into the mechanics of the scour process. The first assumption or deduced characteristic that the rate of scour is equal to the capacity of the flow at the spot under consideration less the rate of supply appears to be rather a definition of capacity.

He called attention to a tacit assumption made in the analysis which is undoubtedly correct for the conditions of the experiments. This assumption is that the fall velocity is the correct representation of the effect of qualities of the bed material. This is correct so long as the mode of transport is by suspension and that suspension is due to vertical components of the flow. He felt that if transport is occurring in close proximity to the bed, investigators should seek a more realistic expression of the pertinent qualities of the material being transported. However, even here the fall velocity may be a sufficiently close approximation.

Mr. Parsons believed that Mr. Laursen had deduced in a convincing manner that there is a finite limit to the extent of scour. Furthermore, he had devised and performed ingenious tests to prove it and to measure the scour at the limit.

Mr. Rouse wished to emphasize the point that neither the velocity of flow nor the size of sediment affects the scour around bridge piers. This significant conclusion, which results from the interrelated dependence on mean velocity of the transport both on the stream bed and in the scour hole, leads to a marked simplification of a complex problem.

Mr. Jetter expressed his interest in the work of Yarnell and Nagler in determining bridge coefficients. He once asked Yarnell what would be the effect of debris caught against the bridge piers on the discharge coefficients. Future studies could be made along the line of the effect of debris on scour and on the design of piers to minimize debris catchment.

Mr. Izzard said that practicing engineers are aware of the drift, or debris, problem, but feel that the general problem must be attacked one factor at a time. There are a number of variables which affect the problem; as some are understood, others will be added. The relationship of the bridge opening to the total flood plain is a geometric variable that may be important. The work which Kindsvater is doing for the U. S. Geological Survey indicates the importance of this part of the general problem even though his work is confined to a fixed bed. There is some indication that the amount of backwater caused by a bridge opening is not changed by piers in the opening. The losses from constriction at the bridge seem to be much more important than the pier losses studied by Yarnell.

Mr. John Dawson said that research performed under his supervision indicated an appreciable movement in the sand two inches below a sand bed in movable beds.

Mr. Albertson reported on a study in his laboratory of the scour resulting from solid and hollow jets directed vertically downward onto a bed of erodible material. With low tailwater elevations the scour hole was small. It increased as the tailwater increased, reaching a maximum and then decreasing. This was contrary to his expectations, but he believed that it was due to changes in secondary circulation.

Mr. Laursen said in conclusion that considerable experimentation remains before the questions raised by the discussors can be answered. As Mr. Rouse pointed out, the absence of effects of velocity and sediment size on the equilibrium depth of scour around bridge piers is a big step in predicting scour in the field from model studies. However, the effect of depth of flow is very probably not a simple geometric effect but may be related to the boundary layer. Furthermore, the sorting that can occur with natural non-uniform sediments may have a great importance in the field. Mr. Izzard has mentioned another important consideration of the bridge scour problem — the constriction effect. This is likely to be especially important if the floodplain carries a high percentage of the flow but not an equivalent sediment load. Scour may also be a factor in the discharge relationship which Kindsvater is studying. Mr. Jetter's question as to the effect of debris cannot be answered at this time.

Debris would certainly change the geometry of the pier. This effect will be studied as the program continues.

Movement as deep as reported by Mr. Dawson has not been noted. In the case of the bridge pier, the shifting moving sand was only a few grain diameters in thickness. In the case of the submerged jet the layer of moving sand was thicker where the jet impinged.

REFERENCES

1. Straub, L. G., "Approaches to the Study of the Mechanics of Bed Movement," *Proc. of Hydraulics Conference*, Univ. of Iowa Studies in Engineering, Bull. 20, 1940.
2. Griffith, W. M., "A Theory of Silt Transportation," *Trans. A.S.C.E.*, Vol. 104, 1939, pp. 1733-1748.
3. Laursen, E. M., "Progress Report of Model Studies of Scour Around Bridge Piers and Abutments," Research Report No. 13-B, Highway Research Board.
4. Laursen, E. M., and Toch. A., "Model Studies of Scour Around Bridge Piers and Abutments — Second Progress Report," Proceedings of 31st Annual Meeting, Highway Research Board, 1952.

SCOUR

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

TRANSACTIONS

Paper No. 3294
(Vol. 127, 1962, Part I)

SCOUR AT BRIDGE CROSSINGS

By Emmett M. Laursen,[1] M. ASCE

With discussion by Messrs. T. Blench; Joseph N. Bradley; D. V. Joglekar; W. J. Bauer; L. J. Tison; S. V. Chitale; A. Rylands Thomas; Mushtaq Ahmad; Pier Luigi Romita; and Emmett M. Laursen.

SYNOPSIS

Relationships are proposed for the prediction of scour at piers and abutments for the case in which sediment is supplied to the scour hole. The relationships were obtained from a combination of an approximate analysis and laboratory experiments, and depend on knowledge of the flow conditions at the bridge site.

GENERAL ASPECTS OF THE PROBLEM

In order to design the foundations of a bridge over an alluvial stream, it is necessary to know the elevation of the stream bed in the vicinity of the piers and abutments. The elevation in question, however, is not the elevation at the time a survey happened to be made, but the lowest elevation which will occur during the anticipated life of the bridge. Unfortunately, engineers are equipped with slide rules rather than crystal balls; therefore this lowest elevation must be coupled with a probability of occurence. On this basis an economic analysis is possible with the cost of construction for one stream bed elevation lower than another considered as an insurance premium for the decreased chance of loss (including interruption of traffic).

Causes of Scour at Bridge Piers and Abutments.—A lowering of the stream bed in the vicinity of the piers and abutments can occur from a variety of causes. A useful distinction can be made by separating the various causes into two general categories; (1) those characteristic of the stream itself, and

Note.—Published essentially as printed here, in February, 1960, in the Journal of the Hydraulics Division as Proceedings paper 2369. Positions and titles given are those in effect when the paper or discussion was approved for publication in Transactions.

1 Assoc. Prof., Dept. of Civ. Engrg., Michigan State Univ., East Lansing, Mich.

(2) those due to the modification of the flow by the bridge crossing. For the past ten years scour due to the modification of the flow has been studied at the Iowa Institute of Hydraulic Research under the sponsorship of the Iowa State Highway Commission and the Bureau of Public Roads. As a result of that investigation means for predicting the scour at bridge piers have been evolved. The relationships which were obtained will be presented in skeleton form in a later section; for a full discussion of the investigation and the design relationships, reference should be made to the final reports which have been published in bulletin form by the Iowa Highway Research Board.[2,3]

Even without the complicating feature of a bridge crossing the bed and banks of an alluvial stream cannot be considered as fixed. Superposed on the very slow changes measured in geological time, there are changes which may occur suddenly in a single flood, periodically during a series of water years, or slowly during the life of a bridge. Past observations at the chosen site, or at similar sites, must be relied upon to estimate the scour which may occur in the future. Each river and each reach must be studied to understand its individual, almost personalized, characteristics.

The reach which includes the chosen bridge site may be degrading: naturally, as the erosional agent of the geological cycle, as a consequence of a dam some distance upstream, or as the result of stream straightening downstream. Old surveys and other records should yield evidence of natural degradation which can be used to predict the future. Degradation caused by the works of man can be analyzed, at least approximately, on the basis of sediment-transport relationships, that is, if the plans of man can be anticipated.

Meandering streams will tend to have the greatest depth at the outside of the bend and shallows between the bends. During high water, the bends will scour and the crossings fill; during low water, the roles will reverse. As the loops of the meanders grow, there may be natural cutoffs with resultant degradation of the reach above and aggradation of the reach below. Usually the meander grows, scouring the outside of the bend, filling the inside; it is not unusual during a flood, however, for a chute to develop across the inside of a bend and even replace the old channel.

Braided streams are characteristically wide and shallow, with deeper channels which may shift erratically. Even if it is not believed that "if anything can go wrong, it will" is a natural law, it must be admitted that there is a fair chance that during the life of a bridge a deep channel may develop at a pier at the time of a flood.

Any contraction, whether of the main channel or of the overbank flow, will result in scour during high stages. As shown by L. G. Straub,[4] the principle of continuity applied to both the discharge and the sediment load permit this case to be solved analytically since the flow conditions in both the contracted and uncontracted reaches can be considered *uniform*. It is likely that this case of scour has given rise to the old riverman's belief that during a flood a plains river will scour its bed as much as the water surface rises. That such

2 "Scour Around Bridge Piers and Abutments," by E. M. Laursen, and A. Toch, Iowa Highway Research Board Bulletin No. 4, May 1956.

3 "Scour at Bridge Crossings," by E. M. Laursen, Iowa Highway Research Board Bulletin No. 8, August 1958.

4 "Approaches to the Study of Mechanics of Bed Movement," by L. G. Straub, Proceedings, 1st Hydraulics Conference, State University of Iowa, Iowa City, Iowa, 1940.

scour cannot be general throughout the stream has been well demonstrated by E. W. Lane and W. M. Borland.[5]

The Long Contraction.—By describing the conditions in the uniform reaches above and in a long contraction by a discharge equation and a sediment-transport equation, it is possible to solve for the ratio of the depths of flow in the two reaches—and, therefore, the depth of scour. The solution will depend, in detail, upon the equations selected to describe the flow and transport. Fig. 1 is a definition sketch of the long contraction in which there is both channel and overbank contraction. The Manning formula can be used to describe the flow conditions

$$Q_c = \frac{1.49}{n_1} B_1 y_1^{5/3} S_1^{1/2} \quad (1)$$

and

$$Q_t = \frac{1.49}{n_2} B_2 y_2^{5/3} S_2^{1/2} \quad (2)$$

An approximate form of the total-sediment-load relationship recently proposed[6] can be used to describe the sediment concentration

$$\bar{c} = \left(\frac{D}{y}\right)^{7/6} \left(\frac{\tau_0'}{\tau_c} - 1\right) b \left(\frac{\sqrt{\tau_0/\rho}}{w}\right)^{a} \quad (3)$$

in which

$$\frac{\tau_0'}{\tau_c} = \frac{V^2}{120 y^{1/3} D^{2/3}} \quad (4)$$

and

$$\sqrt{\tau_0/\rho} = \sqrt{g y S} \quad (5)$$

That portion of the shear associated with the sediment particle of size D is termed τ_0' and τ_c is the critical shear for sediment movement. The coefficient b and the exponent a depend on the mode of sediment movement and are functions of the ratio of shear-velocity to fall-velocity. Only the exponent a is of importance in the final solution and the functional relationship can be expressed as:

a	$\sqrt{gyS}$ / w
1/4	< 1/2
1	= 1
9/4	> 2

For flood flow conditions, the assumption can be made without serious error that $\tau_0'/\tau_c - 1 = \tau_0'/\tau_c$. At equilibrium $Q_t = Q_c + Q_o$ and $\bar{c}_1 Q_c = \bar{c}_2 Q_t$; algebraic manipulation then permits the depth ratio to be expressed as a function of the contraction parameters

5 "River Bed Scour During Floods," by E. W. Lane and W. M. Borland, Transactions, ASCE, Vol. 119, 1954.

6 "The Total Sediment Load of Streams," by E. M. Laursen, Proceedings, ASCE, Vol. 84, No. HY1, February, 1958.

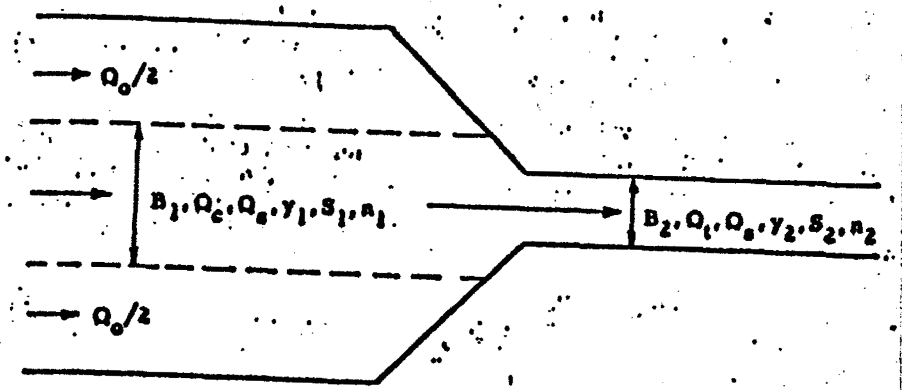


FIG. 1.—DEFINITION SKETCH OF LONG CONTRACTION

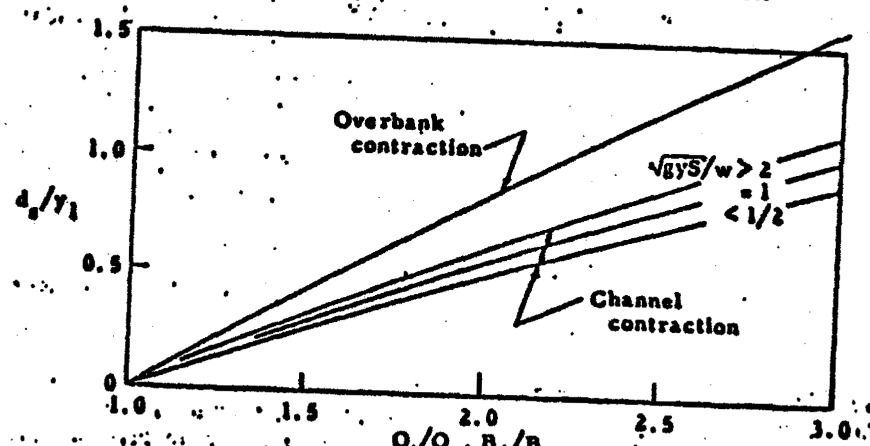


FIG. 2.—SCOUR IN A LONG CONTRACTION

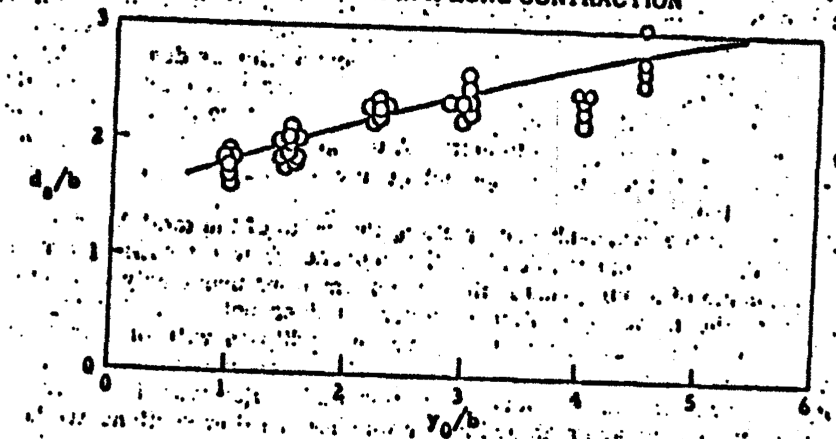


FIG. 3.—EQUILIBRIUM DEPTH OF SCOUR AT A PIER

$$\frac{y_2}{y_1} = \left(\frac{Q_t}{Q_c}\right)^{\frac{6}{7}} \left(\frac{B_1}{B_2}\right)^{\frac{6}{7}\,\frac{2+a}{3+a}} \left(\frac{n_2}{n_1}\right)^{\frac{6}{7}\,\frac{a}{3+a}} \quad \ldots\ldots\ldots (6)$$

Since the ratio of the n values should not be too different from unity and the power is at the most 0.37, this factor can be safely neglected. Assuming that the material scoured out during a flood is redeposited over a large area so that the depth of scour $d_s = y_2 - y_1$, the foregoing equation for an overbank construction reduces to

$$\frac{d_s}{y_1} = \left(\frac{Q_t}{Q_c}\right)^{\frac{6}{7}} - 1 \quad \ldots\ldots\ldots (7)$$

and for a channel contraction to

$$\frac{d_s}{y_1} = \left(\frac{B_1}{B_2}\right)^{0.59} - 1 \text{ for } \frac{\sqrt{gy_1S}}{w} < \frac{1}{2} \quad \ldots\ldots\ldots (8a)$$

$$\frac{d_s}{y_1} = \left(\frac{B_1}{B_2}\right)^{0.64} - 1 \text{ for } \frac{\sqrt{gy_1S}}{w} = 1 \quad \ldots\ldots\ldots (8b)$$

$$\frac{d_s}{y_1} = \left(\frac{B_1}{B_2}\right)^{0.69} - 1 \text{ for } \frac{\sqrt{gy_1S}}{w} < 2 \quad \ldots\ldots\ldots (8c)$$

These relationships are presented graphically in Fig. 2.

The case of the long contraction is of interest not only for its own sake, but for the light its solution sheds on the nature of scour in rivers. The shear-velocity in the contracted and uncontracted reaches of a particular stream should not be very different; so that for a given mode of sediment movement the depth of scour for a channel contraction is, as a first approximation, dependent only on the geometry of the contraction as described by the width ratio and the approach depth. The assumptions which give rise to the approximation are that the n value is the same in the two reaches, and that the ratio τ_0'/τ_c is large compared to unity. Although reasonable and justifiable, these assumptions result in the neglect of any secondary effect of velocity and sediment size.

In the case of the overbank constriction, the ratio of the total discharge to the channel discharge and the depth of the approach flow are sufficient to determine the depth of scour, granted the two assumptions previously mentioned. In this case, the mode of sediment movement does not affect the depth of scour. The detailed geometry of the approach conditions will, of course, determine the flow distribution by determining the velocity and cross-sectional area of the flow in the channel and on the floodplain. However, it matters not whether the velocities are high or low, or whether the flow sections are large or small; but only what discharge ratio results therefrom.

A bridge crossing is in effect a long contraction foreshortened to such an extreme that it has only a beginning and an end. The flow at the crossing cannot be considered uniform, but the solutions for the long contraction can be modified to describe the scour at bridge piers and abutments with the use of experimentally determined coefficients.

LOCAL SCOUR AT PIERS AND ABUTMENTS

Experimental Investigation.—Following the floods of 1947 in which the State of Iowa experienced a considerable monetary loss because of the damage to or the failure of bridges, the Iowa Institute of Hydraulic Research embarked on an investigation of scour around bridge piers and abutments. For the preliminary phase of the study in which the effect of the geometry of the pier or abutment was studied, a flume 10.5 ft wide and 35 ft long was constructed. Two supply lines and two tailgates permitted the wide flume to be operated as two 5 ft flumes by the addition of a center wall. Since neither a sand elevator nor a trap was built into the flume, the time period of the runs was limited and only qualitative results could be obtained. The velocity and depth of flow of the standard run, however, resulted in general movement of the bed, and the observations which were made established the nature of the phenomenon that was occurring.

Albeit qualitatively, the importance of the length-width ratio of the piers, the angle of attack of the stream against the piers, and the length of encroachment of the abutment was established in this flume, as well as the unimportance of small details of the geometry of pier or abutment. It was also demonstrated experimentally that there was an equilibrium or limiting depth of scour. In the preliminary work, piers and abutments representative of current Iowa designs were studied. The study of geometry was continued[7] with more generalized pier forms and scour arrestors. The effect of debris was was also investigated (debris can be considered as an enlargement of the pier somewhat less than the size of the debris mass depending on the permeability of the mass).

For the second phase of the program in which the effects of velocity and depth of flow and of sediment size were to be assessed, a transport flume was constructed which was 5 ft wide and 35 ft long and equipped with a sand elevator and trap. Most of the runs were made with a representative Iowa pier set at an angle of 30° to the flow. The depth of scour was measured by an electrical scour meter which could sense the position of the sand by the difference in conductivity between the water-sand mixture and plain water. As wide a range of velocity, depth, and sediment size was utilized as the characteristics of the flume permitted. Froude numbers approaching the critical and sand jumping the trap limited the maximum value of the velocity and the minimum size of the sand, respectively. The minimum value of velocity and the maximum size of the sand were limited by the requirement that there be general movement of the sediment as bed load. Nevertheless, a sixty-fold change in the rate of sediment transport was obtained—probably the most significant indicator of the range of the tests. The depth of scour at the front of the pier is shown in Fig. 3. No systematic scatter could be detected in the

7 "An Investigation of the Effect of Bridge-Pier Shape on the Relative Depth of Scour," by D. E. Schneible, M. S. Thesis, State University of Iowa, June 1951.

measurements—indicating that the depth of scour was uniquely determined by the geometry.

This trasport flume was also utilized for a model study of a bridge pier on which field measurements were made. The measurements in the field were obtained with an adaptation of the laboratory scour meter. Comparison of the model and prototype data indicated that the depth of scour could be treated as simply another length and that the equilibrium depth of scour obtained in the field.

Measurements on a system of multiple cylinders were also made. The most significant findings of these tests were that the depth of scour did not depend on the degree of contraction (or proximity of an adjacent cylinder) until the scour holes overlapped, and that the *minimum depth of scour* when interference did occur was given by the solution for the long contraction.

For the third phase of the study, the first flume was modified by being lengthened 7 ft, and equipped with a sand elevator and traps. The sand was supplied to five ft on one side of the flume, simulating the river channel. Clear water was supplied to the other side of the flume which simulated the floodplain, or overbank area. The experimental program for the investigation of the scour at the abutment included a variation of the ratio of discharge of overbank and channel, the depth of flow, the position and angle of the pier, and the presence of vegetal screening along the bank line. A few experiments were also conducted on the effect of spur dikes off the end of the abutment parallel to the bank, and of screening on the floodplain in the approach to the abutment.

Design Relationships For Abutments.—The observation previously mentioned, that the local depth of scour does not depend on the degree of contraction until scour holes around neighboring obstructions overlap, suggests an extension of the solution for the long contraction to the case of local scour. For a normal sand, the width of a scour hole at right angles to the flow is about 2.75 times the depth of scour. Fig. 4 is a definition sketch of a long contraction, which it will be assumed approximates the case of the abutment. The scour in the long contraction is assumed to be a fraction $1/r$ of the scour at the abutment d_s. Rewritten for this special case Eq. 7 becomes

$$\frac{1}{r}\ \frac{d_s}{y_0} = \left(\frac{Q_c + Q_o}{Q_c}\right)^{6/7} - 1 \quad \ldots\ldots\ldots\ldots \quad (9)$$

Since the depth of scour is unknown, Q_c is unknown. Therefore, the discharge Q_w over an arbitrary width w is substituted where $Q_w/w = Q_c/2.75\ d_s$. (The width w should be approximately equal to 2.75 d_s and in practice a trial-and-error procedure will be necessary). Algebraic manipulation will now result in

$$\frac{Q_o}{Q_w}\ \frac{w}{y_0} = 2.75\ \frac{d_s}{y_0}\left[\left(\frac{1}{r}\ \frac{d_s}{y_0} + 1\right)^{7/6} - 1\right] \ldots . \quad (10)$$

in which y_0 is the average depth of flow in the width w. Eq. 10, with a value of r of 4.1, is plotted in Fig. 5, together with the experimental data for runs with the approach fill and abutment normal to the direction of flow. Various set-back distances of the abutment and clear distances between the abutment

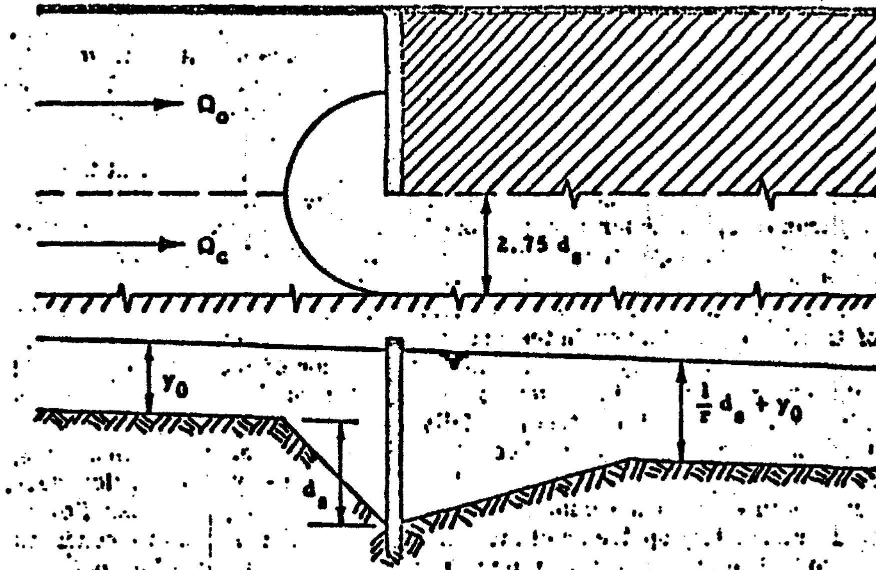


FIG. 4.—DEFINITION SKETCH OF OVERBANK CONSTRICTION

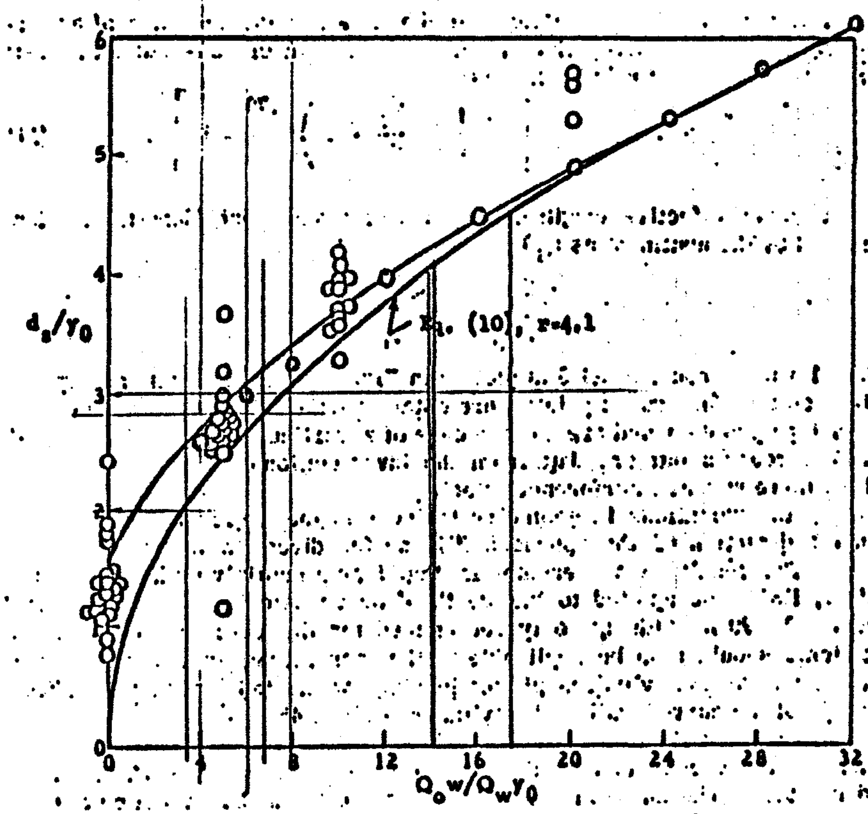


FIG. 5.—SCOUR AT AN OVERBANK CONSTRICTION

and the vegetal screen are represented among the data. The solid points are from the last, and best, series of runs in which the abutments was not set back from the bank line. The high points are from early runs in which the velocity of the approach on the overbank was so great as to be unrealistic. The floor in the overbank area was later lowered to obtain a smaller velocity of approach.

For large values of the parameter $Q_0 w/Q_w y_0$, it is apparent that the approximate solution describes the scour remarkably well. For small values of the parameter, the discrepancy is due to a cross flow from the channel to the overbank in the approach to the constriction. This cross flow could be observed by means of dye streaks. Since the water-surface slope in the channel was greater than in the overbank area, the water surface at the head end of the flume was higher in the channel than in the overbank. Correcting the overbank discharge by an approximate calculation of the cross flow would shift the experimental points over to the analytic curve. This has not been done, to emphasize the importance of fully determining the overbank flow. In practice, the rate of flow on the floodplain will be determined by considering the slope of the water surface, the depth of flow and the area of the flow section, and estimating an n value. If the overbank flow is small, cross flow in the area immediately upstream of the constriction can give rise to a condition similar to that in the laboratory flume. If this cross flow is possible, but cannot be evaluated, the upper curve in Fig. 5 should be used.

A similar approximate solution can be obtained for the case of an encroaching abutment. Using Eq. 8a for the condition of bed-load movement ($\sqrt{gyS}/w < 1/2$), this relationship is

$$\frac{l}{y_0} = 2.75\,\frac{d_s}{y_0}\left[\left(\frac{1}{r}\cdot\frac{d_s}{y_0}+1\right)^{1.70}-1\right] \quad \ldots\ldots \quad (11)$$

where l is the effective length of the abutment such that (denoting the flow obstructed by the abutment as Q_l)

$$\frac{Q_l}{l\,y_0} = \frac{Q_w}{w\,y_0}$$

Eq. 11 with r equal to 11.5 is plotted in Fig. 6 together with some experimental points. The square points were obtained during an investigation of the relief bridge problem and are for the case of a vertical, blunt-ended, normal wall. The round points are data from the investigation of multiple cylinders in which there was no interference effect.

Among the variations in geometry tested was the angle of incidence between the direction of the approach fill and the direction of the flow in the channel. The results are presented in Fig. 7 as a multiplying factor, or coefficient, K_θ to be applied to the depth of scour calculated for the normal crossing ($\theta = 90°$). Although only determined for the overbank case, these coefficients should also be applicable to the encroaching abutment. A check on the points for the cylinders plotted in Fig. 6 would indicate that the effective angle of the curved wall of the cylinder is about 45°.

The effect of setting the abutment back from the normal bank of the stream is difficult to assess. In the laboratory experiments no measurable effect could be noted. The long contraction solution can be further modified to consider that a part of the overbank flow remains on the floodplain, and that the

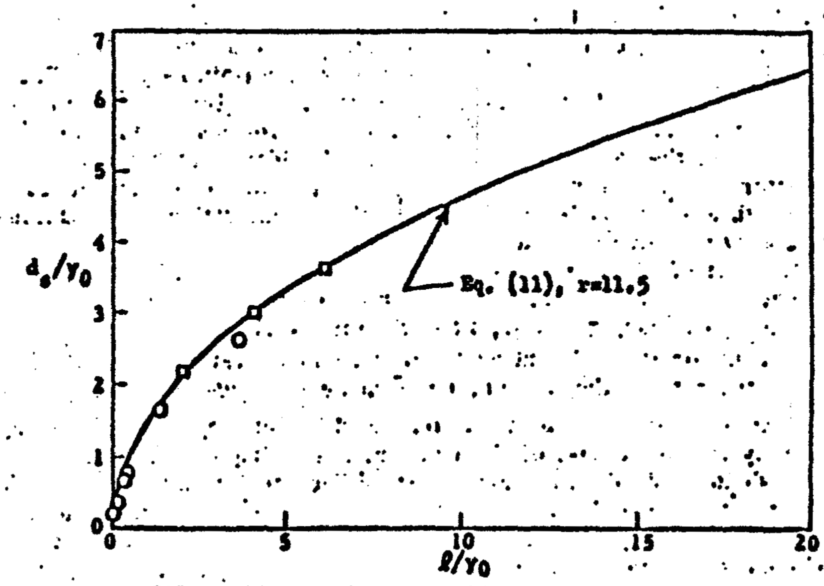


FIG. 6.—SCOUR AT AN ENCROACHING ABUTMENT

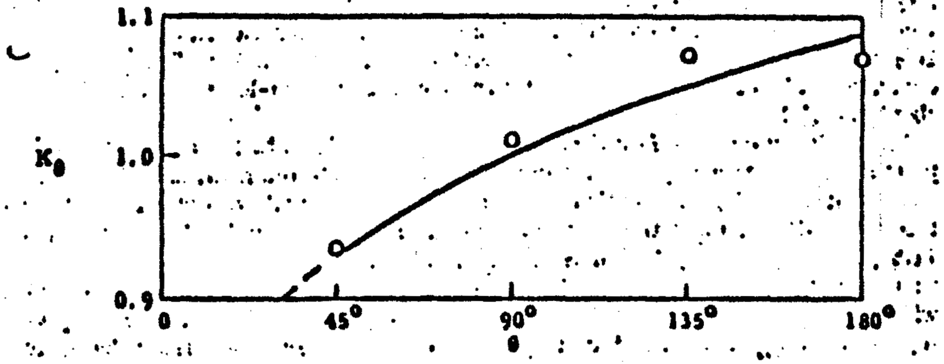


FIG. 7.—EFFECT OF ANGLE OF INCIDENCE

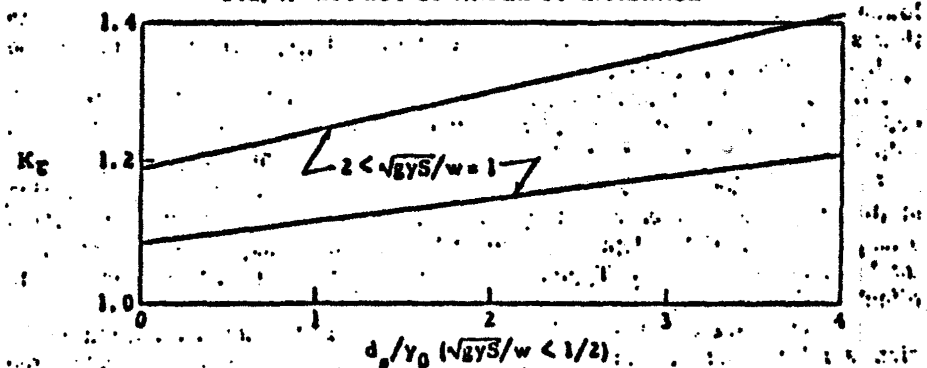


FIG. 8.—EFFECT OF THE RATIO OF SHEAR-VELOCITY TO FALL-VELOCITY

scour hole moves back with the abutment. This approximate solution also indicates that the effect of set back is small. Both the experiments and the analysis, however, do not model a real set back in certain respects. Because of the bank, the depth of flow in the vicinity of the abutment will be less. If the set back is very large, the presence of the river nearby is immaterial. The condition is then a special case of a relief bridge and the scour is due to clear water. If the bank material contains considerable clay or is well covered with a good, well-rooted, tight turf the material near the abutment might be very resistant to scour.

For the case of the encroaching abutment, the mode of sediment movement will affect the depth of scour. An approximate evaluation of this effect can be obtained by comparing Eq. 8a, b and c. For the same values of B_1/B_2 the depth of scour for the different modes of movement were calculated. Defining K_T as the ratio of the depth of scour under suspended-load conditions to depth of scour under bed-load conditions, Fig. 8 was prepared. Note that the fall velocity should be that of the bed material being scoured out. The so-called wash load should be ignored. The effect of a change in the mode of movement should be applicable also to the local scour at a pier. Experiments made with a circular cylinder and a bed material 0.04 mm in diameter confirmed the order of magnitude of the effect. The depth of scour—very difficult to measure and extremely sensitive to unsteadiness of the flow—was about 50% greater than for bed-load conditions. The ratio of shear-velocity to fall-velocity in this experiment was about 20, and the concentration between 5 and 10% by weight, that is, 50,000 and 100,000 parts per million. The ratio between the rate of suspended-load movement and the bed-load movement was about 500 to 1.

Design Relationships For Piers.—All the available data on piers were adjusted to scour around a rectangular pier aligned with the flow, and the design curve shown in Fig. 9 was drawn with conservatism and with due regard for the reliability of the various data. Also shown in Fig. 9 in Eq. 11 with r = 11.5 (note that b = 2 ℓ). The most important aspect of the geometry of the pier was the angle of attack between the pier and the flow, coupled with the length-width ratio of the pier. A family of curves is shown in Fig. 10 as a multiplying factor $K_{\alpha L}$ to be applied to the depth of scour obtained from the basic curve. If the pier is set at an angle to the flow, it is recommended that no allowance be made for shape. If it is certain that the pier is aligned with the flow and will remain so during the life of the bridge, a shape factor can be applied as shown in Table I.

All the experimental work on which the design curve was based involved bed-load movement. If the mode of movement is different, the ratio of shear-velocity to fall-velocity factor K_T in Fig. 8 should be used.

Besides the local scour at the pier, it is possible that the scour centering at the abutment may extend its area of influence out to piers near the bank. Thus, a fraction of the depth of scour at the abutment may have to be added to the local depth of scour. Typical scour holes have been shown whereby this effect can be estimated.[3]

In the bulletins of the Iowa Highway Research Board the application to design is fully discussed, including schematic design examples illustrating the use of all of the design relationships. On the basis of these relationships the prediction of local scour at piers and abutments is simple and straightforward. In order to use the relationships, however, the flow conditions at the

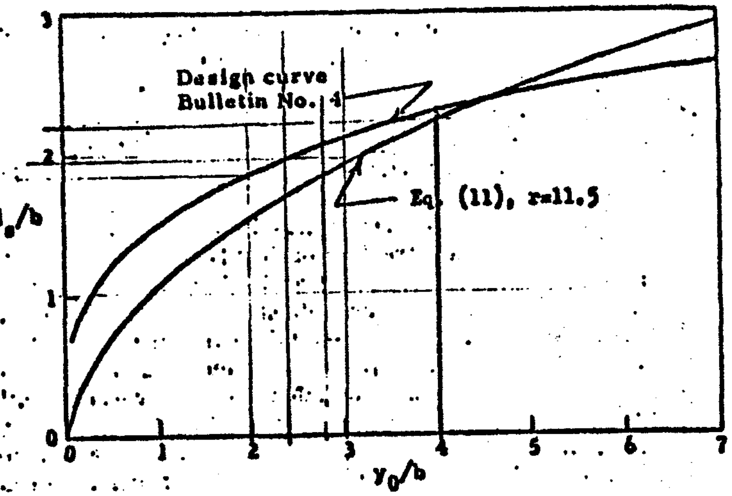


FIG. 9.—SCOUR AROUND BASIC BRIDGE PIER

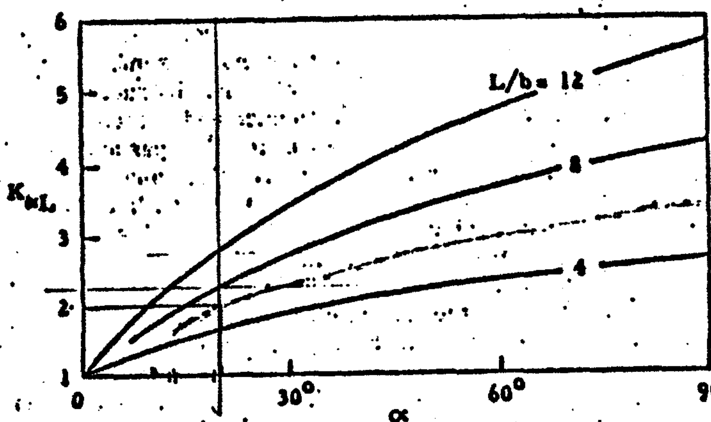


FIG. 10.—EFFECT OF ANGLE OF ATTACK

TABLE I

Shape coefficients K_s for nose forms.
(To be used only for piers aligned with flow.)

Nose form	Length-width Ratio	K_s
Rectangular		1.00
Semicircular		0.90
Elliptic	2 : 1	0.80
	3 : 1	0.75
Lenticular	2 : 1	0.80
	3 : 1	0.70

bridge crossing must be known. As any civil-hydraulic engineer knows, the forecasting of the river flow of the future is not easy. Few gaging sites have been operated for 100 years, and 100 years of record are not sufficient to define precisely the magnitude of a 100-year flood. Unfortunately, even rarer floods during the anticipated life, say 50 years, of a bridge, have a probability of occurence which is large enough to be reckoned with. This problem, of course, is simply the old basic problem of hydrology.

In addition to magnitude, other characteristics of the flow to be expected must be foreseen; the stage, the direction of flow (which may vary with stage), the division of flow between channel and overbank (which will vary with stage and the state of the floodplain). Although these details can never be foretold with complete confidence, they are essential to the prediction of the scour which will occur. The evaluation of the flow characteristics must be judicious, and a calculated risk must be accepted; otherwise, the worst conditions must be assumed and overdesign will usually result. If sufficient control can be exercised over the valley upstream from the bridge crossing, the risk can be reduced to the chance of the flood occurrences. Each bridge crossing will have to be treated individually, and in the final analysis economic considerations will dictate the solution.

Backwater at Bridge Crossings.—When scour occurs at a bridge crossing, the flow pattern which then obtains does not resemble that of a two-dimensional constriction. The flow approaching the obstruction (pier or abutment and embankment) ducks beneath the surface and passes through the constriction as a spiral roller. The remainder of the flow passes over this roller and through the constriction barely noticing the piers and abutments, or that there is a constriction. There is no contracting jet as in a two-dimensional slot; or as in an open-channel constriction with inerodible boundaries. As a result, the velocity increases only as it must to transport the sediment load downstream from the bridge; the energy losses at and due to the bridge crossing are minimal. Since the flow is disturbed, there are, of course, some losses. Fig. 11 shows the measured rise in water surface in the abutment flume. This measured rise is partly due to the velocity-head change between the approach flow in the channel and the flow in the contracted section downstream from the bridge. The average depth in the downstream section can be obtained from Eq. 7 and the curve for the velocity-head differential has been evaluated on this basis. The backwater at the bridge is the difference between the velocity-head differential and the measured rise of the head of the flume. The backwater in a comparable inerodible flume according to Liu, Bradley, and Plate[8] is also plotted for comparison. It is readily apparent that the increase in the flow section due to the scour reduces the backwater to a fraction of the amount that otherwise would occur.

It is likely that a major cause of backwater at a bridge crossing on an alluvial stream might be the resistance to flow of the contracted stream in the reach below the bridge. Depending on bank height and cover on the floodplain the cross flow from the stream back to the floodplain might take place very slowly. If the resistance to flow is greater in the contracted reach than in the normal stream, backwater will result. Moreover, any backwater which does

[8] "Backwater Effects of Bridge Piers and Abutments," H. K. Liu, J. N. Bradley, and E. O. Plate, Civil Engineering Section Project Report CER, 57HKL2, Colorado A. and M. College, Fort Collins, Colorado, 1957.

occur in an alluvial river tends to cause aggradation upstream, and a concomitant lengthening of the backwater curve.

SUGGESTIONS FOR FURTHER STUDIES

Field Measurements.—In order to apply with confidence the relationships proposed for predicting the scour at bridge piers and abutments, field measurements are needed to verify the conformity of model and prototype. Some of these measurements should be made at sites of extremely simple geometry like that of the Skunk River pier near Ames, Iowa. Such sites can be easily modeled in the laboratory, and no questions should arise as to extraneous effects. Other measurements should be made at sites of complex geometry which perhaps cannot be modeled in the laboratory. The scour can be pre-

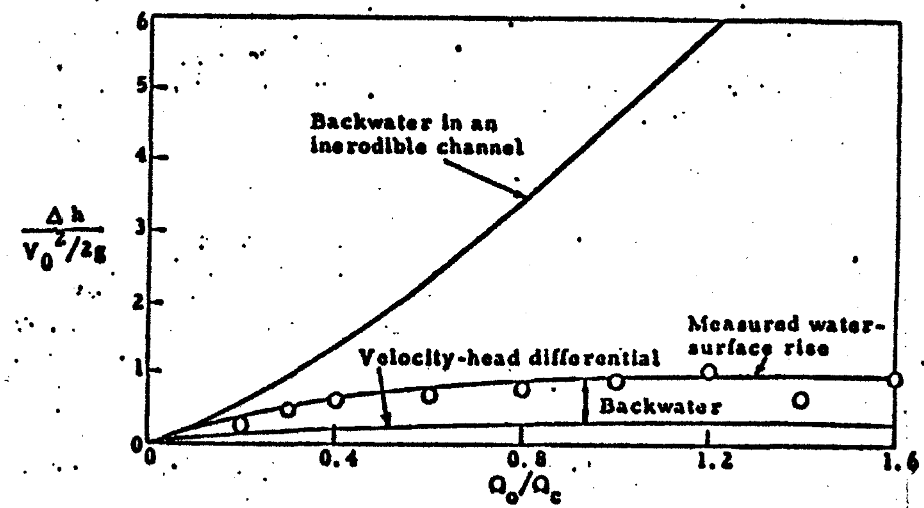


FIG. 11.—BACKWATER IN AN ALLUVIAL CHANNEL

dicted with the aid of the relationships presented, and any discrepancies may indicate factors that should receive further study.

The experience of the past can be used to substitute for field measurements. By hindcasting, the scour which occurred around piers and abutments at selected crossing during major floods can be evaluated. If bridges that failed and that did not fail during the same flood are checked, a correlation should be possible among hindcasted depth of scour, permissible depth of scour (based on foundation design), and failure or non-failure.

Laboratory Investigations.—The one general aspect of scour at bridge crossings which has not been discussed is the scour at relief bridges. Relief bridges are openings placed in the highway embankment as it crosses the floodplain of the valley. For design purposes the flow through the relief bridge must be considered to consist of clear water. If there is a sediment load it will tend to be much finer in size than the material which will be scoured out. The depth of scour due to clear water is most assuredly a function of the velocity of flow and the sediment size as well as the geometry of the opening and its environs. The limiting condition for this type of scour is a boundary shear that is equal to the critical for the material of which the boundary is composed. Since there is no supply of sediment coming into the scour hole, at the limit there can be no sediment going out of the scour hole. This limit is finite, but is reached asymptotically in infinite time. It is very difficult in the laboratory to scale all factors controlling clear water scour and a combination of experiment and analysis is indicated.

For an abutment which is set back from the normal bank less than the depth of scour, the relationships proposed herein should be applicable. An abutment set back over three times the depth of scour should be considered a relief bridge. Following a successful investigation of the relief bridge problem, it would seem in order to study the effect of set backs between these limits.

One could, of course, go on studying the effect of various geometries of piers, abutments and overall crossing. However, the results of this investigation indicate that the effects of these details are minor, and of less importance than the error that can be expected in the evaluation of the flow conditions at the site. Further studies of geometry should probably be deferred until field measurements indicate they are of sufficient importance.

ACKNOWLEDGMENTS

The writer wishes to acknowledge the contributions of the many others who were associated with him in this investigation of scour at bridge crossings. On the one hand there was the continued support and encouragement of Dr. Hunter Rouse, Director, Iowa Institute of Hydraulic Research; Mr. Mark Morris, Director of Highway Research, Iowa State Highway Commission; and Mr. Carl Izzard, Chief, Hydraulic Research Branch, Bureau of Public Roads; and on the other, the aid of the many research assistants who labored so long and so well, and especially Mr. Arthur Toch, Research Engineer, I.I.H.R. who, as a colleague for years, contributed in many ways, and Dr. Phillip G. Hubbard, Research Engineer, I.I.H.R., who was responsible for the development of the scour meter.

DISCUSSION

T. BLENCH,[9] F. ASCE.—The writer draws attention to the collection of data of scour around bridge piers and other obstacles made by C. C. Inglis, from 1924 to 1942.[10] The data of severe scour around piers numbered 17 and were for floods from 30,000 cfs to 2,600,000 cfs. They appeared, with descriptions, as Table 8-1 in the reference and were plotted in its Fig. 8-1 to show the relation of scour depth (measured from water surface to bottom of scour) to the Lacey regime depth for the river at the flood discharge; the line in Fig. 8-1 was an excellent fit and indicated that, all other things being equal, scour depth varies as the cube root of discharge exactly as the regime theory based on controlled canals shows; the ratio to regime depth averaged almost exactly 2.0. The writer has replotted[11] the Inglis data along with model data of Inglis, of the present author (some time ago), and of workers at the University of Alberta. The coordinates were chosen in terms of estimated discharge intensity for the sake of uniformity and future applications; allowing for the discharge intensities of the rivers being averages over the whole river whereas for models they are discharge intensities of attack, the river and model data were in good accord with each other and with the regime theory prediction of variation as the two thirds power of discharge intensity—all other factors being equal. What to do when other factors vary has been explained.[11]

Inglis has described,[10] also, experiments on model bridge piers, similar to the recent Iowa ones, has plotted the results, and has given a formula to fit the graph. The writer has reduced this formula to a rather simpler form:

$$d_s/d_r = 1.8\left(b/d_r\right)^{\frac{1}{4}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (12)$$

where d_s is scour depth from water surface, d_r is regime depth of the river, and b is the breadth of the pier projected at right angles to the direction of attacking flow.

It is interesting to note that the mean depth of flow between piers (as distinct from the local scour round them) is shown by Leopold[12] to follow the regime law of variation as the two thirds power of discharge intensity in the bridge over the Powder River at Arvada, Wyoming.

The writer feels that Eqs. 7 and 8 should not have been derived from considerations of total load since it is bed load that is mainly effective in determining depth and that the formula is rough; furthermore, he notes that y seems to be unrelated to regime depth so is open to objection as a standard for comparison. Considering the evidence of regime theory, plus the Inglis and Leopold observations on rivers, the writer would replace Eqs. 7 and 8 by their equivalents from

$$\text{regime depth} \propto \sqrt[3]{q^2/F_b} \quad \ldots\ldots\ldots\ldots\ldots (13)$$

and then apply geometric coefficients to various obstacles to obtain scour;[11] q is discharge intensity and F_b is the appropriate bed factor.

JOSEPH N. BRADLEY,[13] *M. ASCE.—The model results on scour at bridge* abutments and piers have been treated in a logical and commendable manner. The consistency with which the model results plot could be misleading how-

---

[9] Prof. of Civ. Engrg. and Cons. Engr.

[10] "The Behaviour and Control of Rivers and Canals," (with the aid of models), by Sir C. C. Inglis, Govt. of India, Central Water Power Irrigation and Navigations Research Sta., Poona, 1949.

[11] "Regime Behaviour of Canals and Rivers," by T. Blench, Butterworths Scientific Publications, London and Toronto, 1957.

[12] "The Hydraulic Geometry of Steam Channels and Some Physiographic Implications," U.S.G.S., Prof. Paper 252, 1953, Fig. 4.A.

[13] Hydr. Engr., Internatl. Engrg. Co., Inc., Dacca, East Pakistan.

ever, by making the prediction of scour appear as a simple routine procedure. Since Mr. Laursen has said little with regard to the limitations of his results, a few remarks on this phase of the subject may be appropriate.

Figs. 5 and 6 represent the results of model studies on abutment scour made under essentially ideal conditions, for example, with bed of granular material free to move, a rectangular flow cross section, and a uniform velocity distribution. One can compare the author's results with those of a completely independent set of experiments made with different size and gradation of bed material, a constant depth of flow and a uniform velocity distribution, performed at Colorado State University,[8] and find that the two are in close agreement. This indicates that the model results are consistent and easy to duplicate.

Where streams meet similar specifications in nature as those of the models, there should be a reasonable correspondence between model and prototype. There are streams in India and Pakistan which do approach these so-called ideal conditions; the stream beds present an unlimited depth of alluvial material, the river channels are extremely wide with more or less constant width-to-depth ratio, and the velocities are low due to an unusually flat gradient (the average is 2 ft in 4 miles in East Pakistan). Under such conditions the velocity distribution cannot vary greatly across the stream. Field measurements of scour at bridge abutments, spurs, and guide banks for the rivers of India and Pakistan are on record[14] and these show surprisingly good correlation with the model results.[15]

Limited experience with scour on rivers in the United States show less favorable comparisons. The reason is obvious; the gradients are steeper, the cross sections are irregular, the velocities are higher, and the velocity distributions are far from uniform. Under these conditions the greatest scour does not necessarily occur at the abutments but is more likely to be found in the portion of the channel where the depth of flow and velocity are greatest. Records of the United States Geological Survey of bridge sites in Mississippi, where the beds are generally of alluvial material, show this to be true. There is also evidence from past floods in various sections of the United States that the settlement of piers in the deeper portion of the channel is more common than abutment failures due to scour. Yet if Figs. 3 and 6 are consulted, in order, it is found that prediction from the model gives scour depth up to three times the pier width for the center of the channel, while scour up to six times the depth of flow is supposedly possible at abutments. The latter can result in fantastic figures, which are true in the case of the model with rectangular cross section, where all scour is concentrated at the abutments; but such predictions are unreasonable when applied to irregular cross sections in the field.

This discussion was not written to confuse the issue or discredit the model results (which are valid for the conditions tested). Rather it is to point out some of the remaining unknowns and (1) encourage investigators to make a

14 "The Behavior and Control of Rivers and Canals," by Sir Claude Inglis, Research Publication 13, Part II, Central Water Power Irrigation and Navigation Report, Poona Research Sta., 1949.

15 "Field Verification of Model Tests on Flow Through Highway Bridges and Culverts," by Carl F. Izzard and Joseph N. Bradley, Proceedings, 7th Hydr. Conf., Iowa, 1958, p.

concerted effort to take soundings of streams at constrictions both before and during floods for the purpose of better understanding the field problem and making better utilization of the model results; and (2) to warn engineers to not use the model information blindly but to treat each river crossing as an individual problem, using the model results as a guide rather than a definite solution. Returning to item (1), further model studies will not produce the answer desired; field measurements are the only alternative. A further comment on item (2) is that model results applied to abutment scour in the United States will certainly fall on the side of safety. The extra cost of unduly deep footings may, therefore, be sufficient to finance a comprehensive field study in a very short time.

D. V. JOGLEKAR,[16]—It is agreed that each river and each reach must be studied to understand its individual, almost personalized, characteristics and that scour at bridge piers is closely related to (a) the river conditions upstream and downstream (such as indicated by flood hydrographs from year to year); (b) the meandering tendency of the river that, in turn, depends on the detritus load carried by the river during floods, (c) whether the river is flowing in its alluvial plain or has its sides and bed resistant to scour, (d) whether the river is flashy or has sustained floods, and (e) the constriction of the river section caused by the bridge. The reach of the river also has an important bearing on scour at bridge piers, for example, bridges on hill torrents, bridges on alluvial plains with mild slopes, and bridges on the tidal reaches of the river. In a braided river near a hilly region the river changes are so rapid that it is very difficult to estimate the waterway required for bridges across various streams of the braided river. An illuminating case of violent movement of Manas River[17] in Assam can be cited. In 1909, three bridges were provided on the three tributaries of this river:

1. on the Manas River which had 10 spans of 100 ft,
2. on the Bholookadoba Branch - 2 spans of 75 ft,
3. on the Beki Branch - 4 spans of 12 ft.

Due to changes in river courses, the bridge had to be rebuilt several times on the Bholookadoba Branch between the period 1909 to 1945:

1. 2 of 75 ft,
2. 3 of 75 ft,
3. 1 of 250 ft and 2 of 150 ft.

Changes in the Beki Branch were more violent and larger waterways had to be provided during this period as follows:

1. 4 x 12 ft,
2. 4 x 20 ft,
3. 9 x 20 ft,
4. 1 x 50 ft, 1 x 30 ft, 6 x 19 ft,
5. 1 x 50 ft, 2 x 40 ft, 6 x 19 ft,
6. 2 x 40 ft, 3 x 100 ft,
7. 7 x 150 ft.

16 Adviser, Central Bd. of Irrig. and Power, Poona, India.

17 "River Training," Railway Board Technical Paper No. 334.

ove the low-flow bed of the stream. It is possible to imagine circumstances which the length of the bridge might be cut in half, the waterway area re-aining the same. This has obvious economic significance.

The natural filling of the scour hole during the recession of the flood tends minimize the depth of the pool of water that might be left standing under e bridge during low flow. As the beds of such streams are commonly made of alternating pools and bars anyway, the addition of another small pool neath the bridge is of little consequence. Fig. 12 shows the type of design oposed.

L. J. TISON.[30]—The relationship proposed by Mr. Laursen for the prediction scour at piers and abutments for the case in which sediment is supplied to e scour hole, is extremely interesting.

The author presents an analysis in which he compares the flow without ntraction and the flow due to a long contraction. He then uses, for both ows, an approximate form of the total sediment load relationship recently oposed by himself and considers a bridge crossing as a long contraction, reshortened in such an extreme that it has only a beginning and an end.

The writer has found that a scour at piers and abutments also took place ien the contraction of the flow was without significance and introduced the ea that the scour at piers may be affected by the curvature of the stream-ies around the piers.

When a pier with an arbitrary shape is placed in a stream, it produces 'ig. 13) a first curvature with a center 0_1 and a radius ρ_1, followed by a cond curvature 0_2 ρ_2. In the neighborhood of the bank of the river, the pier erts no action on the direction of the streamlines, cd. But, considering the gion of the first curvature in the neighborhood of the surface, a line such 3, tangent in each point to the principal normals of the streamlines, the llowing relationship may be written:

$$z_B + \frac{p_B}{\gamma} + \frac{1}{g}\int_A^B \frac{v^2}{\rho}\,ds = z_A + \frac{p_A}{\gamma} \ldots\ldots\ldots\ldots\ldots \quad (16)$$

which z is the vertical height, p is the pressure, v is the velocity and γ is e specific weight of the liquid. The first supposition was that the motion is rallel to the bottom.

In the neighborhood of the bottom, another relationship of the same nature ves:

$$z_{B'} + \frac{p_{B'}}{\gamma} + \frac{1}{g}\int_{A'}^{B'} \frac{v'^2}{\rho}\,ds = z_{A'} + \frac{p_{A'}}{\gamma} \ldots\ldots\ldots \quad (17)$$

and B' are on a same line perpendicular on the bottom.

The variation of the pressure between B and B' follows the hydrosatic law,

[30] Prof., Univ. of Ghent, Ghent, Belgium.

so that:

$$z_B + \frac{p_B}{\gamma} = z_B' + \frac{p_B'}{\gamma} \ldots\ldots\ldots\ldots\ldots\ldots \quad (18)$$

From these three relationships

$$z_A + \frac{p_A}{\gamma} - \left(z_A' + \frac{p_A'}{\gamma}\right) = \frac{1}{g}\left[\int_A^B \frac{v^2}{\rho}\,ds - \int_{A'}^{B'} \frac{v'^2}{\rho}\,ds\right] \ldots \quad (19)$$

with v greater than v' (v' is the velocity in the neighborhood of the bottom).

A consequence of Eq. 19 is that the motion cannot be parallel to the bottom, the trajectories must dive.

This diving motion will have another consequence : a local attack of the bottom under the influence of the first curvature 0_1 ρ_1. Evidently, the importance of scour will depend on the value of the vertical component of the diving motion, and this vertical component will depend on the value of the second member of Eq. 19.

For example, small values of ρ will increase the depth of scour. Experiments with different models of piers, with the same length and breadth and with the same discharges and heights, gave a confirmation of this result.

Rectangular piers (reduced values of ρ) produced a scour of 113 mm with a discharge of 30 l per sec in a flume with a breadth of 70 cm and a height of 10.5 cm. The length of the pier was 24 cm and its breadth was 6 cm.

The simple rounding of the edges of the piers reduced the scour to 91 mm, whereas a triangular shape reduced it 70 mm.

An aerodynamic shape gave no further reduction, but the shape of a lens produced a reduced scour of 54 mm. (large values of ρ).

The protection realized by a single pile before the lens-shaped pier reduced the scour to 38 mm. This pile produced bigger values of ρ and worked as a reduction of the width-length ratio.

A change in the repartition of the velocity will have an influence on the second member of Eq. 19 and, therefore, on the value of the scour. A higher roughness of the bottom on a distance before the pier (with pebbles on the bottom) is realized. In Eq. 19, v' was reduced whereas v was increased, with the conclusion that the erosion had to be increased. That is what was found with an erosion of 71 mm with the lens-shaped pier (54 mm with the fine sand).

The consideration of Eq. 19 shows that it would be possible to considerably reduce the erosion by adapting the variation of the radius ρ with that of v^2.

Therefore, the repartition of the velocity from the bottom to the surface was measured and the lens-shaped pier was given a variable radii of curvature corresponding with the values of v^2. The result was a non-prismatic pier with a larger base. The erosion around this new model was reduced to 5 mm maximum.

It is easy to see that the second curvature 0_2 ρ_2 will give a relation (Eq. 19) with a negative second member. A rising motion will, therefore, The writer agrees with the author that afflux caused by a bridge depends so much on the erodibility of the bed material. Thus in the case of the railway bridge on the River Ganges[25] at Mokameh, the afflux caused by the bridge was only a couple of inches for a flood of 18,000,000 cu sec. This was because the flood was sustained and the river scoured its bed as the discharge increased. In the case of rivers with flash floods or with inerodible bed, full afflux has to be allowed for in the design according to standard formulas.

As emphasized by the author, more field experiments are necessary to improve our method of estimating scour at bridge piers.

W. J. BAUER,[29] M. ASCE.—The writer was in attendance at the University of Iowa, when the research described by Mr. Laursen was getting under way, and is, therefore, aware of its pioneering nature. The purpose of this discussion is to consider the application of the results presented by the author to a particular approach to waterway design.

The fundamental purpose of a bridge over a stream is to separate the human traffic and the flowing water. It is commonly accepted that provision for the passage of the maximum conceivable discharge through the waterway beneath the roadway is economically undesirable. It is good practice to provide for the safe, although infrequent, overtopping of the roadway by a rare flood. If the design is adequate, after the passage of such a flood, the highway is immediately ready for service with only minor repairs being required during subsequent routine maintenance.

In order to achieve this objective, the backwater produced by the structure must be small at the stage of incipient overtopping. The writer has used values of 0.5 ft to 1.0 ft as being reasonably small for the backwater at this stage. The design problem then becomes how to provide for the passage of the flood corresponding to the stage of incipient overtopping without exceeding an allowable backwater.

Some of the flow area required will exist beneath the elevation of the low-flow bed of the stream in accordance with the reasoning set forth by Mr. Laursen under the heading "Local Scour at Piers and Abutments: Backwater at Bridge Crossings." The extent to which such area is available to the flow during flood may be controlled by suitable scour protection.

At bridge sites in West Virginia, at which scour protection of broken rock is readily available, the writer has used a design velocity beneath the bridge of between two and three times the typical velocity in the stream during flood. This was accomplished without exceeding the allowable backwater at the stage of incipient overtopping. Such a relationship, between the velocity in the contracted section and that in the approach flow, gives sufficient assurance that the material deposited over the broken rock fill during low flow will be scoured out during flood, provided the depth of scour required is not excessive. Fig. 6 indicates the relationship between depth of scour at an encroaching abutment and a parameter dependent on the degree of contraction of the natural waterway by the structure. Fig. 6 indicates the attaining of depths of scour equal to the depth of flow at relatively minor amounts of contraction. It is, therefore, not difficult to imagine circumstances in which it would be possible to provide as much waterway area beneath the low-flow bed of the stream as above it.

[29] Cons. Engr., Chicago, Ill.

In the instances that the writer has in mind, the waterway area provided beneath the low-flow bed of the stream was about 20% of the total waterway area at the stage of incipient overtopping. The depth of the waterway area

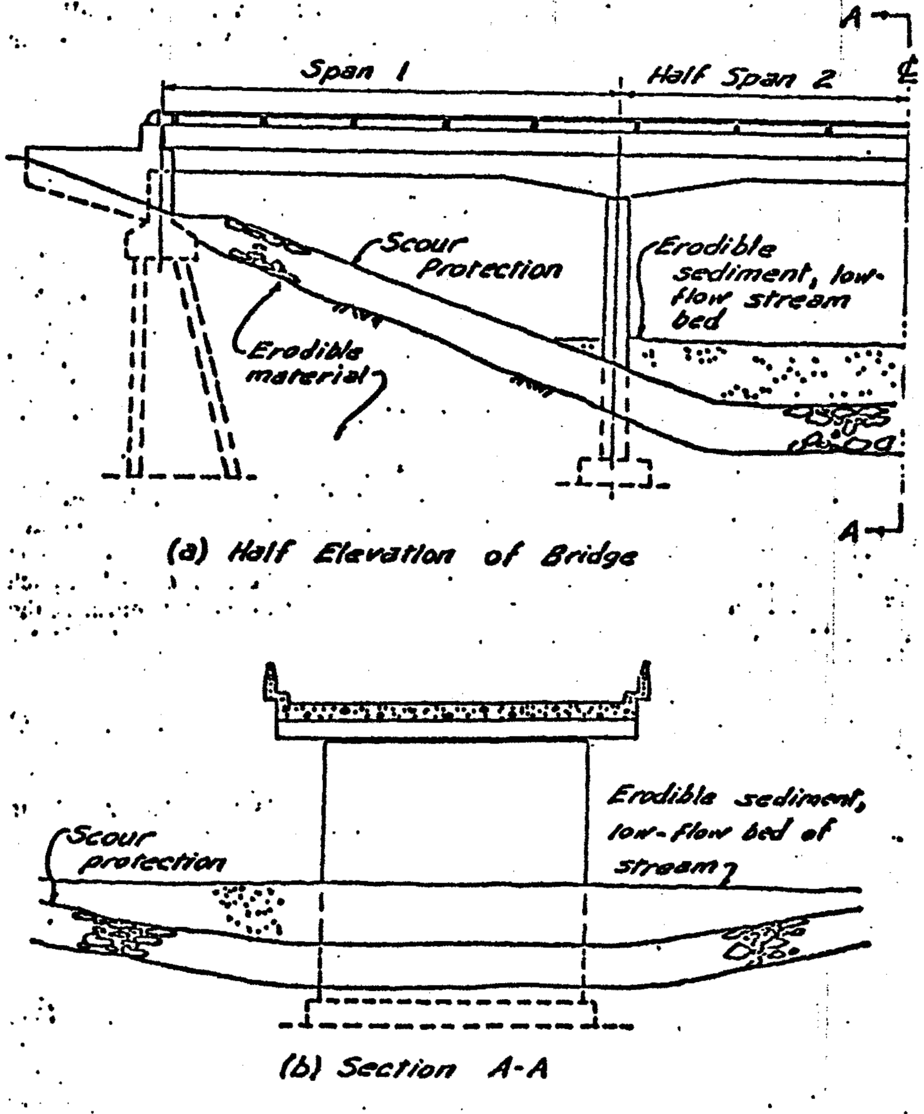


FIG. 12.—PROPOSED DESIGN

beneath the low-flow bed was only about 40% of the nominal depth of flow at the stage of incipient overtopping. Nevertheless, significant cost savings were achieved compared to a longer bridge with equal waterway area entirely e river is still uncontrollable in spite of various training measures.

Experiments[18,19] were carried out at the Central Water and Power Research ation, Poona, India, in 1938 and 1939 for finding scour round a single pier aced in the center of a parallel sided flume, embedded in sand of mean diamer 0.29 mm. The following relation was worked out:

$$\frac{d_s}{b} = 1.70 \left(q_c^{\frac{2}{3}}/b\right)^{0.78} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (14)$$

which b is the width of the pier, q_c is the discharge per foot in the center of e flume upstream of the pier, and d_s is the maximum depth of scour at the ier below water level. As it was difficult to correlate this with the depth of cour at prototype piers, due to q (intensity of discharge per foot) depending pon the curvature of the river upstream - which varies from river to river, t was considered desirable to study cases of actual scour in prototypes and vork out a general, empirical relationship. Besides, it has to be remembered hat the angle of repose of bed material in the model and the prototype is the same, hence, the extent of scour in plan in the vertically distorted model is ound always relatively greater than in the prototype. This in effect reduces he discharge intensity at the pier due to greater dispersion of flow and hence he depths of scour obtained in the model would be relatively less. Data[19] were, therefore, collected for scour round bridge piers of various bridges contructed in India and a general relationship[19,20] was worked out as follows:[21]

$$d_s = (2)\ 0.473 (Q/f)^{\frac{1}{3}} = 2\ D(\text{Lacey}) \quad \ldots\ldots\ldots\ldots\ldots \quad (15)$$

in which Q is the maximum flood discharge in cu sec; d_s is the maximum depth of scour below highest flood level; f is = $1.76\sqrt{m}$, and m is the mean grade of bed material in millimeters. In Eq. 15, a representative f value has to be used. From bore data, values of f for each strata is to be worked out to ascertain that the anticipated depth is not based on the f value which is higher than that appropriate at that depth. Recent advances in foundation engineering have made it possible to take the pier foundation sufficiently below the maximum probable depth of scour to provide adequate grip length. Where this cannot be done, high level stone protection, though costly in the long run, has to be employed. Another advantage of deep foundation is that, because of increased side friction on the pier sides embedded in sand, the load bearing capacity of the pier increases considerably as compared to that of a pier with shallow foundation. Generally this additional load bearing capacity is not taken into consideration in the design but is kept as a margin of safety. The previous railway practice was to work out the depth of bridge foundations according to the observations

18 Technical Annual Report, Central Water and Power Research Station, Poona, India, 1938-39.

19 "Behaviour and control of rivers and canals with the aid of models - part II," Research Publication No. 13, Chapter VIII, Central Water and Power Research Sta., Poona, India.

20 Technical Annual Report, Central Water and Power Research Station, Poona, India.

21 "Stable Channels in Alluvium," by G. Lacey, Journal of the Institution of Civil Engineers, Paper No. 4736, 1929.

made by Spring[22] and Gales.[23] In the case of shallow foundation, protection has to be provided by stone pitching and if this is at high level, a lot of the stone (stone used generally weighs 80 to 120 lb and is called one man stone) is washed away downstream due to turbulence and has to be replenished, even during floods, to ensure the safety of the bridge. Due to this turbulence very deep scour occurs downstream of the piers as in the case of Hardinge Bridge[24] on the river Ganges, depth of scour being of the order of 200 ft.

The current railway practice is to provide a grip length for the pier equal to half the depth of scoured bed below H.H.F.L. so that the total length of piers below H.H.F.L. is 3D(Lacey).

Experiments[25,26] were carried out at the Central Water and Power Research Station, Poona, India, for testing the design of training works, waterway, and length of piers of a railway bridge at Mokameh on the river Ganges near Patna (Bihar State). The pier foundation of this bridge has been taken to a depth of about 200 ft below H.H.F.L. which is equal to 3D(Lacey). As the foundations are deep enough, protection by way of stone pitching round piers is not provided.

Various cases of bridges, for which rivers had to be trained to ensure the safety of piers, have been experimented on. These experiments are described in the Technical Annual Reports of the Central Water and Power Research Station, Poona, India, for the year 1937-38, 1938-39, 1939-40, 1940-41, 1944 to 47, 1949 and 1952 to 1958.

In the case of flash flood type rivers, fixing the waterway is much more difficult. In such cases the flood rises and falls so rapidly that the river has no time to scour its bed with the result that the afflux (difference in water surface upstream and downstream of bridge) increases enormously; the bridge is likely to fail by outflanking. A railway bridge on Luni River in Rajasthan State[27] failed in this manner.

In the case of inerodible bed material, it is difficult to estimate the maximum depth of scour. Hydraulic model experiments are unable to reproduce this scour due to obvious limitations. Recourse has, therefore, to be taken to field data. In some cases, the maximum flood level is underestimated and the waterway provided is insufficient. If the bed is inerodible afflux increases beyond the safe limit, with the result that standing wave conditions prevail downstream of the bridge, which lead to undermining of bridge piers. The railway bridge on Yeshwantpur River[28] in Andhra Pradesh failed and collapsed for similar reasons.

22 "River Training and Control of the Guide Bank System," by F. J. E. Spring, Railway Board's Technical Publication No. 153.

23 "Principles of River Training for Railway Bridges and their Application to the Case of the Hardinge Bridge over the Lower Ganges at Sara," by R. Gales, Journal of the Institution of Civil Engineers, December, 1938, Paper No. 5167.

24 Investigations carried out by means of models at the Khadakwasla Hydrodynamics Research Station, near Poona in connection with the protection of the Hardinge Bridge which spans the river Ganges near Paksey, by C. C. Inglis and D. V. Joglekar, East Bengal Railway, Public Works Department, Bombay, India, 1936, Technical Paper No. 55.

25 Technical Annual Report, Central Water and Power Research Sta., Poona, India, 1950.

26 "Manual on River behaviour, control, and training," Ch. VI, Pub. No. 60, Central Board of Irrigation and Power.

27 Technical Annual Report, Central Water and Power Research Station, Poona, India, 1954.

28 Technical Annual Report, Central Water and Power Research Station, Poona, India, 1955.

llow the first diving motion. This was observed on all the models. For the ctangular pier, the small radii of curvature around the upstream edges oduced a quasi-vertical rising motion.

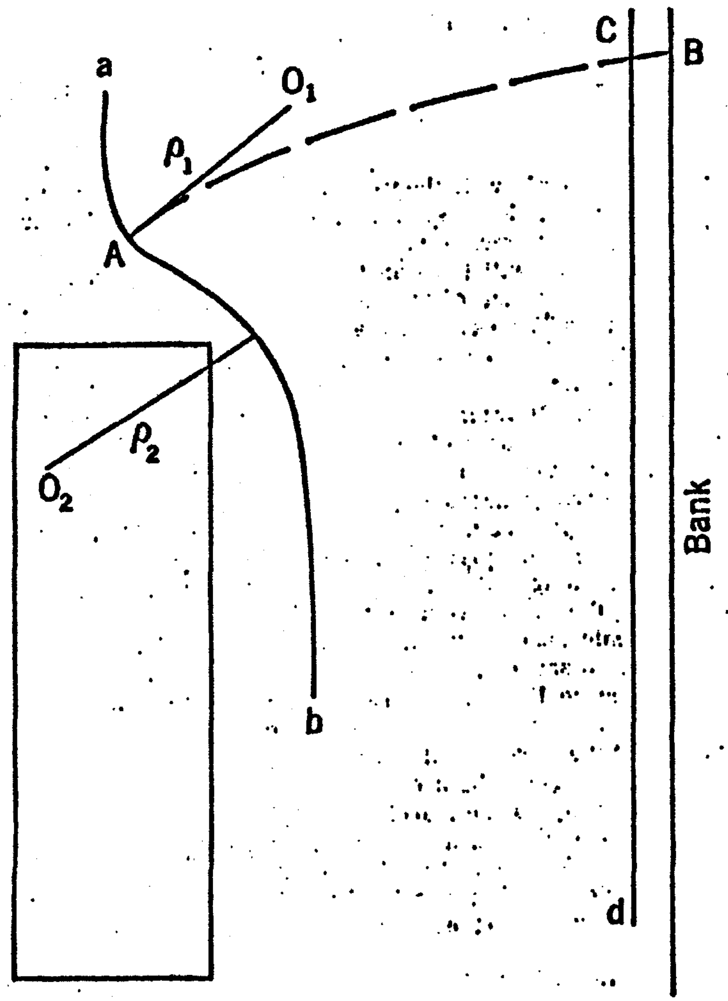


FIG. 13

Behind the pier, with the lens-shaped section, the curvatures of the type 2 was followed by another curvature of type 1, and the formation of a secondary smaller scour was to be observed just behind the pier.

Many other results could be deduced from three first considerations and also from the consideration of the spiral motion induced by the diving motion

deviated by the reaction of the bottom. Further results can be found in some of the writers publications.

The same theory can be used for the study of the action of groynes and of the motion of sediments in derivations.

REFERENCES

1. Tison, L. J., "Affouillement autour de piles de ponts en riviere," Academie Royale de Belgique, Bulletin de la Classe des Sciences, 5e Serie XXIII, 1937.
2. Tison, L. J., "Recherçhes de Laboratoire sur l'Erosion dan les cours d'eau," Bulletin Société belge d'Etudes Géographiques, T. VIII, No. 2., 1938.
3. Tison, L. J., "Erosion autour de piles de ponts en riviere," Annales des Travaux Publics de Belgique, December, 1940.
4. Tison, L. J., "Au sujet du mode d'action des obstacles en saillie sur les rives et particulièrement des epis," Association Française pour l'Avancement des Sciences, Liege, Belgium, 1939.
5. Tison, L. J., "Erosion à l'aval des barrages," Association Internationale des Travaux hydrauliques, Stockholm, Sweden, 1948.
6. Tison, L. J., "Répartition du débit solide d'un cours d'eau se divisant en plusieurs bras naturels ou artificiels," Association Internationale des Congres de Navigation, Congrès de Rome, 1953.

S. V. CHITALE.[31]—In connection with estimation of scour at piers and abutments of bridges, Mr. Laursen has stated that comparison of model and prototype data indicated that the depth of scour could be treated as simply another length and that equilibrium depth of scour obtained in the field. This concept of equilibrium depth is due to the author and more clearly enunciated by the following quotation:[2]

> "At least as a first approximation the equilibrium scour depth, with certain qualifications as to the flow conditions, appears to be a function only of geometry, i.e. the relative depth of flow, the shape of the pier and the angle of attack. . . velocity of flow and sediment size (and, therefore, rate of transport and intensity of boundary shear) do not influence the equilibrium depth of scour. . . ."

Some investigations in models have been made on the subject of scour depth at bridge piers in India at the Central Water and Power Research Station, Poona which the writer thought would be of interest in context with the author's findings.

The first series of experiments were conducted in connection with the Hardinge Bridge over Ganga River and were reported in the Annual Reports of the Station for the years 1938 to 1942. A geometrically similar replica of Hardinge

[31] Chf. Research Officer, Flood Control Div., Central Water and Power Research Station, Poona, India.

[2] "Scour Around Bridge Piers and Abutments," by E. M. Laursen and A. Toch, Iowa Highway Research Board Bulletin No. 4, May, 1956.

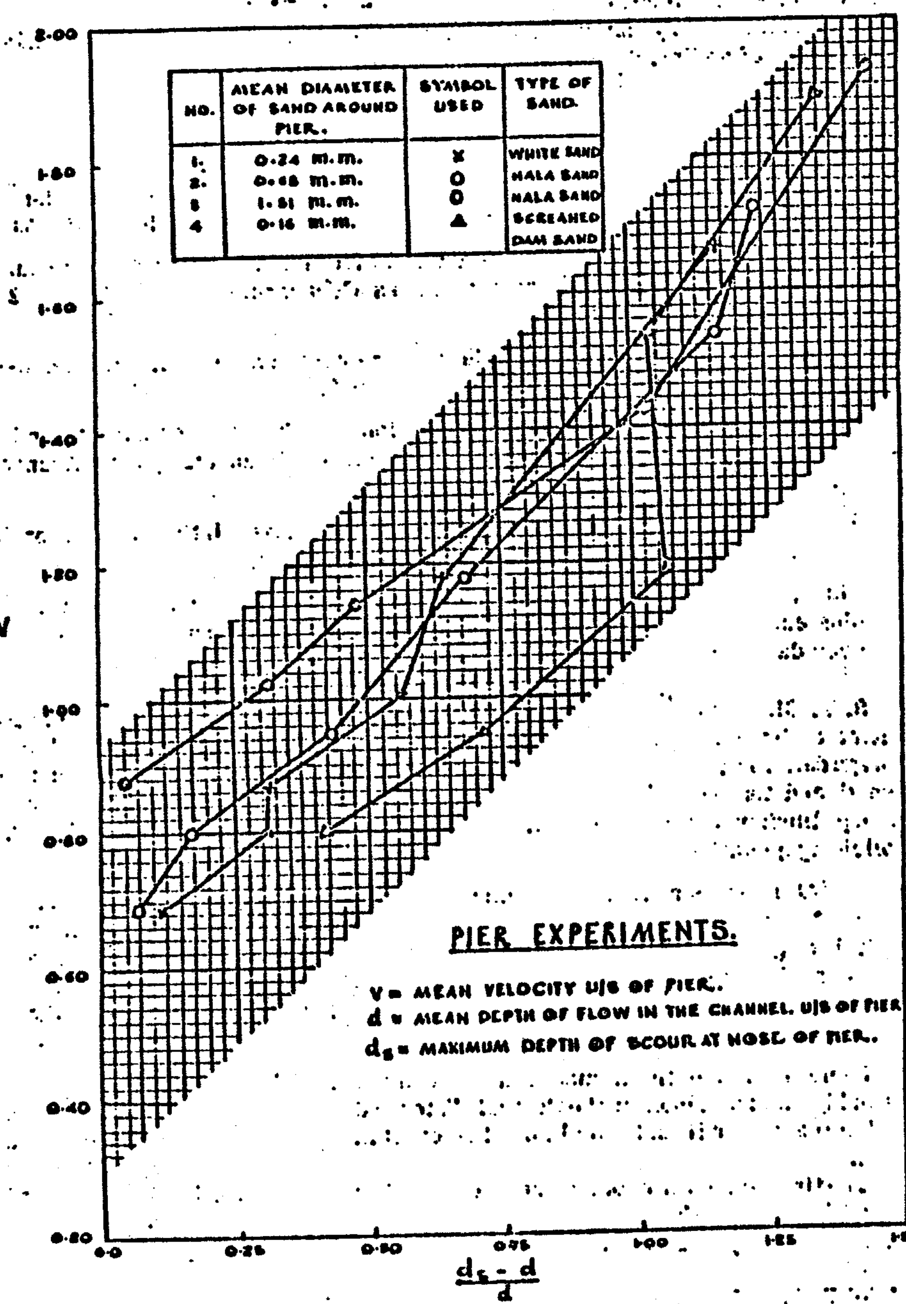


FIG. 14.—PIER EXPERIMENTS

CHITALE ON SCOUR

Bridge pier was embedded in Ganga sand of mean diameter of 0.20 mm in a parallel sided channel. The results of these experiments gave the relation

$$\frac{ds}{d} = 1.70 \left(\frac{q_c^{2/3}}{b}\right)^{0.78} \qquad (20)$$

in which ds = maximum depth of scour below H.F.L.; d = depth of flow in the flume; b = width of pier; and q_c = discharge per foot run upstream of the piers.

TABLE 2.—RESULTS OF EXPERIMENT

Experiment No.	$V = \frac{q}{d}$ in ft per sec	ds in ft	d in ft	$\frac{V}{\sqrt{gd}}$	ds − d in ft	$\frac{ds - d}{d}$	Mean diameter of sand around pier, in mm
1	2	3	4	5	6	7	8
1	0.69	1.60	1.45	0.1005	0.15	0.103	0.24
	0.80	1.63	1.25	0.1261	0.38	0.304	
	0.87	1.51	1.16	0.1428	0.36	0.313	White
	1.01	1.55	0.99	0.1789	0.56	0.563	V
	1.18	1.40	0.85	0.2254	0.55	0.647	sand
	1.67	1.30	0.60	0.3799	0.70	1.170	
2	0.69	1.35	1.45	0.1005	0.10	0.069	0.68
	0.80	1.45	1.25	0.1261	0.20	0.160	
	0.95	1.50	1.05	0.1631	0.45	0.428	nala
	1.16	1.43	0.85	0.2254	0.58	0.682	sand
	1.54	1.41	0.65	0.3366	0.76	1.170	
	1.73	1.30	0.58	0.4002	0.72	1.240	
3	0.88	1.13	1.08	0.1400	0.05	0.046	1.51
	1.03	1.28	0.97	0.1841	0.31	0.320	
	1.14	1.30	0.88	0.2140	0.42	0.478	nala
	1.43	1.42	0.70	0.3012	0.72	1.030	sand
	1.93	1.28	0.52	0.4762	0.76	1.460	
4	0.80	1.77	1.25	0.1261	0.52	0.417	0.16
	0.95	1.60	1.05	0.1631	0.75	0.715	
	1.18	1.75	0.85	0.2254	0.90	1.060	
	1.54	1.33	0.65	0.3366	0.68	1.040	
	1.89	1.25	0.53	0.4575	0.72	1.360	

Pier experiments: In the 8 ft channel with sand of m = 0.32 mm

Scale of pier: 1/65 b = width of pier = 0.57 ft ≡ 37 ft
l = length of pier = 0.925 ft ≡ 63 ft
with constant discharge q = 1 cfs per ft
ds = maximum depth of scour round the pier
d = flow depth in the flume

This relation is not dimensionally correct and cannot therefore be adopted for general application.

Basic experiments were subsequently conducted in 1941 with the object of testing the influence of upstream depth and sand diameter on scour round piers. The pier tested in these experiments was also 1/65 scale model of that of the Hardinge Bridge. It was rectangular in section, of length 1 ft, width 0.6 ft and micircular cut and ease waters. The bed of the flume in these experiments s laid with sand of 0.32 mm while the following materials were used just ound the pier in succession.

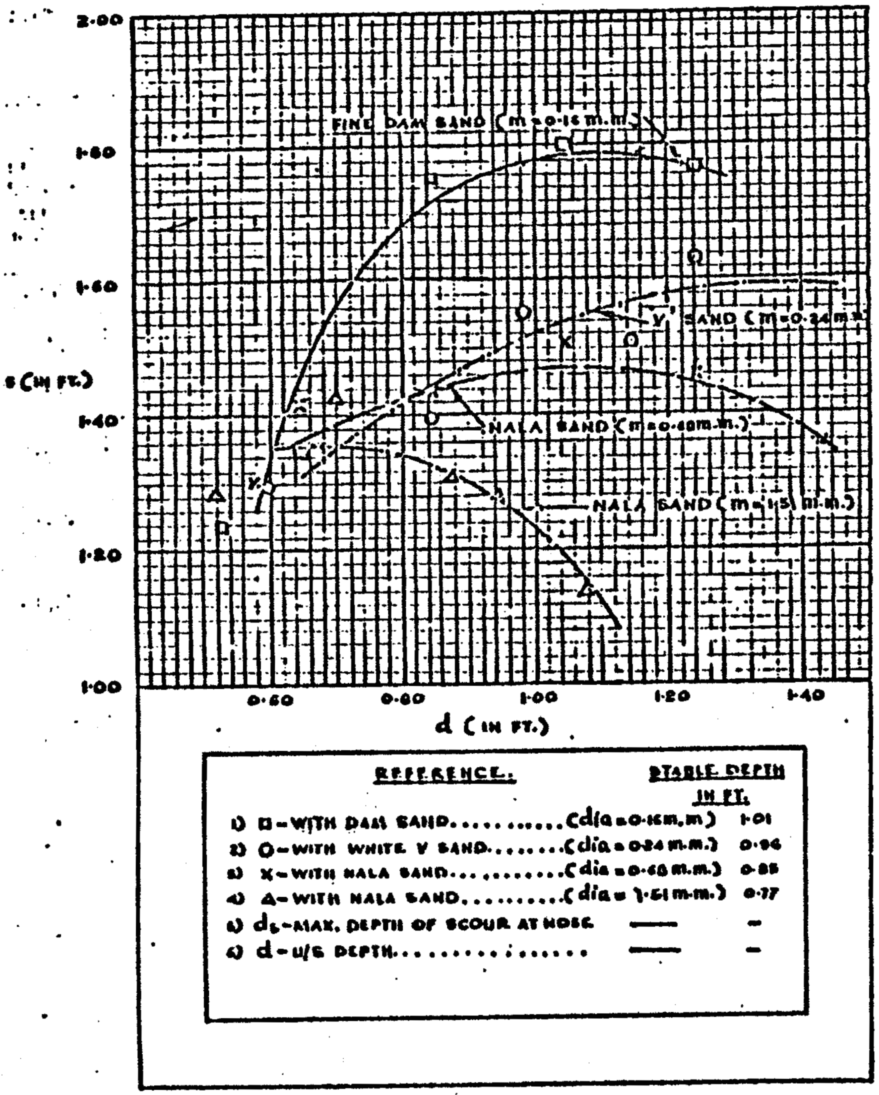


FIG. 15.—1/16 SCALE PIER EXPERIMENTS, ds : d FOR q = 1 cfs

Sieve number	Type of sand	Mean size, in mm
1	screened dam sand	m = 0.16
2	White 'V' sand	m = 0.24
3	Nala sand	m = 0.68
4	Nala sand	m = 1.51

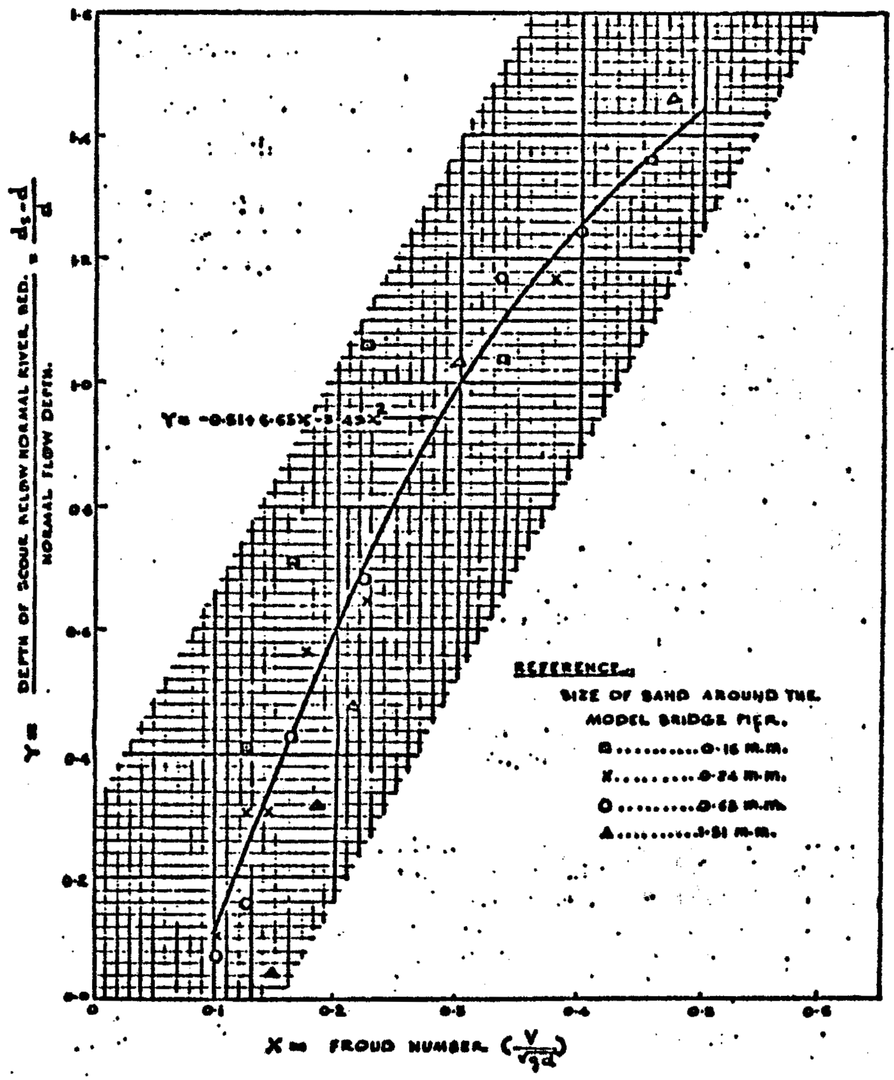


FIG. 16.—RESULTS OF BASIC EXPERIMENTS

The sand round the piers was laid flush with the upstream bed level. A constant discharge of $q = 1$ *cfs per ft* was run and water level was adjusted to get a particular depth, the depths varying from 0.5 to 1.45 ft. Each experiment was continued until final maximum scour was obtained round the pier.

In a few tests in which the upstream depth was less than stable depth the upstream bed scoured and blanketed the scour pit around the pier. In such cases, the maximum depth of scour at the nose was measured just before deposition in the scour hole of sand from upstream occurred.

In experiments when sand around pier was coarser than the bed material upstream, the bed around the pier was laid higher than upstream level, to get scour round the pier for the upstream depth laid.

Table 2 shows the results obtained important conclusions, as follows:

1. With axial flow, maximum depth of scour was always at the nose of the pier, scour at sides being less by 5 to 15%.
2. The ratio of scour at the nose and depth of flow in the channel bears a simple relation with the approach velocity in the channel (see Fig. 14).
3. The depth of flow on the upstream has also an influence on scour at the pier nose (see Fig. 15).

It will be seen that correlation of depth of scour either with upstream depth (Fig. 15) *does* not appear to be satisfactory. The writer, therefore, further analyzed the experimental data and found that the Froud number provides a better criterion. Fig. 16 shows plot of depth of scour against the Froud number. Although some scatter of points is evident in this plot, the general trend is remarkable, statistical equation obtained over the range of experimental data being

$$y = -0.51 + 6.65\,x - 5.49\,x^2 \quad \ldots\ldots\ldots\ldots\ldots \quad (21)$$

It is thus seen that experiments at the Research Station do not lend support to the author's equilibrium scour theory, and it appears that further investigations both in the field and in the laboratories are desirable before it can be accepted unreservedly and with *confidence*.

Grateful acknowledgment is made of the kind permission given by M. G. Hiranandani, Director, Central Water and Power Research Station to refer to experiments previously conducted at the Station and also for useful suggestions *offered* by him.

A. RYLANDS THOMAS,[32] F, ASCE.—In Fig. 17, the author's design curve for piers (Fig. 9), with a factor of 0.9 for application to piers with semicircular nose form, is compared with results of experiments with models of piers carried out by the writer in association with Sir Claude Inglis.[33,34] These piers were models of the piers of the Hardinge Bridge over the Ganges River (now in East Pakistan) which were 37 *ft* wide and 63 *ft* long, including the semicircular nose and tail. The model piers were fixed in a parallel-sided channel with a bed of incoherent Ganges *sand* about 0.3 mm mean diameter and the procedure was to run a constant discharge without sediment load until scour had ceased.

It will be seen from Fig. 17 that the author's design curve, and still more his curve from the Iowa data, Fig. 3, lie well above the Poona results. The difference appears to be too great to be explained by sediment load,[21] though this cannot be ruled out as the upstream depth may be reduced more by sediment load than is the depth of scour at the pier. It would be of value if the author presented the data on which he based his design curve, showing particularly the effect of load.

It is also most desirable to compare small-scale results with observations made under full-scale conditions. Such data are difficult to obtain because of

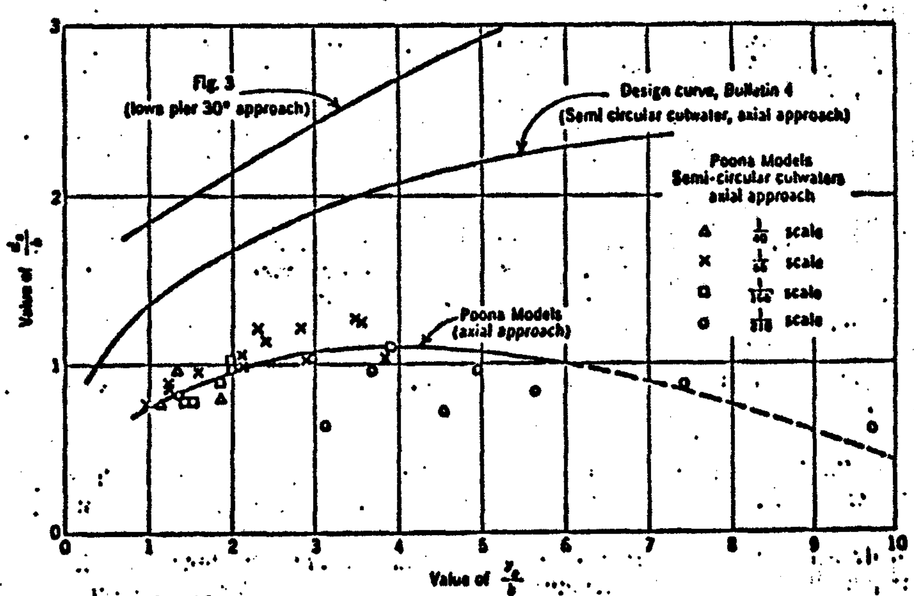


FIG. 17.—SCOUR AT BRIDGE PIERS

the need to take observations during high floods which are very often of short duration. It is easier to measure depth of scour at the pier than in the channel upstream, where a number of observations must be taken to average out *the effect of* bed waves. Even when this is done it is not certain that the bed was in equilibrium at the time of observation. The Poona experiments showed that a reduction in channel depth, due for example to resistence to scour, would increase the depth of scour at the nose of the pier.

The relationship between depth of scour at the nose of the pier and the depth of channel upstream is, therefore, perhaps not the most suitable one for comparison of full-scale data. Nor is it generally the most suitable for practical application of a design formula, because the upstream depth during a maximum

---

32 Cons. Engr., London, England.

33 Annual Reports (Technical), Central Irrigation and Hydrodynamic Research Station, Poona, India, 1938-39, p. 39; 1939-40, p. 33; 1940-41, p. 35.

34 "The Behaviour and Control of Rivers and Canals," by C. C. Inglis, Central Waterpower Irrigation and Navigation Research Station, Poona, India, 1949, p. 327.

Table IV. Computation of Total Sediment Load for Underlying Layer

d(mm)	p	$(d/y)^{7/6}$	$(\tau_o'/\tau_c - 1)$	$f(\sqrt{gyS}/w)$	$\bar{c}$(% by weight)	% of total load
			q = 80 cfs/ft, d_m = 17.2 mm			
110	0.2	0.0346	0	6.5	0	0
48	0.2	0.0132	0.64	7.1	0.012	1.2
17.2	0.2	0.00398	3.60	8.2	0.024	2.6
3.2	0.2	0.00057	23.8	15	0.041	4.2
	0.2	0.000052	188	450	0.880	92.0
					0.957	

at the head of the reach drops. As shown in Figure 9, the backwater curve for a rate of flow of 80 cfs/ft, an n value of 0.03, and a reach degraded six feet is only a little over 1000 feet. If the degradation was triangularly shaped; six feet at the upstream end, zero at the downstream end, the backwater curve would be longer, but not long enough to decrease the competence of the flow to the extent necessary to permit the bed to be armoured.

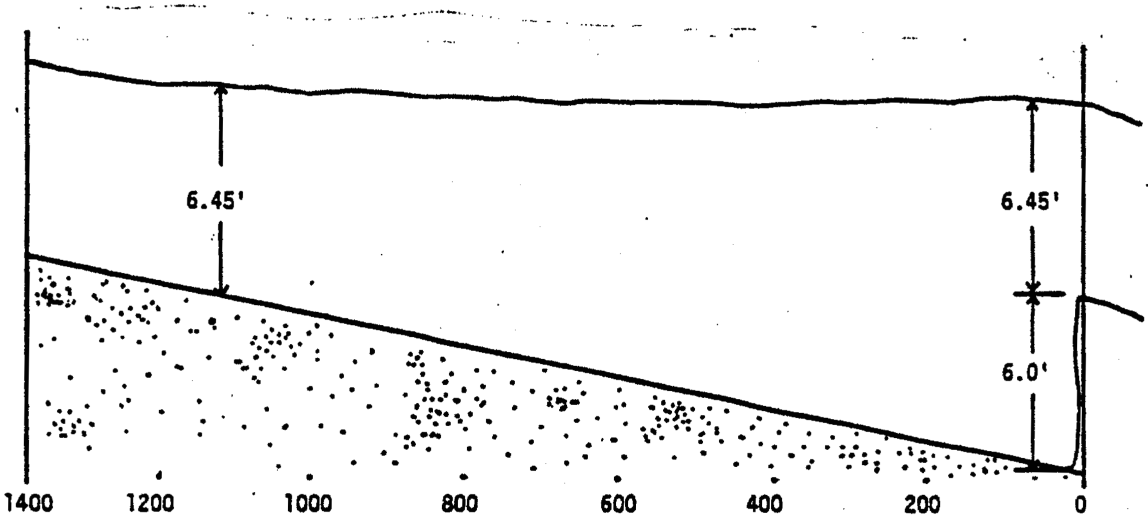


Figure 9. Backwater in degraded reach.

The important number in Table IV is the indication that at q = 80 cfs/ft the coarsest 20% should not move and the bed should armour. Degradation of two feet should leave a layer of 4-inch and larger cobbles. Thus, a total degradation of eight feet--the six foot thick surface layer plus two feet of the underlying layer--should be added to the expected local scour of four feet. The total probable maximum scour at the Queen Creek emergency spillway would then be twelve feet. One cannot be absolutely certain of this limit, but it is difficult to imagine it would be deeper if the unit discharge of about 80 cfs/ft is the maximum probable flow to be expected. The final design studies should refine and confirm these estimates.

The reach immediately below the spillway will quickly degrade the first six feet and should be a rectangular incised channel almost equal to the depth of flow. The flow should be straight out from the spillway tending to keep the flow and degrading channel straight. The flow that escapes to the side will lesson the sediment-transporting capacity of the flow, therefore, the time of degradation will be somewhat longer. The flow that escapes to the side will also be reduced in its capacity and competence to transport and will tend to drop a part of its load and build a natural levee, which in turn helps to keep the flow in its initial width.

Severe bank scour and widening or instability of the bank line is only probable if the flow becomes supercritical because dunes and roughness disappear and the n value goes down to about 0.02. Sediment-transporting, supercritical flow is unstable because an oblique supercritical wave results in a lowered transport capacity and deposition beyond the oblique wave--which makes the wave worse and then the deposition worse. Even this worst conceivable state of affairs, however, should not result in a migration of the bank line over to the principal spillway.

RECOMMENDATIONS FOR FURTHER STUDY BEFORE FINAL DESIGN

Local Scour

The Queen Creek emergency spillway structure is so wide that the basic model studies can probably be performed on a slice of the spillway several baffles wide; arranged so that the end slice with wing wall can be checked in the final design. The generalized studies that have been made of the baffled-apron spillway seem to leave little room for improvement of the recommended design geometry, but several questions should be answered regarding the local scour to be expected.

1. *What is the limiting, equilibrium scour?* With a two-dimensional model with one side of glass, a time history of the development of the scour hole should answer this question. The 0.5 mm sand used in previous studies should be used again. Does any self-sorting occur, creating an armour layer?

2. *What is the effect of sediment size on the depth of local scour?* Several sands should be used to trace out the function between relative scour depth and the nominal boundary shear/critical tractive force. If a 1:16 model and the 0.5 mm sand are used, then the other sands might be 0.2 mm, 1/16 inch, 1/8 inch, 1/4 inch and 1/2 inch. These sands should not be single sized but neither should they contain such a wide range of particles that self-sorting is an important factor. A pair of tests with 1/4 inch sand laid on a fine sand in the geometry the 1/4 inch sand takes by itself should be interesting; one of the sands should be the one used in determining the effect of sediment size, the other should simulate the coarsest

quarter of the coarse sand-gravel-cobble layer at Queen Creek. It would be much more convincing and satisfying if a 1:4 model with comparable sands could also be run; it is not good policy to change only one variable in a dimensionless grouping--if possible all variables should be varied.

3. *Should the depth of scour be measured from the original ground surface or from the tailwater surface?* Usually the depth of scour measured from the streambed is most meaningful because this is the dimension of the hole. In this case, however, the jet is slowed down in the pool and it might be better to measure the scour from the tailwater (or pool water) surface. The downstream half of the scour hole, and therefore the original stream bed evaluation might have only a small effect on the scour depth. A few runs with the tailwater constant and several original stream bed elevations should answer this question. Care should be taken to observe any self-sorting that could effect the results.

4. *Does the position of the baffles relative to the tail-water surface or the scoured streambed have any significant effects?* Anywhere in the pool, the baffle piers can serve as deflectors turning the jet toward the horizontal. Does this happen? Does it matter where it happens?

Degradation

Further computations of expected degradation might not be especially useful, but there are some further questions that could be investigated with possibly useful results.

1. *Have there been any field observations of degradation for comparable flows and sediment sizes?* Even if the flows have not be as large, if the

τ_o'/τ_c ratio is similar, it should be possible to interpret the field measurements.

2. *Is there going to be severe degradation downstream of the CAP aqueduct due to flows less than the 100-year flood?* The controlled releases should not result in degradation greater than the limit expected in the extreme floods over the emergency spillway, but the possible effect should be examined.

3. *Could the CAP aqueduct be built incorporating a rock dike that would fix the bed and water surface at that point and thereby limit the degradation below the Queen Creek spillway?* If the alignment of the CAP is changed to bring it closer to the Queen Creek structure, this would be a possibility but whether the savings on the Queen Creek structure could offset the increased cost for the CAP canal, would also have to be determined.

4. *How deep is the underlying sand-gravel-cobble layer?* This question is not related to degradation, but the total scour at the spillway. The test pits went down about twelve feet and this is the predicted depth of total scour. If a finer layer is beneath the sand-gravel-cobble layer at twelve feet, the bottom of the scour hole should be carefully lined with cobble to be sure it stays in place. If a finer layer is exposed at this elevation, the local scour could go down quite a bit and undermine the structure.

APPENDICES

flood is not often known beforehand and would have to be calculated. An error in assessing this depth would lead to a corresponding and greater error in estimating the level to which scour is liable to occur.

A more basic relationship is that between depth of scour below water level, D_s, width of pier, b, and upstream discharge per unit width, q. The relationship indicated by the Poona results is

$$\frac{D_s}{b} = 1.70 \left(\frac{q^{2/3}}{b}\right)^{0.78} \qquad (22)$$

in ft-sec units, applicable to scour in fine sand within the limits of $q^{2/3}/b$ or y_o/b at least from 1 to 6 (see Fig. 17) and not greatly affected by the grade of sand. It is necessary to take into account the effect of obliquity of approach flow, which may be done by using for b the projected width of pier on a plane normal to the direction of approach. A factor of safety should also be used in design.

The discharge per unit width to be used for design must, in most cases, be greater than the mean value because in rivers the deep channel meanders from one bank to the other. It is therefore advisable to use, at least as a check, the relationship proposed by Inglis[34] giving the maximum depth of scour around a pier as approximately twice the Lacey regime depth, that is,

$$D_s = (Q/f)^{1/3} \qquad (23)$$

in which D_s is the depth of scour in feet below water surface, Q is the discharge in cu sec, and f is the Lacey silt factor (approximately $1.8\sqrt{m}$ where m = mean diameter of bed sand in mm). Eq. 23, derived from data of several bridges in India and Pakistan, takes into account obliquity of approach and concentration of discharge but not the width of pier, which is clearly an important factor. The data related to normal widths of pier and to depths of scour ranging from 25 ft to 117 ft.

MUSHTAQ AHMAD.[35]—The problem of correct estimation of depth, shape, and extent of localized scour is important from the point of view of establishing sound design practice and safety of hydraulic structures such as bridge piers, abutments, spur dikes, groins, or pitched islands. This problem is much more important for hydraulic structures in alluvial rivers of West Pakistan where fine sand is found for hundreds of feet in depth, and scour depths of 40 ft to 80 ft below water level are common. The author has, therefore, dealt with a subject of special importance and great utility to this region.

It is proposed to discuss the author's approach to the problem with special reference to field and laboratory experience in West Pakistan. The author has assumed the depth of scour as another length and has related it to the normal depth or the width of a pier, and maintains that the depth of scour does not depend on the degree of concentration until scour holes around neighboring obstructions overlap. He maintains that there is equilibrium or limiting depth and believes that for a given mode of sediment movement, the depth of scour depends only on the geometry of contraction and the approach depth. He holds that the effect of velocity and sediment size can be neglected as that of secondary order. The writer agrees that there is an equilibrium and limiting depth of

[35] Dir., Irrigation Research Inst., West Pakistan.

scour and that the effect of sediment size can be neglected. Tests made on depth of localized scour at spur dikes[36] also showed that localized scour depth does not vary with grain size in the range (0.1 m to 0.7 m) usually met with in alluvial plains of West Pakistan. However, this concept may not be valid for the entire range of bed material sizes ranging from fine sand to gravel and boulders.

The author's view that scour depth is another length connected with normal depth or width of pier implies that it varies with the discharge, or more correctly with the discharge per foot run. The author's Eq. 8b and c holding for rivers and streams in West Pakistan are:

$$\frac{d_s}{y_1} = \left(\frac{B_1}{B_2}\right)^{.64} - 1 \quad \text{for } \frac{\sqrt{g\,y\,s}}{w} = 1 \qquad (24a)$$

and

$$\frac{d_s}{y_1} = \left(\frac{B_1}{B_2}\right)^{.69} - 1 \quad \text{for } \frac{\sqrt{g\,y\,s}}{w} < 2 \qquad (24b)$$

where the exponent varies between 0.64 to 0.69, or on the average the relation can be written as

$$\frac{d_s + y_1}{y_1} = \left(\frac{Q/B_1}{Q/B_2}\right)^{.64-.69} \sim \left(\frac{q_2}{q_1}\right)^{2/3} \qquad (25)$$

$$\frac{y_2}{y_1} = \left(\frac{q_2}{q_1}\right)^{.64-.69} \sim \left(\frac{q_2}{q_1}\right)^{2/3} \qquad (26)$$

and

$$\frac{y_2}{q^{2/3}} = \frac{y_1}{q^{2/3}} = K \qquad (27)$$

in which K may be a function of boundary geometry of the contraction at the bridge, abutment shape and thickness and shape of the nose of the piers, and so forth. Because the depth is a dependent variable on discharge intensity, the latter may be used in preference to depth. Here only lies the difference in approach between the author and the writer in studying this problem. As shown previously the two approaches are not very different. The writer prefers to study the variation of $D_s/q^{2/3} = K$ as a function of boundary geometry, shape of pier nose, and abutment, characteristics of bed material, and distribution of velocity in the cross section at the piers representing the concentration of flow. The functional relation for study may be of the type:

$$\frac{D_s}{q^{2/3}} = K f\left(\theta, \frac{V^{1/2}}{V}, \frac{\sqrt{g\,y\,s}}{w}, S, \frac{B}{B_t - nt}\right) \text{ etc.} \qquad (28)$$

in which θ is the angle between the current and the spur dike, an abutment or a pier, $V^{1/2}/V$ is the ratio of mean velocity in half the channel width on the side of spur dike or abutment to the mean velocity in full section. It will be a

[36] "Experiments on Design and Behaviour of Spur Dykes," by Mushtaq Ahmad, Proceedings, Minnesota Internatl. Hydr. Convention, September 1-4, 1953.

measure of concentration of flow; W is fall velocity of bed material; B is the river width upstream of the bridge; B_t is distance between the abutments; n is the number of piers,; and t is thickness of each piers.

The writer does not agree that the depth of scour does not depend on the degree of concentration. It has been found that by changing the concentration or

TABLE 3.—VALUES OF $V_{\frac{1}{2}}/V$ AND K

Approach condition (1)	$V_{\frac{1}{2}}/V$ (2)	K (3)
Scour below a severe bend on the concave side accompanied by a swirl on the convex bend.	1.25	2 - 2.25
Moderate Bends.	1.15	1.5 - 1.75
Straight obstruction placed at any angle of 30° to 90°	1.0	1.2 - 1.5
Straight obstruction placed at an angle of 90° - 150° to the flow	1.0	1.5 - 1.75

TABLE 4.—MAXIMUM SCOUR DEPTHS OBSERVED AT BRIDGE PIER AND ABUTMENTS ON DIFFERENT MODEL STUDIES AT IRRIGATION RESEARCH INSTITUTE, LAHORE.[b]

River	Site	Q Max, in cu. sec	q	Scour depth: Observed	Scour depth: Scale distortion factor.	Scour depth: Corrected scour depth.	$K = \frac{D_s}{q^{2/3}}$	Y_o	Thickness of pier (b)	$\frac{Y_o}{b}$	$\frac{ds}{b}$	$D_s = ds + Y_o$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
Ravi	Shahdara Bridge	332,000	237	52	1.23	64.0	1.68	26.4	10	2.64	2.06	47
Jhelum	Jhelum bridge[a]	600,000	128	30	1.3	39	1.60	23	10	2.3	1.98	42.8
	Mangla New bridge	275,000	429	90	1.0	90	1.60	28	abutment	..	..	..
Deg	S-harakpur	40,000	200	34	1.34	46	1.34	21.6	6.25	3.45	2.22	35.6
Rohi	Kasur	35,000	147	30	1.3	39	1.4	14	6.25	2.23	1.97	26.3
Sohan	Dhok Pathan	150,000	172	36	1.17	42	1.36	18.6	9.0	2.07	1.94	36.1
Indus	Thatha Sujjawal proposed bridge.	1,100,000	275	52	1.31	68	1.6	25	7.0	3.57	2.25	40.8

[a] Computation from Fig. 9, Bulletin No. 4.
[b] Bed material - five sand.

velocity distribution at bridge site by training works upstream, the scour depth can be considerably reduced. The variation of scour depth as a result of change in velocity and flow distribution due to the training works is explained in Figs. 18, 19, and 20. Figs. 19 and 20 depict a model of the Ling Stream showing a bridge below a curve. The concentration of flow near the left flank is notice-

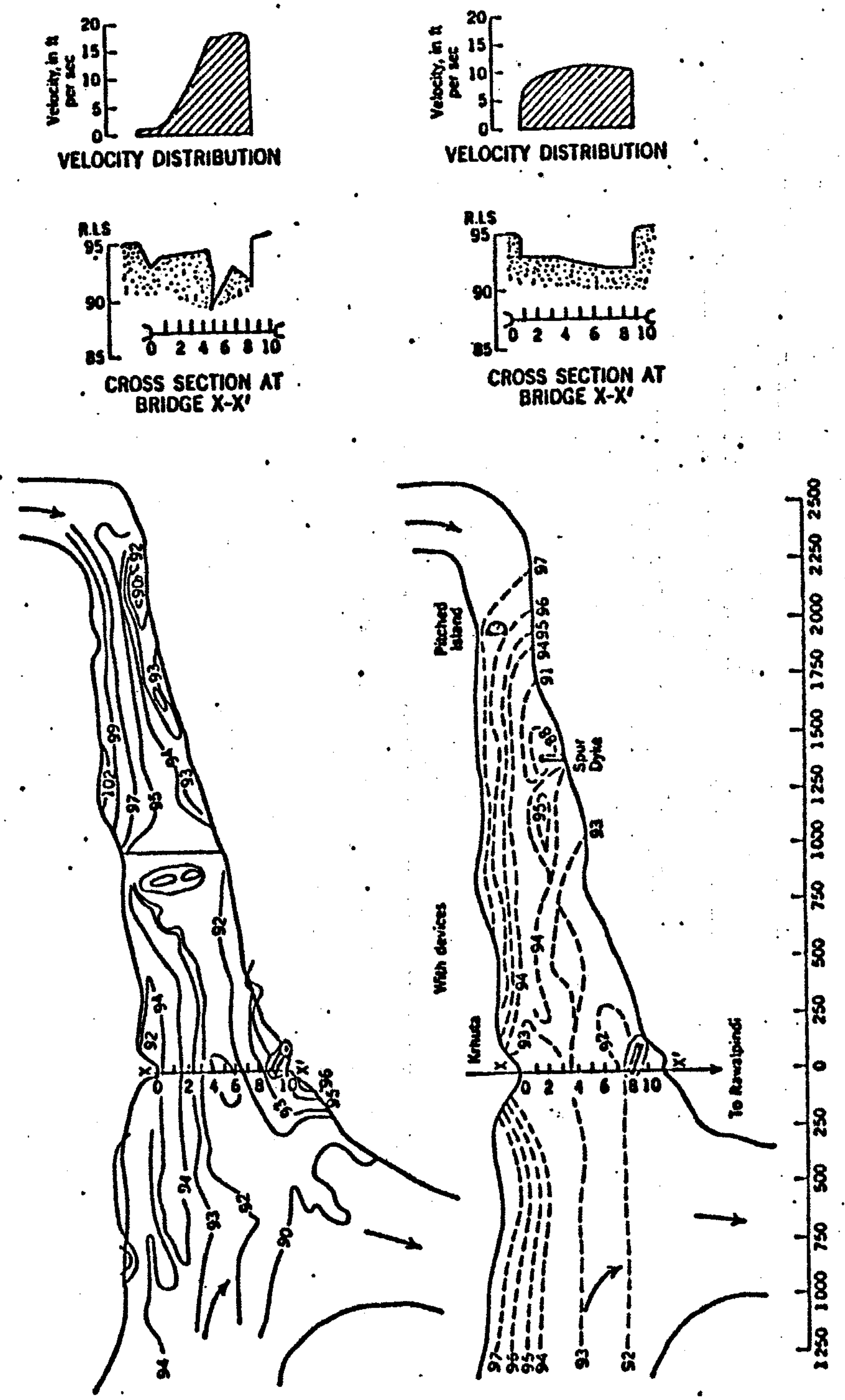


FIG. 18.—LING NALLAH BRIDGE

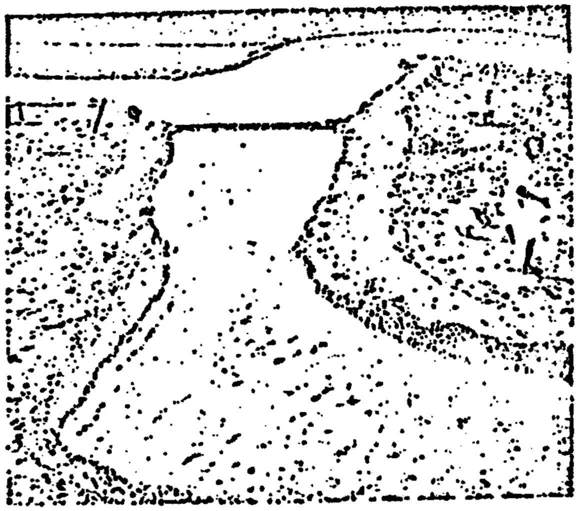
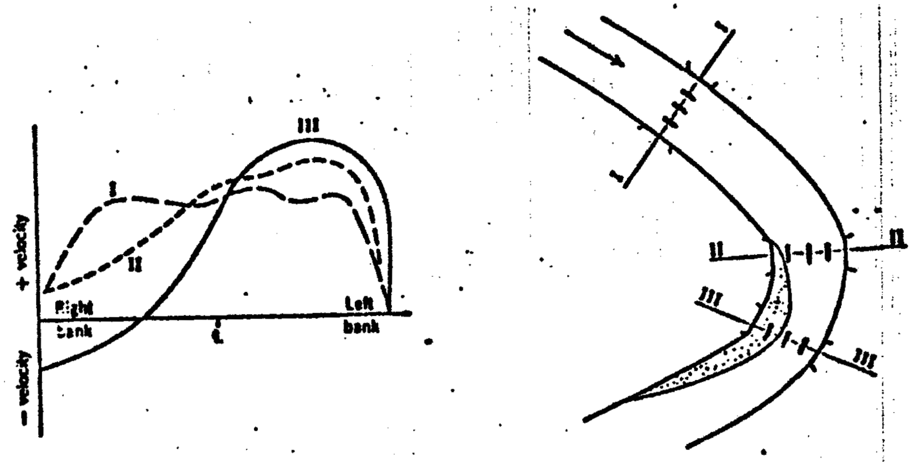


FIG. 19

FIG. 20.—FLOW AT BRIDGE SITE



(a) VELOCITY DISTRIBUTION AT BRIDGE SITES I, II, AND III

(b) THREE BRIDGE SITES IN A CURVE

FIG. 21.—THREE BRIDGE SITES IN A CURVE

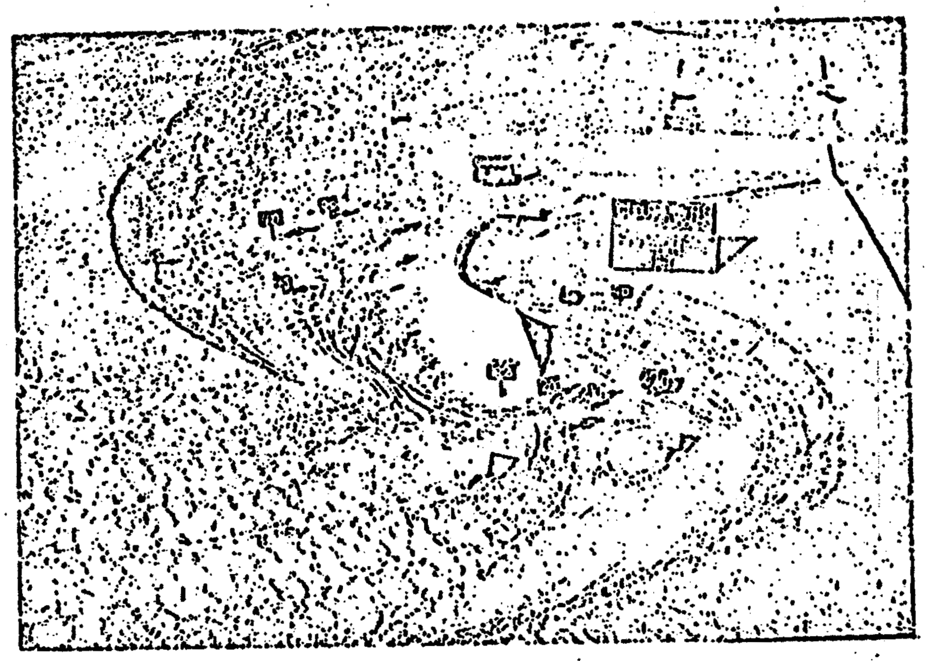


FIG. 22.—ABNORMAL SCOUR AT A GUIDE BANK HEAD

able in Fig. 19. A pitched island and a spur on the left bank fans out the flow at the bridge site to obtain more uniform velocity and lesser scour as shown in Fig. 20.

In the method of plot adopted by the author in which scour depth in relation to U/S depth has been studied, the non-dimensional scour perimeter may not have significant relation with velocity, for, in open channels as the velocity or

TABLE 5.—SHAHDARA RAILWAY BRIDGE ON RIVER RAVI[a]

S. No.	Year	Q	H.F.L.	Gauge B	Water-way at the bridge B, in feet	q	Max. Scour R.L.	Max. Scour Ds	$K = \frac{DS}{q^{2/3}}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
1.[b]	1948	86000	688.0	11.7	12 x 90 = 1080	79.6	651.14	36.86	1.989
2.[c]	1949	52000	686.0	9.7	8 x 90 = 720	72.2	657.14	28.86	1.664
3.[c]	1950	1,93000	692.0	15.7	15 x 90 = 1350	142.9	651.2	40.8	1.49
4.[c]	1951	44000	686.3	10.0	7 x 90 = 630	69.8	656.14	30.16	1.77
5.[c]	1952	56000	687.2	10.9	9 x 90 = 810	69.13	655.0	32.0	1.63
6.[c]	1953	83000	689.1	12.8	11 x 90 = 990	83.75	651.64	37.46	1.95
7.[c]	1954	1,72000	691.8	15.5	15 x 90 = 1350	127.5	655.0	36.8	1.453
8.[c]	1955	250,000	695.0	18.7	15 x 90 = 1350	105.18	660.0	35.0	1.078
9.[c]	1956	87000	688.3	12.0	12 x 90 = 1080	80.5	657.22	31.08	1.66
10.[c]	1957	1,92000	692.3	16.0	15 x 90 = 1350	142.2	654.64	37.66	1.383
11.[c]	1958	1,52000	691.2	14.9	14 x 90 = 1260	120.61	662.64	28.56	1.17

[a] Part of the water way is marked by a semi erodable island at the bridge and hence the difference in the water way for different years. [b] No training works a river bed u/s. [c] A spur constructed u/s to current approach.

discharge increases so does the depth, although not necessarily by erosion of the bed. For a uniform approach velocity, a mean depth can be assumed for use in the author's relations. But, generally, abnormal scour depths at piers or abutments are associated with concentration of flow resulting from the variation of velocity and depth in the approach section and the selection of representative approach depth for use in the author's relation and the estimation of

In fact, the author's term $(Q_c + Q_o)/Q_c$ in Eq. 9 is, also, a measure of concentration of flow. However, the estimation of Q_c, Q_o, or Q_w is difficult for practical use in the computation of maximum scour depth in case of a curved approach. The writer has shown[37] that change in flow concentration above a spur dike, can be depicted in terms of velocity distribution and common types are depicted in Fig. 21 by I, II and III. The type III velocity distribution gives the maximum concentration on the outside of a bend as due to negative velocity on the inside, a part of the water way is blocked by reverse flow. For the three types of velocity distribution depending on different types of approaches, the values of V_1/V and K as determined from writer's studies on spur dikes (which may be applicable to abutments), are given in Table 3. Since maximum scour can occur on any one or a group of piers in a meandering alluvial channel, the depth of foundation of piers has to be computed from the values of K as fixed by the river curvatures likely to occur, keeping in view the restraints imposed on river by the upstream training works. The method commonly used in West Pakistan supported by Laboratory studies and field observations consists in working out $q = \frac{Q_{max}}{B}$.

At the bridge site, the value of K can be selected from the maximum curvature likely to occur. This will give scour depth below water level, and the depths below bed level can then be worked out. Maximum depth of scour from model studies at bridge piers and abutments of different rivers and streams, after correcting the results for the effect of scale distortion on geometry of scour,[37] are summarized in Table 4. In this table, scour depth has been computed from the author's design curves of Bulletin No. 4, in Fig. 9. The scour depths obtained after correcting for the model scale distortion are generally higher than those of the author. In computing D_s value by his method, the actual mean value of Y_o has been taken from the cross section above the bridge. The value of K does not exceed 1.7 because severe bends are not possible in these cases due to the presence of guide banks.

Fig. 22 shows abnormal scour at a guide bank head. A heavy embayment is noticeable on the right and construction due to silting caused by roller on the left. The K value of 2.3 can be obtained in such a case.

The heavy embayment resulting in abnormal scours illustrated in Fig. 22 with K value greater than 2 are possible only on spur dikes or guide bank heads and are not to be allowed on bridge piers or abutments. In fact, for alluvial rivers or streams of West Pakistan, guide banks at least equal to the length of the bridge, with curved heads and expanding water way on the upstream side, are provided to shift the maximum embayment and abnormal localized scour to the guide bank heads instead of allowing it to occur near the bridge with short abutments. Under these conditions, the values of K for estimating the maximum probable depth of scour can safely be taken between 1.7 and 2.0. In case of a short abutment length before the conditions for abnormal scour where K greater than 2 are obtained, the approach roads or railway will be threatened and cut by the embayment formed by an alluvial meandering river. It is therefor necessary to design pier and abutment depths for scour calculated from K 1.7 to 2.0 and provide proper guide bank to keep the road or railway ap-

37 "Effect of Scale Distortion, Size of Model Bed Material and Time Scale on Geometrical Similarity of Localized Scour," by Mushtaq Ahmad, Proceedings, Internatl. [illegible]

proaches safe from embayment and to keep abnormal scour away from the main bridge crossing. Actual scour depths observed on the railway bridge on Ravi near Lahore and the computed values of K as recorded in Table 5 also supported the recorded values of K.

PIER LUIGI ROMITA.[38]—The author's long lasting efforts to cast light upon the phenomenon of scour around bridge piers and abutments are to be greatly commended because of the great practical importance of the problem. The clear and condensed presentation in this paper of the results of these efforts is a substantial step towards a generalized solution of the problem, and will be of great use to designers and to the administrations responsible for the construction and maintenance of roads and railways. To this end a coordinated effort should be made, in order to insure the necessary verification on prototypes of the proposed relationships.

There is no doubt that one of the most important factors of scour, when piers have a sufficiently high length per width ratio, is the angle of attack between the pier and the flow. Even small deviations of this angle from the zero value are responsible for fast increases of the scour; this usually represents a much greater danger to the stability of the pier than any underevaluation of the maximum possible flood. The collapse of many piers during floods is, in fact, to be ascribed more to the angles of attack, which may occur due to unusual cross-currents and deformations of the river bed, than to the unexpected entity of the discharge.

In view of all this, the writer carried out, some years ago, a systematic model investigation of the influence of the angle of attack upon the depth of scour. The experiments were carried out in a glass-flume of the Hydraulic Laboratory of the Polytechnic Institute of Milan, supplied with clear water, using uniform sand of 1 mm diam as bed material, and in such conditions that there was no general bed movement but only localized scour around the pier. The shape of the pier was not particularly studied, and it reproduced a rather widely used type of pier; its length per width ratio was about 5. Two series of tests were carried out with different values of the discharge, while the angle of attack was varied from 0° to 90°.

The data obtained at a zero angle of attack are in rather good agreement with the curve representing Eq. 11 Fig. 9, but for the fact that they consistently lie a little below this curve. An increase by 60% in the discharge (which corresponded to an increase by 20% in the average velocity of flow) brought around only a 20% increase in the maximum depth of scour, at the same angle of attack.

The tests with varying angle of attack have shown the basic importance of this factor. For an angle of 15° the maximum depth of scour was 1.9 times that corresponding to a zero angle; for an angle of 30° the increase of scour depth was in the ratio 2.6:1 in respect to the zero angle, for an angle of 45° in the ratio 3.0:1, and for an angle of 60° in the ratio 3.3:1. Increases of the angle of attack beyond 60° and up to 90° did not bring around any further appreciable increase in the scour depth. These values are considerably in excess of those indicated by Fig. 10.

Another interesting observation was that the point of maximum scour depth always occurs very close to the pier wall, so that, if the depth of the foundation is not sufficient, the pier will easily be undermined, and collapse.

[38] ... Politecnico di Milano, Milan, Italy.

As a conclusion, the fundamental importance of avoiding any angle of attack between the pier and the flow should again be stressed. In order to obtain this, the river banks should be stabilized with adequate measures for a sufficiently long stretch upstream of every bridge crossing. In braided rivers, however, the possibility that an angle of attack occurs should always be taken into account, and the pier foundations designed accordingly.

EMMETT M. LAURSEN,[39] M. ASCE.—The geographical distribution of the discussers and the quality of the discussions indicate the widespread interest in and attest to the import of the problem of scour at bridge piers and abutments and similar obstructions in a stream. Without doubt the crucial issue raised by the analyses is the question of the effect of the velocity of flow on the depth of scour. The position of the writer is that there is a fundamental difference in this regard depending on whether the approach flow supplies or does not supply sediment to the scour hole; that under conditions of no supply, such as a relief bridge, the velocity and the sediment size are important in determining the depth of scour; that under conditions of supply by the approach flow well above the critical tractive force, the velocity and sediment size have little effect except insofar as they determine the mode of sediment movement.

The position of Joklekar, Chitale, Thomas, Ahmad, Blench, and Bradley (either explicitly or by reference) is that the depth of scour is proportional to the two-thirds power of the discharge per unit width. Because the discharge per unit width is the product of the velocity and the depth, for a given geometry (including the depth of flow), the depth of scour should vary with velocity. Alternately, one may rewrite the Poona equation so that $d_s/b = f(y_0/b, F)$. Again indicating that, for a constant geometry, the depth of scour is a function of the velocity. Although the position of Tison and Romita is not entirely clear, one may infer from their analyses that the velocity has an effect on the depth of scour. Interestingly, none of them seemed to stress an effect of sediment size.

The case of the long contraction can be used to illustrate the fundamental difference between clear-water scour and scour in a sediment-transporting stream. The merit of this case for illustrative purposes is that the flow conditions and sediment-transporting competence and capacity can be evaluated with relative confidence and agreement, especially if the complicating features of the zone of non-uniformity and of the ripple and dune formation are disregarded. If one now considers a contraction of some given geometry, two widths and a depth of flow, it is possible for the velocity of flow to be so small that there is no movement of the sediment anywhere. At or below this velocity, that is dependent on the sediment size, there will be no scour, and the flow will behave as if there were a rigid bed. If the velocity is increased, but not to such an extent that there is sediment movement in the uncontracted approach, the contraction will scour. The limit of the depth of flow (or scour) will be a velocity (or tractive force) that will not move the bed sediment. For this case of clear-water flow in the long contraction, it is readily apparent that the velocity of flow and the sediment size, as well as the geometry, will affect the depth of scour. The similar argument for the case of sediment-transporting flow in a long contraction culminated in Eqs. 6, 7, and 8 in which the velocity and depth of flow enter only insofar as they affect the mode of movement.

[39] Assoc. Prof., Dept. of Civ. Engrg., Michigan State Univ., East Lansing, Mich.

The confused flow in the area of scour around a bridge pier or abutment cannot be as well described, but one can expect that qualitatively the effect of velocity and sediment size would be similar to that in the case of the long contraction. Thomas explicitly states that the Poona experiments that resulted in Eq. 22 were run "without sediment load." Eq. 22 rewritten so that

$$\frac{d_s}{b} = 4.05\ F^{0.52}\left(\frac{y_0}{b}\right)^{0.78} - \left(\frac{y_0}{b}\right) \dots\dots\dots\dots \quad (29)$$

will approximate the curve through the Poona data in Fig. 17 with a Froude number of 0.2. The curve for a Froude number of 0.4 would cross the writer's proposed design curve at approximately y_0/b equal to unity and would rise to a value of d_s/b equal to 4.2 at y_0/b equal to 6. Higher values of the Froude number would indicate even greater scour. Chitale, after describing these same Poona experiments, cites further experiments which evidently involved uncontrolled sediment movement at the higher velocities: "In a few tests in which the upstream depth was less than the stable depth the upstream bed scoured and blanketed the scour pit around the pier. In such cases the depth of scour at the nose of the pier was measured just before deposition in the scour hole of sand from upstream occurred."

The data for the higher velocities from Chitale's Table 2 if plotted in Fig. 9 would fall between the design curve (drawn conservatively) and the analytic curve, Eq. 11, as a group of points with d_s/b between 1.19 and 1.33 and y_0/b between 0.91 and 1.23. Blench presents a relationship, Eq. 12, reduced from the Poona equation and based on the same experiments without sediment load, that would also pass through the Poona data in Fig. 17. Romita cites an increase in the depth of scour with an increase in velocity, but again the tests at Milan were run ". . . in such conditions that there was no general bed movement but only localized scour around the pier."

Correspondence with Ahmad established the fact that his experiments were conducted so that there was good general movement of the bed. However, he used the average depth of the stream to obtain the depth of scour by the writer's proposed method and admits that a small angle of approach between the flow and pier can have a considerable effect. A modest increase in depth of flow and a small angle of attack could easily account for the discrepancy between the observed and predicted depths of scour in Table 4.

Blench objects to the use of a total load equation for the development of Eqs. 7 and 8 on the grounds that it is bed load that is mainly effective in determining depth. He feels that the formula is rough, and implies that y is not the regime depth. Eqs. 7 and 8 are for the case of the long contraction. If there is to be mass conservation in regard to the sediment load, a total load relation must be used. The difference in depth of scour is not large for different modes of movement as can be inferred from Eq. 8 and seen in Fig. 2. The writer's sediment load relationships were used because they permitted the evaluation of this effect. Any other sediment - transport equation will give approximately the same results because the load is not treated absolutely but in ratio. The depth y is the regime depth in the sense that it is the equilibrium depth that will not change with time, although it may not be equal to Lacey's regime depth that does not consider the effect of load. If the equilibrium depth does not obtain, one should also consider the first category of scour mentioned as "those characteristics of the stream itself."

The Milan experiments are cited by Romita as differing from the Iowa results with the depth of scour for zero angle of attack lying below the curve representing Eq. 11 on Fig. 9 and the effect of angle of attack being greater than indicated by Fig. 10. Because the conditions of the experiments were different, the Milan experiments being run without sediment load, one can only speculate on the reason for the discrepancy. The lesser scour noted at zero angle of attack may be because of the velocity effect of clear-water scour. Romita, in fact, mentions an increase in depth of scour with an increase in velocity. The greater effect of angle of attack may be related to the affect of turbulence level on the critical tractive force. The critical tractive force is not really the mean value of the shear that will just move the sediment particles (except in laminar flow), but the mean value of the shear when the maximum shear due to turbulent fluctuations will just move the sediment particles. In the disturbed, confused flow in a scour hole one might expect that the turbulence level would be greater than normal, and that the increase would be related to the degree of obstruction. Because the limiting condition for clear-water scour is the critical tractive force on the boundary of the scour hole, it is possible that the effect of geometry such as angle of attack would be magnified over the effect found for the case in which there is a sediment supply.

Similarly the considerable effect of shape found by Tison may be due to the lack of supply. A lesser velocity than used would probably have resulted in all scour depths being reduced, and if a small enough velocity were used, there might not have been any scour at all around the better shaped piers. Thus, the relative effect of shape would have been magnified even more. Tison's demonstration of the causation of secondary flow as a consequence of the horizontal curvature of the stream-lines and the vertical velocity distribution is sound albeit qualitative. How this argument can be extended to the drastic change in the flow pattern after a scour hole has developed, however, is difficult to envision.

Bauer's crossing design should achieve the two ends he obviously intends; limiting the depth of scour to insure the safety of the bridge, and permitting a controlled depth of scour to reduce the backwater. A few words of caution are in order with such a design. The rock layer must be composed of sufficiently large material so that it will not be scoured out during a flood; this is a clear-water scour problem that is not solved as yet (1961). The deeper the layer is placed the smaller the rock can be. The scour will be concentrated around the piers and abutments and may not extend over the entire crossing. An approximate idea of the lateral extent of the scour can be obtained by equating the cross-sectional areas of the arrested and unarrested scour holes.

Bradley's cautionary comments should be heeded by all engineers seeking to use the proposed methods of predicting scour. The relationships proposed have been tested at only one real bridge pier, and this at a site of simple geometry. His plea for field measurements cannot be seconded with too strong a voice.

AN ANALYSIS

Journal of the

HYDRAULICS DIVISION

Proceedings of the American Society of Civil Engineers

AN ANALYSIS OF RELIEF BRIDGE SCOUR

By Emmett M. Laursen,[1] M. ASCE

SYNOPSIS

Based on the proposition that the limit of clear-water scour is a boundar shear equal to the critical tractive force, a relationship for the scour in long contraction as a function of the geometry, the flow, and the sediment ha been obtained. Certain assumptions were made to describe the determinin factors, but any other methods of evaluating these factors should produce simi lar relationships. The solution for the long contraction was modified for th abutment and the pier by assuming that the scour holes forming around thes obstructions would be a multiple of the scour in an imaginary long contractio of the width of the scour hole. An approximation of the modified solution fo the abutment and an approximate sediment-transport equation were used t describe the development of the scour hole with time.

Comparisons of the relationships obtained by these approximate analyse with measurements from several laboratories indicate that the predictions ar reasonably satisfactory and that the method of attack is promising.

STATEMENT OF PROBLEM

A simple crossing of an alluvial valley consists of embankments on t floodplain and a bridge over the river channel. During a flood, the embankmen

Note.—Discussion open until October 1, 1963. To extend the closing date one mon a written request must be filed with the Executive Secretary, ASCE. This paper is pa of the copyrighted Journal of the Hydraulics Division, Proceedings of the Americ Society of Civil Engineers, Vol. 89, No. HY3, May, 1963.

[1] Prof. and Head, Dept. of Civ. Engrg., The Univ. of Arizona, Tucson, Ariz.

obstruct the flow on the floodplain, forcing the entire flow through the waterway opening of the main channel crossing. On the upstream side of the embankments, the water-surface elevation will be greater than normal (in the absence of the crossing), partly because of the backwater due to the valley crossing and partly because a gradient is required for the flow to move laterally across the floodplain to the main channel. On the downstream side of the embankments, the water-surface elevation will tend to be lower than normal because the water is slack for some distance until the flow on the floodplain is re-established.

A secondary bridge placed on the floodplain will divert a part of the flow from the main channel crossing; the "relief" thus obtained presumably permits a reduction in the length of the bridge over the main channel and in the height of the fills.

Flow on the floodplain is liable to be at a low velocity and, therefore, to have a small competence for the movement of sediment. Moreover, the cover on the floodplain will tend to inhibit sediment movement. Thus, a supply of sediment to the vicinity of the relief bridge of the size of the material that could be scoured out is not to be expected. Because of the large difference in elevation of the water surface on the two sides of the embankment, the velocity through the relief bridge opening will tend to be large, and the competence of the flow will be sufficient to move the alluvial boundary material. Assuming such increased competence, scour will occur. As the opening under the bridge is enlarged, the flow through the opening will tend to increase; as the flow increases, the difference in the water-surface elevations on the two sides of the embankment will tend to decrease. The velocity of flow through the opening, therefore, tends to decrease as the scour hole enlarges. The shear forces on the boundary will also decrease—both because the velocity decreases and because the flow section increases. The limit of the scour will be reached asymptotically with time as the boundary shear decreases to a value that will no longer move the boundary material. Thus, the problem of the scour at a relief bridge is a case of clear-water scour in which the criterion for the limit is the critical tractive force, and in which the velocity of flow and the sediment size, as well as the geometry, will play important, and similar, roles.[2]

Certain aspects of scour at the piers and abutments of the main channel bridge can be considered as variants of the relief bridge problem. If an abutment is set back so far from the bank of the stream that there is no sediment supplied to the scour hole, the situation is essentially that of a relief bridge. If rip-rap is placed on or below the bed of a stream around a pier or abutment to limit the depth of scour, the sediment supplied to the scour hole will be much finer than the rip-rap material and the problem is that of clear-water scour.

Notation.—The letter symbols adopted for use in this paper are defined where they first appear and are arranged alphabetically in the Appendix.

[2] "Observations on the Nature of Scour," by E. M. Laursen, Proceedings, 5th Hydraulics Conf., State Univ. of Iowa Studies in Engrg., Bulletin No. 34, Iowa City, Iowa, 1953.

LIMITING CLEAR-WATER SCOUR

Long Contraction.—Perhaps the simplest geometry in respect to scour is the long contraction, because the characteristics of the flow and the competence and capacity for sediment movement can be described with at least a modest measure of confidence and agreement. This is especially true if the nonuniform flow of the transition from one width to the other is ignored, and it is assumed that there are no dunes or ripples on the bed. With reference to the definition sketch, Fig. 1, the flow approaches the contraction with a

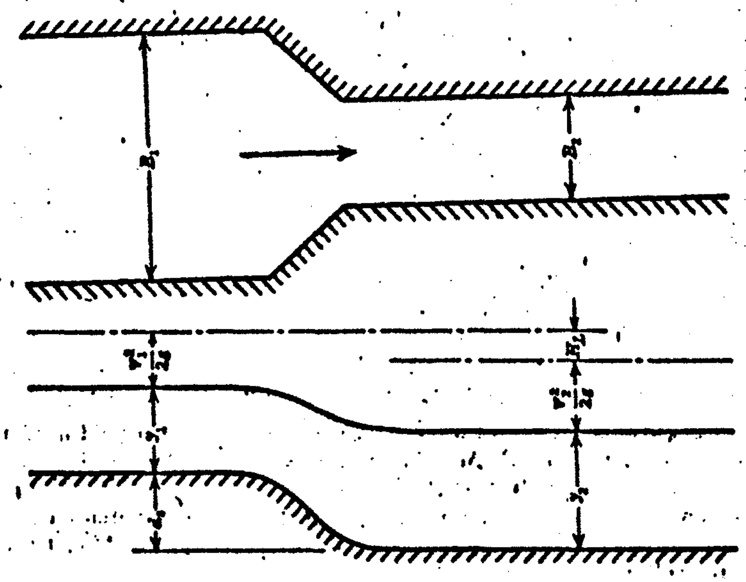


FIG. 1.—DEFINITION SKETCH FOR LONG CONTRACTION

depth y_1 and a width B_1; in the contraction, the depth is y_2 and the width is B_2. The same discharge is characteristic of both the contracted and uncontracted reaches; therefore,

$$Q = V_1 \, y_1 \, B_1 = V_2 \, y_2 \, B_2 \quad \ldots\ldots\ldots\ldots\ldots (1)$$

Assuming that the differences in slopes in the two reaches can be neglected, but that there is a loss H_L through the transition, the depth of scour is

$$d_s = (y_2 - y_1) + (1+K)\left(\frac{V_2^{\,2}}{2g} - \frac{V_1^{\,2}}{2g}\right) \quad \ldots\ldots\ldots (2)$$

in which

$$H_L = K\left(\frac{V_2^2}{2g} - \frac{V_1^2}{2g}\right) \qquad (3)$$

or in dimensionless form

$$\frac{d_s}{y_1} = \frac{y_2}{y_1} - 1 + \frac{1+K}{2} F_1^2 \left[\left(\frac{B_1}{B_2}\right)^2 \left(\frac{y_1}{y_2}\right)^2 - 1\right] \qquad (4)$$

in which

$$F_1^2 = \frac{V_1^2}{g\, y_1} \qquad (5)$$

In the contracted reach when the scour has developed fully, the boundary shear will be the critical tractive force given approximately as

$$\tau_c = 4\, D \qquad (6)$$

in which D is the mean diameter of the sediment, in feet. The factor of 4 gives fair agreement with both C. M. White[3] and A. Shields[4] for silica sands and fully turbulent flow at the bed level.

In the uncontracted reach, the boundary shear can be evaluated by means of the Manning equation and Strickler's relation for n as a function of the particle diameter D as

$$\tau_0' = \frac{V_1^2\, D^{1/3}}{30\, y_1^{1/3}} \qquad (7)$$

in which the prime is used to signify the shear of interest (that associated with the sediment particle, rather than that associated with the total resistance to flow).

The ratio of the shear, or tractive force, in the two reaches is then

$$\frac{\tau_0'}{\tau_c} = \frac{V_1^2}{120\, D^{2/3}\, y_1^{1/3}} \qquad (8)$$

[3] "Equilibrium of Grains on Bed of Stream," by C. M. White, Proceedings, Royal Soc. of London, Series A, Vol. 174, 1940, pp. 332-334.

[4] "Anwendung der Aehnlichkeitsmechanik und der Turbulenz-forschung auf die Geschiebebewegung," by A. Shields, Mitteilungen der Preuss, Versuchsanst. fur Wasserbau und Schiffbau., Berlin, Heft 26, 1936.

or, rearranging

$$F_1^2 = 3.74 \left(\frac{D}{y_1}\right)^{2/3} \left(\frac{\tau_0'}{\tau_c}\right) \qquad (9)$$

An expression for the depth ratio y_2/y_1 can be obtained because the shear in the contraction can also be evaluated in the same manner as the shear in the approach. The ratio of the shear in the two reaches is

$$\frac{\tau_{01}'}{\tau_{02}'} = \left(\frac{V_1}{V_2}\right)^2 \left(\frac{y_2}{y_1}\right)^{1/3} = \frac{\tau_0'}{\tau_c} \qquad (10)$$

Combining this expression with the equation of continuity, Eq. 1, in order to eliminate the velocity ratio results in

$$\frac{y_2}{y_1} = \left(\frac{\tau_0'}{\tau_c}\right)^{3/7} \left(\frac{B_1}{B_2}\right)^{6/7} \qquad (11)$$

The expression for the depth of scour, Eq. 4, can now be written as

$$\frac{d_s}{y_1} = \left(\frac{\tau_0'}{\tau_c}\right)^{3/7} \left(\frac{B_1}{B_2}\right)^{6/7} - 1 + 1.87\,(1+K) \left(\frac{D}{y_1}\right)^{2/3} \left[\frac{(B_1 B_2)^{2/7}}{\left(\frac{\tau_0'}{\tau_c}\right)^{6/7}} - 1\right] \left(\frac{\tau_0'}{\tau_c}\right) \qquad (12)$$

If the difference in the velocity heads and the loss through the transition are neglected, the expression for the depth of scour is greatly simplified

$$\frac{d_s}{y_1} = \left(\frac{\tau_0'}{\tau_c}\right)^{3/7} \left(\frac{B_1}{B_2}\right)^{6/7} - 1 \qquad (13)$$

or, making use of Eqs. 1 and 8,

$$\frac{d_s}{y_1} = 0.13 \left(\frac{Q}{D^{1/3}\, y_1^{7/6}\, B_2}\right)^{6/7} - 1 \qquad (14)$$

The dimensionless scour in a long contraction (neglecting kinetic terms) is shown in Fig. 2 as a single-valued function of either of the two forms of the combined parameters linking the geometry, the flow, and the sediment (Eqs. 13 and 14). To clarify the relationship, however, it is instructive to plot a family of curves as in Fig. 3 so that the separate effects of the shear ratio and the width ratio can be observed. For values of the shear ratio less than unity, there is some ratio of the widths greater than unity below which there is no scour. Below this minimum ratio of the widths (associated with a given shear ratio), the boundary shear would be less than the critical tractive force and there would be no movement of the bed material. On the other hand, for values of the shear ratio greater than unity, scour is indicated for a width ratio of unity (no contraction). The only way in which it would be possible to have such a shear ratio and no supply would be for the sediment movement in the approach to be inhibited by a vegetal or clay cover. Then, if the inhibiting cover is removed in the "contraction," there would be movement of the sediment until the limit of scour is obtained.

The dashed curve in Fig. 3 is the solution for the scour in a long contraction for sediment-transporting flow where the shear ratio is considerably greater than unity, but where the mode of movement is still bed load. The analysis leading to this curve was similar to the present analysis.[5] It is interesting to note that the curve for sediment-transporting flow lies well below the curve for the threshold of movement, or a shear ratio of unity.

A nominal velocity in the contraction

$$V_n = \frac{Q}{y_1 B_2} \qquad (15)$$

may be inferred from Eq. 14 as an indication of the maximum velocity for which there will be no scour

$$V_n = 10.8\ D^{1/3}\ y_1^{1/6} \qquad (16)$$

For a depth of 5 ft and a sediment size of 0.25 mm, the permissible velocity would be only 1.3 fps. For a velocity of 10 fps and a depth of 5 ft, 4-in. rip-rap would be needed. For slightly higher velocities in the long contraction than this nominal value, the scour might be so slow as not to be noticed; but for a relief bridge where there would be nonuniform flow, the permissible velocity should probably be lower or the requisite rip-rap size larger.

The effect of the difference in velocity heads and the loss through the transition on the clear-water scour in a long contraction is presented in Fig. 4. A value of $y_1/D = 100$ was selected as being probably as small as would be encountered even in model studies. This value corresponds to a depth of approach flow of approximately 0.5 ft and a particle diameter of 1.5 mm. Larger values of y_1/D will have even less effect, as is readily apparent from Eq. 12. For the loss term, a value of $K = 1$ was assumed. The absolute value of the added scour because of the kinetic terms is approximately the same for both the

5 "Scour at Bridge Crossings," by E. M. Laursen, Transactions, ASCE, Vol. 127, Part I, 1962, p. 166.

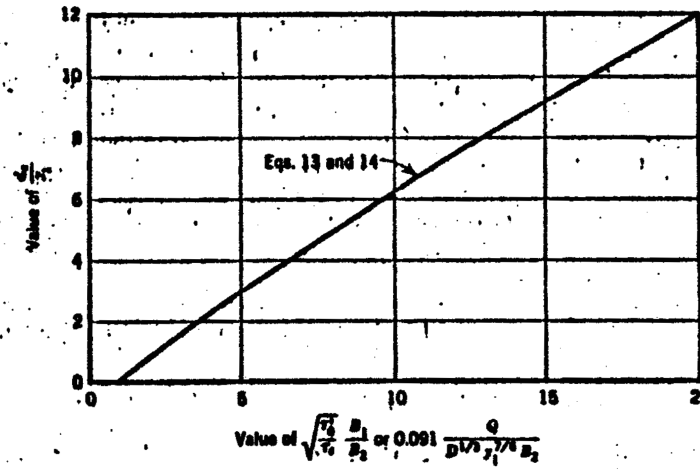


FIG. 2.—CLEAR-WATER SCOUR IN A LONG CONTRACTION (NEGLECTING KINETIC TERMS)

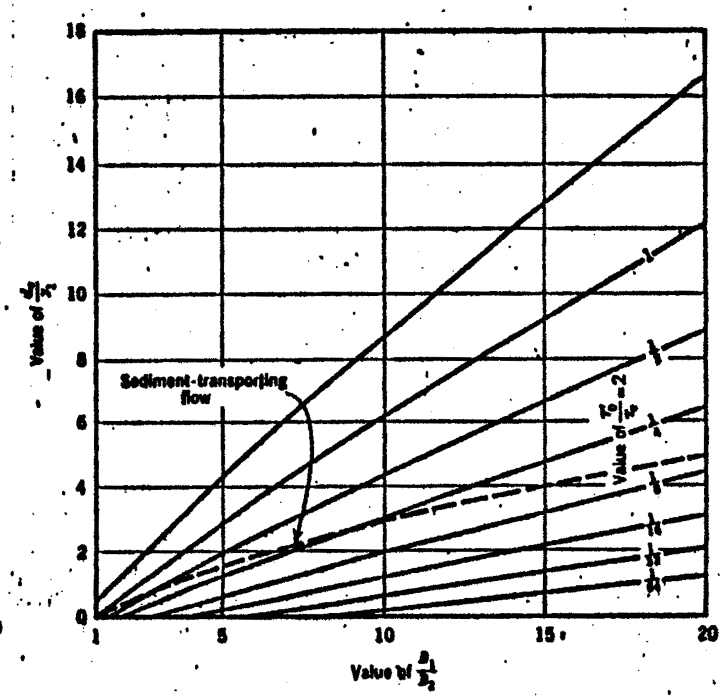


FIG. 3.—EFFECT OF SHEAR RATIO AND WIDTH RATIO ON SCOUR IN A LONG CONTRACTION

shear ratios, $\tau_0'/\tau_c = 1$ and $\tau_0'/\tau_c = 1/64$. For a width ratio of 20, the added scour is approximately $\Delta d_s/y_1 = 0.15$. The relative increase for the shear ratio of unity is, at the most, a few percent; the much smaller scour depths associated with the shear ratio of 1/64 result in an increase of 12% at a width ratio of 20 and quite large percentage increases at smaller width ratios. However, an appreciable percentage increase can only be expected at small scour depths, and the effect would be less for larger y_1/D-values. The error involved in neglecting the kinetic terms is probably no greater than those that result from the approximations of the particle shear and the critical shear, or those that could be expected in estimating the flow conditions (depth and discharge) of a future flood in the field.

Abutment and Pier.—The solution for the long contraction serves only as a minimum estimate of the scour to be expected at a relief bridge. However,

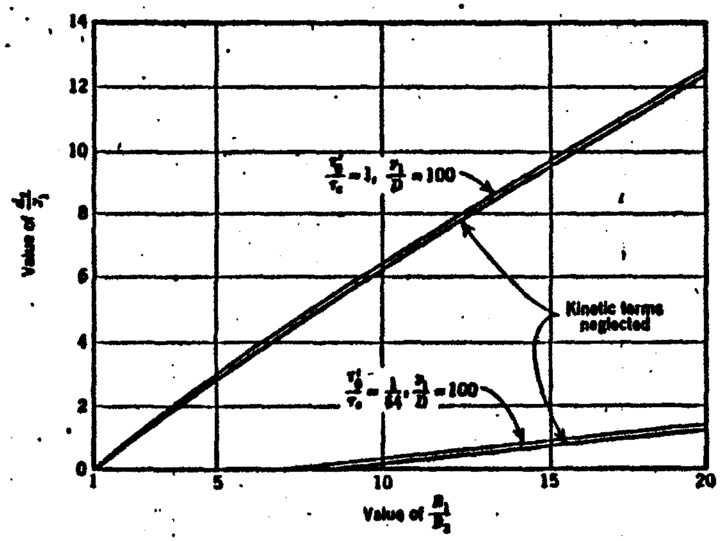


FIG. 4.—EFFECT OF KINETIC TERMS ON SCOUR IN A LONG CONTRACTION

if the same assumptions can be made concerning the nature of the flow in the clear-water case as in the case with sediment supply by the stream, the solution for the long contraction can be adapted to the case of the abutment (and the case of the pier). The key observations in the case of sediment-transporting flow were that the flow approaching the obstruction dived beneath the surface and passed through the constriction in a somewhat distorted conical scour hole centered at the upstream corner of the abutment, and that the flow approaching the clear opening was little disturbed. As a result, the scour holes at opposite abutments develop independently and do not effect one another until they are so large as to overlap physically. If the scour holes do not over-

lap, the flow in the zone between them is not affected noticeably by the constriction. (Incidentally, these observations imply that the backwater, or loss, caused by the constriction will be small if scour holes develop.) Obviously, these are bold assumptions in the case of the relief bridge where the bulk of flow is approaching the embankments and only a small fraction is approaching the clear opening. It is a conservative view, however, for if not satisfied, the scour should be larger in lateral extent but would probably be of less depth than indicated by the solution adapted from the long contraction.

The schematic model for the case of the abutment and pier is shown in Fig. 5. For the abutment model, the width of the contraction is taken as $2.75\ d_s$,

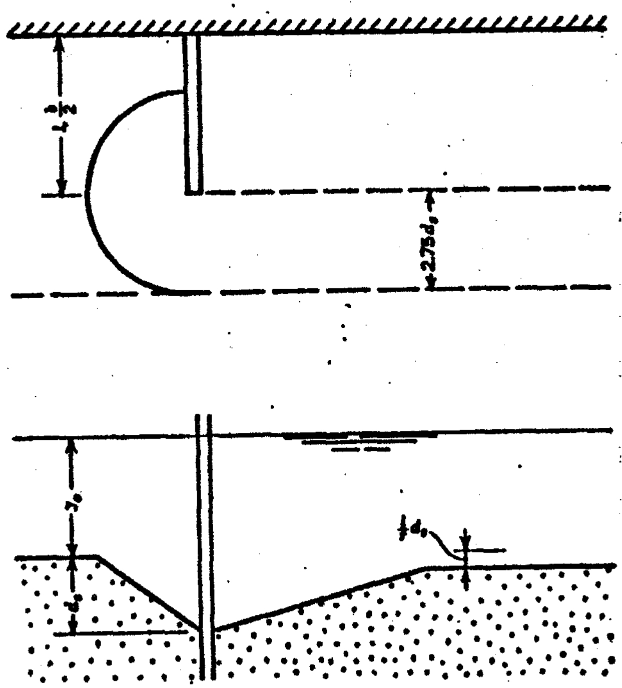


FIG. 5.—DEFINITION SKETCH FOR PIER AND ABUTMENT

the width of the approach flow as $1 + 2.75\ d_s$, the depth of approach flow is called y_0 to distinguish these cases from the long contraction, and d_s is defined as the depth of scour at the abutment (or pier) with the depth of scour in the imaginary long contraction being a fraction d_s/r of the scour at the abutment. The nominal length of the embankment is the width required for the flow approaching the embankment at the nominal velocity and depth of flow on the floodplain. The only difference for the pier model is that the half-width of the pier b/2 is substituted for the length of the embankment.

For the case of the abutment, Eq. 12 can now be rewritten as

$$\frac{1}{r}\frac{d_s}{y_0} = \left(\frac{\tau_0'}{\tau_c}\right)^{3/7}\left(\frac{\frac{l}{y_0}}{\frac{2.75\,d_s}{y_0}}+1\right)^{6/7}$$

$$+\ 1.87\,(1+K)\left(\frac{D}{y_0}\right)^{2/3}\left(\frac{\tau_0'}{\tau_c}\right)\left[\frac{\left(\frac{\frac{l}{y_0}}{\frac{2.75\,d_s}{y_0}}+1\right)^{2/7}}{\left(\frac{\tau_0'}{\tau_c}\right)^{6/7}} - 1\right] \quad \text{(17)}$$

or, neglecting the kinetic terms and rearranging the equation,

$$\frac{l}{y_0} = 2.75\,\frac{d_s}{y_0}\left[\frac{\left(\frac{1}{r}\,\frac{d_s}{y_0}+1\right)^{7/6}}{\left(\frac{\tau_0'}{\tau_c}\right)^{1/2}} - 1\right] \quad \text{(18)}$$

in which

$$\frac{\tau_0'}{\tau_c} = \frac{V_0^2}{120\,D^{2/3}\,y_0^{1/3}} \quad \text{(19)}$$

In terms of pier geometry, the half-width b/2 is substituted for l in Eq. 18.

Eq. 18, neglecting the kinetic terms, is presented graphically in Fig. 6 for the case of the abutment, and in Fig. 7 for the case of the pier. The factor r has been assumed as 12 in accord with experience for similar situations in sediment-transporting flow. The comparable solutions for sediment-transporting flow (bed load movement) are shown in each figure, and it is interesting to note that again the scour with sediment supply is less than for the condition of incipient sediment movement in the approach.

Further insight into the relationships is provided by examining the limits as d_s/y_0 becomes very small or very large. If d_s/y_0 is very small, d_s/y_0

+1 ≈ 1 and the relationship is approximated, for the abutment, by

$$\frac{d_s}{y_0} = \frac{1}{2.75\left[\frac{1}{\left(\frac{\tau_0'}{\tau_c}\right)^{1/2}} - 1\right]}\ \frac{l}{y_0} \quad \text{(20)}$$

or, for the pier, by

$$\frac{d_s}{b} = \frac{1}{5.5\left[\frac{1}{\left(\frac{\tau_0'}{\tau_c}\right)^{1/2}} - 1\right]} \quad \text{(21)}$$

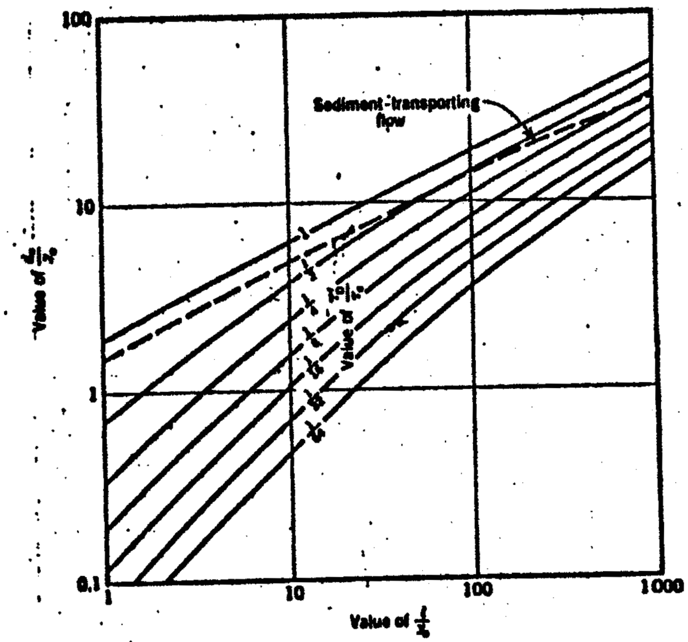


FIG. 6.—CLEAR-WATER SCOUR AT AN ABUTMENT

It is interesting to note that, at this limit (when d_s/y_0 is small), the value of r disappears, the scour at the abutment or pier varies linearly with the length of the embankment or the width of the pier, and the depth of flow is a factor only in its influence on the particle shear τ_0'. This limit might not be completely real, because if the scour is small the flow pattern may not be that visualized but instead that of a two-dimensional slot orifice. Nevertheless, Eqs. 20 and 21 illustrate a tendency to be compared to the other limit.

As the scour becomes very large, $(d_s/ry_0) + 1 \approx d_s/ry_0$ and the relationship at this limit is approximated, for the abutment, by

$$\frac{d_s}{y_0} = \frac{r^{7/13}}{1.6} \left(\frac{\tau_0'}{\tau_c}\right)^{3/13} \left(\frac{1}{y_0}\right)^{6/13} \qquad (22)$$

or, for the pier, by

$$\frac{d_s}{b} = \frac{r^{7/13}}{2.2} \left(\frac{\tau_0'}{\tau_c}\right)^{3/13} \left(\frac{y_0}{b}\right)^{7/13} \qquad (23)$$

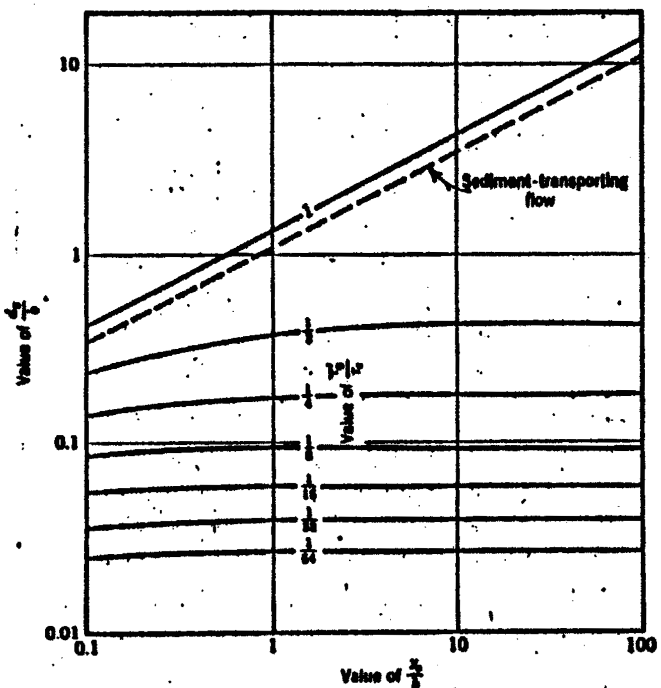


FIG. 7.—CLEAR-WATER SCOUR AT A PIER

Because the depth of scour is independent of the factor r at one limit and varies with the 7/13 power of r at the other limit, it is apparent that the depth of scour is surprisingly insensitive to the ratio of the depth of scour at pier or abutment to the depth of scour in the following imaginary long contraction. To illustrate

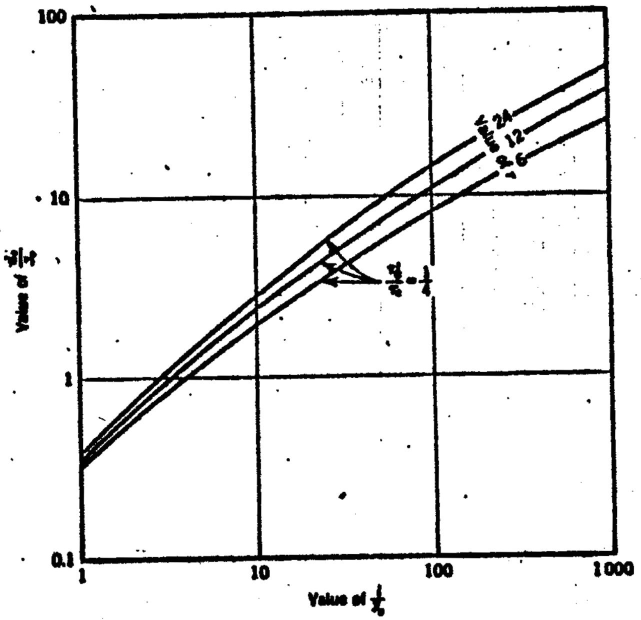


FIG. 8.—EFFECT OF PARAMETER r ON SCOUR AT AN ABUTMENT

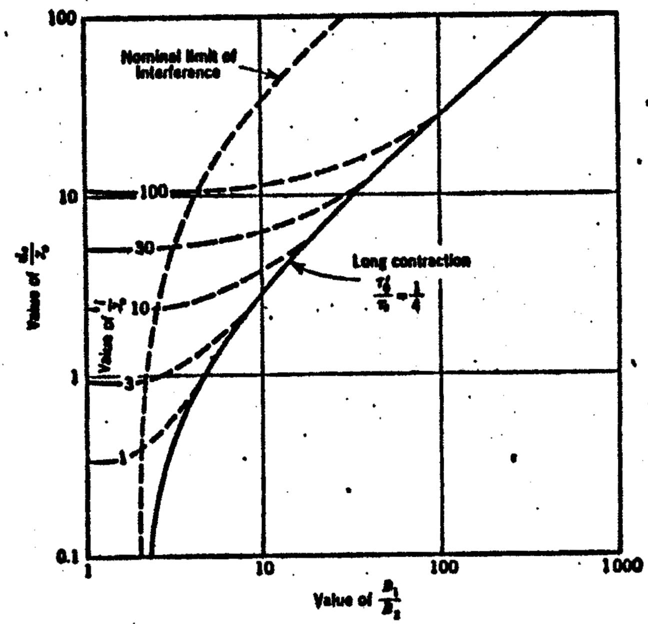


FIG. 9.—POSSIBLE INTERFERENCE EFFECT ON SCOUR AT AN ABUTMENT

further the relative unimportance of the value of r used in Eq. 18, Fig. 8 shows the depth of scour at the abutment for a shear ratio of 1/4 and values of r of 24, 12, and 6. In the range of greatest interest, $d_s/y_0 < 10$, the effect of the probable variation of r is remarkably small.

For large values of clear opening, it is possible that the envisioned flow pattern could exist and the analytic solution be at least qualitatively correct. If the clear opening is not large, however, the scour holes at the two opposite abutments may overlap. The flow pattern might be quite different; at the least, the flow would be confined laterally. Borrowing again from experience with sediment-transporting flow, the scour depth with maximum interference is that of the long contraction. Some interference would be expected if the clear opening were less than 5.5 d_s. A comparison of scour at an abutment and scour in a long contraction is shown in Fig. 9 for a shear ratio of 1/4. The dashed transition curves are a best guess based on similar curves found experimentally for the case of multiple cylinders in sediment-transporting flow.[6]

ACTIVE PHASE OF CLEAR-WATER SCOUR

In the region of moderate $1/y_0$-values, from 1 to 10, the solution for clear-water scour at an abutment, Eq. 18, can be approximated by

$$\frac{d_s}{y_0} = 0.8\ \frac{1}{y_0}\left(\frac{\tau_0'}{\tau_c}\right)^{2/3} \qquad (24)$$

in which this range of $1/y_0$ has been selected as being representative of values that could be expected in laboratory experiments. A similar analysis, for larger $1/y_0$-values that would be more representative of field conditions, could be carried out at such time as it might appear to be warranted.

At the limit, the shear on the boundary of the scour hole will be equal to the critical tractive force. In the active phase when the depth is less than the limit, the shear on the boundary is greater than the critical and is, therefore, able to transport material out of the scour hole. Assuming that the geometry during the active phase is similar to the geometry at the limit, the depth of scour at any given time is the limiting depth of scour for some coarser sediment for which the prevailing boundary shear is equal to the critical shear for that coarser sediment; therefore, the following may be written:

$$\frac{d_t}{y_0} = 0.8\ \frac{1}{y_0}\left(\frac{\tau_0'}{\tau_t}\right)^{2/3} \qquad (25)$$

in which τ_t is the boundary shear $\left(> \tau_c'\right)$ at the transient depth of scour d_t less than the limiting depth of scour d_s.

6 "Scour Around Bridge Piers and Abutments," by E. M. Laursen and A. Toch, Iowa Highway Research Bd., Bulletin No. 4, May, 1956.

The ratio of the active tractive force (or boundary shear) to the critical tractive force as a function of the ratio of the limiting depth of scour to the transient depth of scour is then simply

$$\frac{\tau_t}{\tau_c} = \left(\frac{d_s}{d_t}\right)^{3/2} \qquad (26)$$

If the scour hole is assumed to be a cone with base radius 2.75 d_t and height d_t, the volume of the scour hole can be approximated as $V = 8\ d_t^3$ and the rate of transport out of the scour hole is

$$Q_s = \frac{dV}{dt} = 24\ d_t^2\ \frac{dd_t}{dt} \qquad (27)$$

in which Q_s is a bulk volume rate of sediment transport.

An expression for the sediment transporting capacity of the flow in the scour hole is needed, and the following extreme simplification of a relationship for the bed load proposed by the writer is convenient for this purpose[7]:

$$\bar{c} = 8\left(\frac{D}{y_0}\right)^{7/6}\left(\frac{\tau_t}{\tau_c} - 1\right) \qquad (28)$$

in which $\bar{c}$ is the sediment load, in percent by weight.

Assuming that the discharge involved is the flow obstructed by the embankments, that $Q = V_0\, y_0$, that the material in bulk weighs 100 lb per cu ft, and that y_0 in Eq. 28 is the depth of the approach flow, and using the minute instead of the second as the unit of time for the rate of transport, then

$$Q_s = 3\ \frac{V_0\ D^{7/6}}{y_0^{\ 1/6}}\left(\frac{\tau_t}{\tau_c} - 1\right) \qquad (29)$$

in which Q_s is the capacity of the flow in the scour hole for transport out of the scour hole, in cubic feet per minute of bulk volume of sediment.

Equating the two expressions for Q_s, substituting for the tractive force ratio its evaluation as a function of the fraction of the limiting scour, Eq. 26, and using the approximate expression for the limiting scour, Eq. 24, a differential equation for the rate of scour is formed

$$\frac{x^{3.5}}{1 - x^{1.5}}\ dx = 0.48\ \frac{\left(\frac{D}{y_0}\right)^{1.5}\left(\frac{g}{y_0}\right)^{0.5}}{\left(\frac{1}{y_0}\right)^{2}\left(\frac{\tau_0'}{\tau_c}\right)^{1.5}}\ dt \qquad (30)$$

7 "The Total Sediment Load of Streams," by E. M. Laursen, Journal of the Hydraulics Division, ASCE, Vol. 84, No. HY2, Proc. Paper 1530, February, 1958.

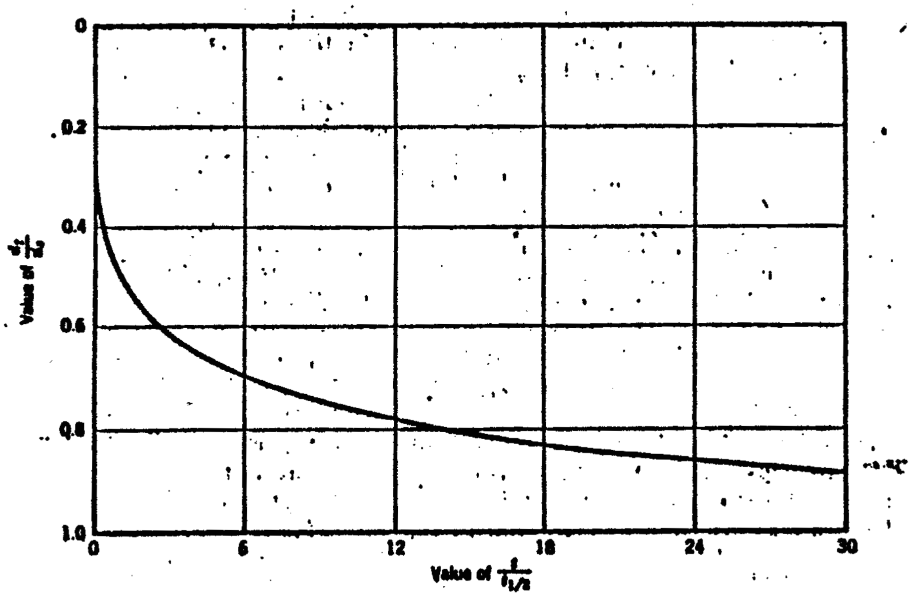


FIG. 10.—ACTIVE PHASE OF CLEAR-WATER SCOUR (ARITHMETIC TIME SCALE)

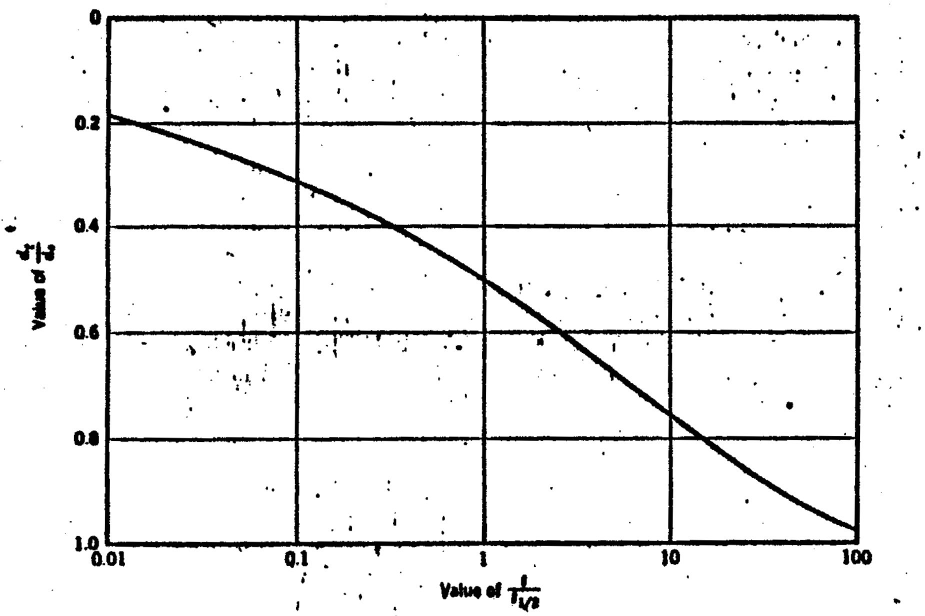


FIG. 11.—ACTIVE PHASE OF CLEAR-WATER SCOUR (LOGARITHMIC TIME SCALE)

in which

$$x = \frac{d_t}{d_s} \quad \ldots\ldots\ldots\ldots\ldots\ldots (31)$$

and the factor g has been inserted arbitrarily to obtain dimensional homogeneity (the argument could be used that the shear is evaluated from the Manning equation which lacks g).

Numerical integration of Eq. 30 results in a relationship between the depth of scour and time, which is presented graphically in Fig. 10 with an arithmetic time scale, and in Fig. 11 with a logarithmic time scale. Use of the arithmetic time scale emphasizes how rapidly the scour proceeds initially and how slowly it approaches the ultimate limit. Use of the logarithmic time scale illustrates the reason why a limited segment of the total curve, such as is likely to be obtained from experimental data, might appear to plot as a straight line on semi-log paper.

Note that in both Figs. 10 and 11 the unit of time is the time required to reach one-half of the limiting scour depth

$$t_{1/2} = 0.03 \frac{\left(\frac{1}{y_0}\right)^2 \left(\frac{\tau_0}{\tau_c}\right)^{1.5}}{\left(\frac{D}{y_0}\right)^{1.5} \left(\frac{g}{y_0}\right)^{0.5}} \quad \ldots\ldots\ldots\ldots (32)$$

in which $t_{1/2}$ is the time required to reach 0.5 d_s.

The relationship that has been obtained for the active clear-water scour would indicate that, if half the limiting depth of scour were reached in 1 hr, in 1 min the depth of scour would be 0.21 d_s (as fast or faster than flow could be established in a laboratory flume), in 8 hr the depth of scour would still only be 0.73 d_s, and to reach 0.96 d_s would take 72 hr. Some of the difficulty of experimental measurements should be readily apparent.

The depth-time relationship of Fig. 10 is qualitatively of the correct nature, giving a rapid rate of scour in the beginning and a slow rate of scour at the end. However, the assumptions that were required to obtain the relationship raise a question as to how much reliance can be given to any numbers that might be obtained from it. The least of these assumptions are (1) the similarity of scour holes during the active phase of scour, (2) the volume of the scour hole, and (3) the approximation for the limiting depth of scour; more important is the assumption regarding (4) the tractive force in the scour hole, and (5) the capacity of the flow for transport out of the scour hole.

COMPARISON OF ANALYSIS AND EXPERIMENT

In order to test the ability of the tentative relationships to predict the depth of scour, it has been necessary to use the published measurements of others. Laboratory experiments on a model pier of the Hardinge Bridge performed at the Central Water and Power Research Station, Poona, India,[8] on the circular cylinders performed at the, Laboratorie National D'Hydraulique, Chatou, France,[9] and on abutments performed at the Hydraulic Laboratory of Colorado State University (CSU)[10] have been used.

The model pier of the Poona experiments was 0.925 ft long, 0.57 ft wide, and had semi-circular cut and ease waters. The flume was 8 ft wide and the discharge was 8 cfs for all runs; four sands were used with mean diameters of 0.16 mm, 0.24 mm, 0.68 mm, and 1.51 mm. Five or six different depths

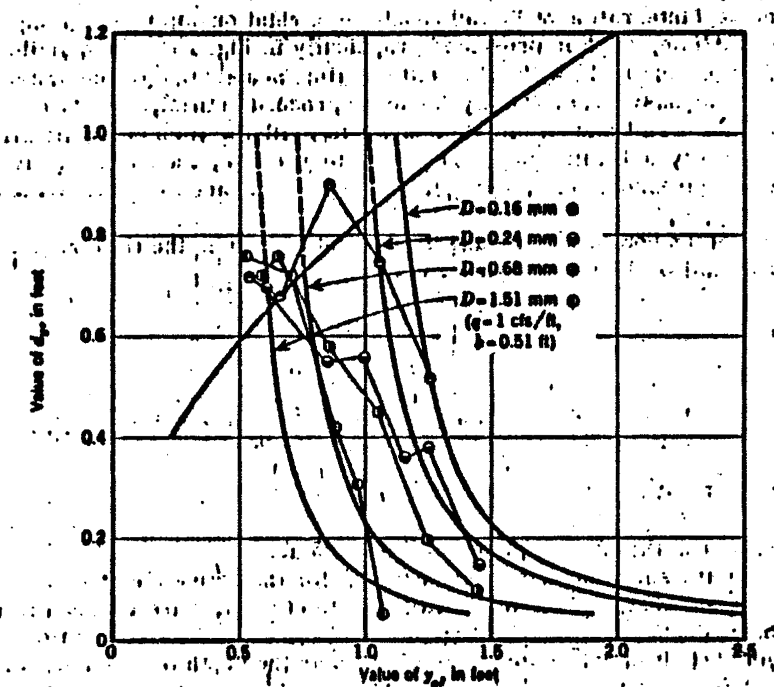


FIG. 12.—POONA DATA FOR SCOUR AT A PIER (HARDINGE BRIDGE, 1941)

of flow were used with each sand. The following description of the experiments is taken from S. V. Chitale:[8]

"The bed of the flume in these experiments was laid with sand of 0.32 mm while the following materials were used just around the pier in succession. (Table of sands follows). The sand round the piers was laid flush with the upstream bed level. A constant discharge of 1 cfs

[8] Discussion by S. V. Chitale of "Scour at Bridge Crossings," by E. M. Laursen, Transactions, ASCE, Vol. 127, Part I, 1962, p. 191.

[9] "Etude des Affouillements Autour des Piles de Pont," by J. Chabert and P. Engeldinger, Laboratoire National D'Hydraulique, Chatou, October, 1956.

[10] "Effect of Bridge Constrictions on Scour and Backwater," by H. K. Liu, F. M. Chang, and M. M. Skinner, CER60HKL22, Civ. Engrg. Sec., Colorado State Univ., Fort Collins, Colo., February, 1961.

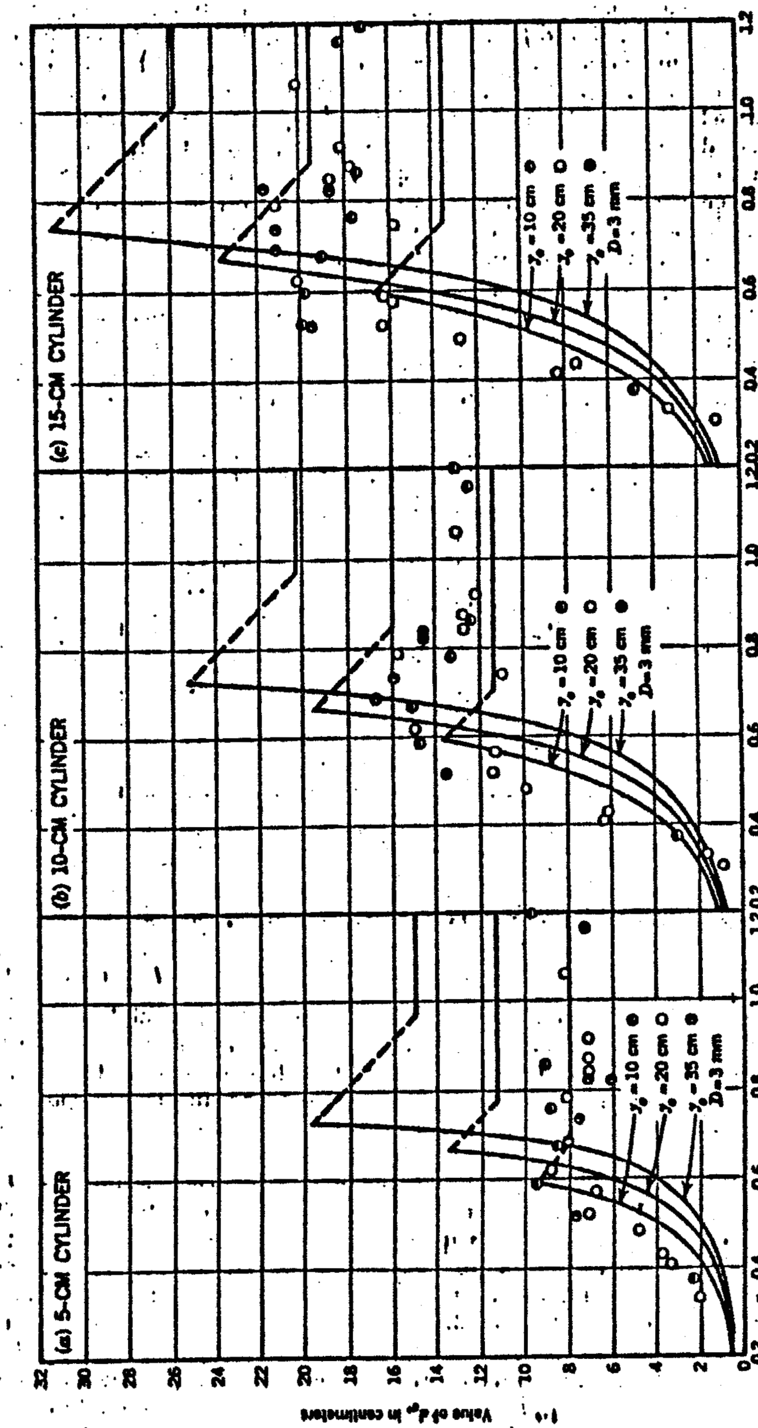


FIG. 13.—CHATOU DATA FOR SCOUR

per ft was run and the water level adjusted to get a particular depth, the depths varying from 0.5 to 1.45 ft. Each experiment was continued until the final maximum scour was obtained round the pier. In a few tests in which the upstream depth was less than stable depth the upstream bed scoured and blanketed the scour pit around the pier. In such cases, the maximum depth of scour at the nose was measured just before deposition in the scour hole of sand from upstream occurred. In the experiments when sand round the pier was coarser than the bed material upstream, the bed around the pier was laid higher than upstream level to get scour round the pier for the upstream depth laid."

It is always difficult to interpret another's experiments adequately, but it is clear that some of the runs were with clear water and others were in the range of at least light sediment movement. The measurements are plotted in Fig. 12 as depth of scour against depth of flow with curves predicted by Eq. 18

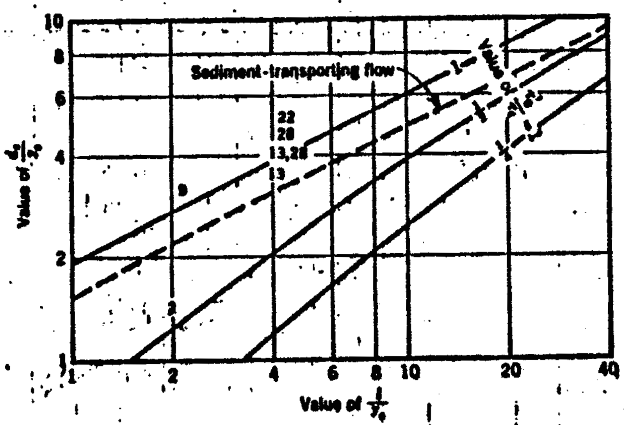


FIG. 14.—CSU DATA FOR MAXIMUM 6-HR SCOUR AT VERTICAL WALL ABUTMENTS

and the curve for sediment-transporting flow. The agreement between the predicted and measured depths of scour, especially for the two finer sands, seems reasonably satisfactory. It is interesting to note that as the depth of flow is reduced, the depth of scour increases, until conditions of supply are reached and then the depth of scour decreases with a further decrease in the depth of flow.

Of the Chatou experiments, only the series with a sand of 3 mm has been used because this series involved the most runs with clear-water scour. The three piers were 50 mm, 100 mm, and 150 mm in diameter, and depths of flow of 100 mm, 200 mm, and 350 mm were used. For each depth of flow, a number of runs were made with velocities varying from 0.3 m per sec to approximately 1.2 m per sec. The results are plotted in Fig. 13 as depth of scour against velocity of flow.

The predicted scour for the conditions of the Chatou experiments is less than the measured. However, the pattern of variation with velocity of flow

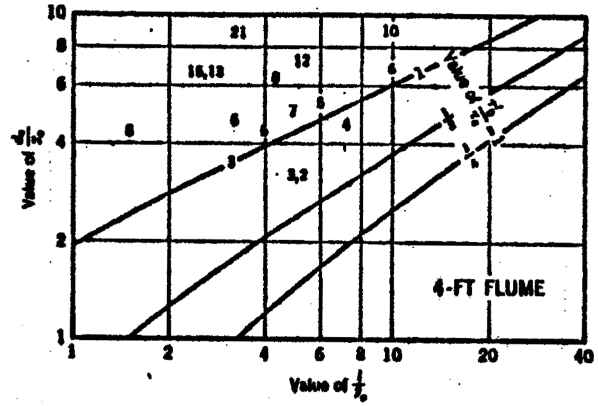


FIG. 15.—CSU DATA FOR MOVEMENT IN PREFORMED SCOUR HOLES, VERTICAL-WALL ABUTMENTS

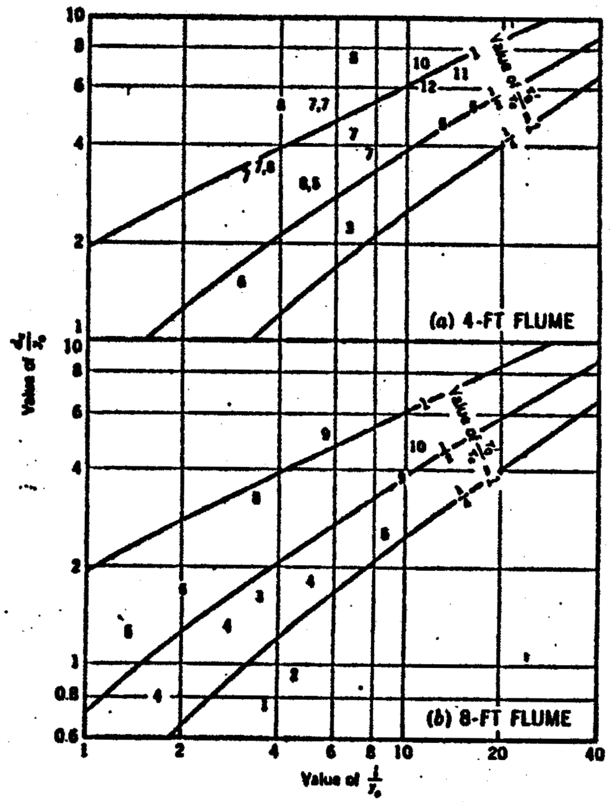


FIG. 16.—CSU DATA FOR MOVEMENT IN PREFORMED SCOUR HOLES, SPILL-THROUGH ABUTMENTS

and with diameter of cylinder is encouraging; although the evidence of a variation with depth of flow is not sufficient to draw any conclusions. Especially gratifying is the demonstration of the difference between clear-water scour and scour with sediment supply.

Because neither depth, velocity, nor discharge were kept constant for a series in the Colorado State University experiments, their data have been pre-

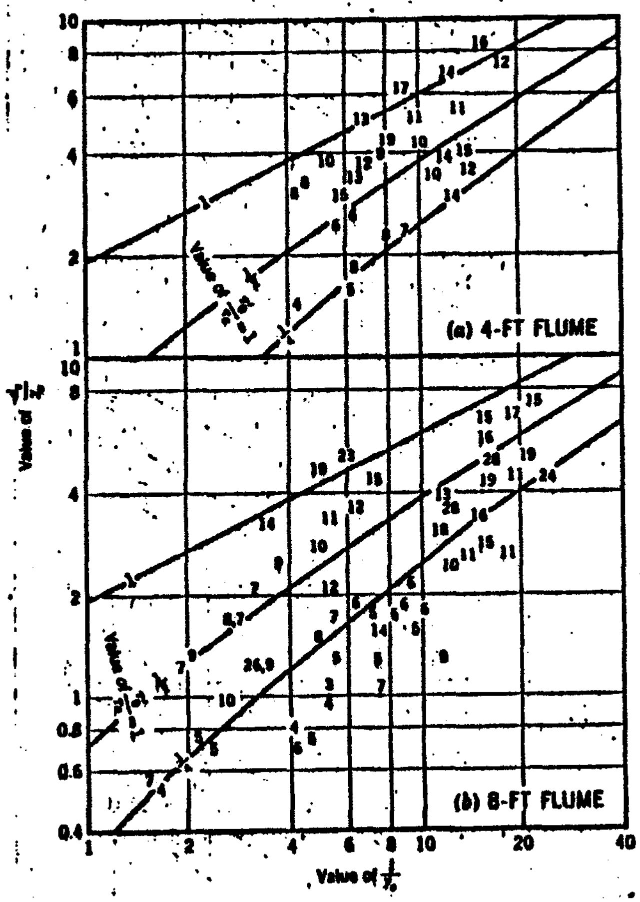


FIG. 17.—CSU DATA FOR MOVEMENT IN PREFORMED SCOUR HOLES, SPILL-THROUGH ABUTMENTS

sented in dimensionless form plotting d_s/y_0 against l/y_0 with the points numbers to the nearest tenth value of the calculated shear ratio. In Fig. 14, the maximum scour in a 5-hr period for a vertical-wall abutment in a 4-ft flume is shown. Beyond the fact that the predicted scour is less than the measured, little can be concluded from Fig. 14. Fig. 14 is included primarily because one of the runs in this series was used to test the time relationship for active

scour, but also to serve as a check on the other data that were obtained in a different manner.

For the data presented in Figs. 15 through 18, a scour hole was preformed and the depth and discharge varied "until noticeable scouring (movement) of the bed material was observed in preshaped scour hole." A vertical-walled abutment in a 4-ft flume was used in the runs of Fig. 15; a wing-wall abutment in a 4-ft and an 8-ft flume in the runs of Fig. 16; and a spill-through abutment in a 4-ft and an 8-ft flume in the runs of Fig. 17. In the runs with the 4-ft flume, the upstream bed was covered for the higher velocities in order to prevent movement into the scour hole. For the runs of Fig. 18, a vertical-wall abutment was placed on a 6-ft wide floodplain with a 2-ft river channel 0.5 ft. deeper on the opposite side of the flume. The velocity on the flood-

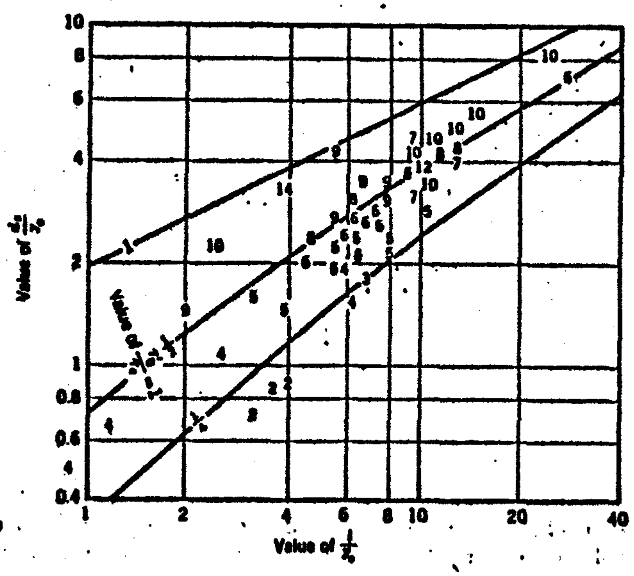


FIG. 18.—CSU DATA FOR MOVEMENT IN PREFORMED SCOUR HOLE AT SET-BACK ABUTMENTS

plain was estimated by assuming that the velocities on the floodplain and in the river channel were proportional to the 2/3 power of the depths of flow.

Despite the scatter and a shift with the geometry of the abutment, the comparison between analysis and experiment is promising. The relative scour depth increases with the length of the embankment and with the shear ratio in accord with the analysis. The predicted depth is quite satisfactory for the wing-wall abutment, too small for the vertical-wall abutment, and too large for the spill-through abutment. As the shift is in harmony with the "streamlining" of the abutment, it might have been expected.

In Fig. 19, the time history of CSU run 72 is compared to the predicted behavior of the active phase of scour. The solid curve is based on the relationship of Fig. 11 and Eq. 32. The time values are reasonable, but the limiting scour is only approximately half of that measured. The dashed curve represents a modification of the previous analysis in which the coefficient of 0.8 in Eq. 24 is doubled; the limit of scour is more realistic, but the time to reach any given depth is approximately half of that measured. That a credible pre-

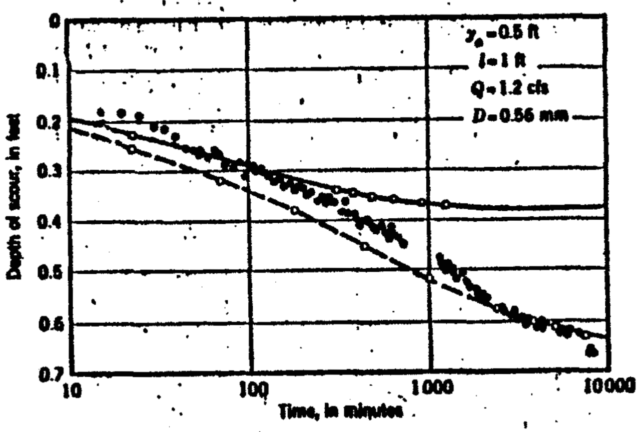


FIG. 19.—HISTORY OF SCOUR, CSU RUN 72

diction of the time history of scour is possible (given better assumptions) would seem to be a reasonable hope.

CONCLUSIONS

A truly rigorous analysis of the clear-water scour problem demands a detailed description of the flow pattern, especially the fluid forces on the boundary, in the complex geometries that will be obtained, and a knowledge of the resistance to movement, and of the rate of transport, of the material composing the boundary for the prevailing conditions.

For the case of the limiting scour in a long contraction, these two requirements can be reasonably satisfied, and the relationships obtained through the analysis should be sound. Any other expressions that might be used to evaluate the boundary shear and the critical tractive force should result in comparable relationships. If selective sorting occurs during the scouring process, a dilemma arises because the material originally in place is not the material finally composing the boundary. This is not a difficulty in regard to the relationship for the limiting scour depth, but it is in the selection of the sediment size that will govern, or in solving for the time history of scour.

For the case of the limiting scour around a pier or abutment, the three-dimensional nonuniform flow pattern cannot be described in detail with any confidence. The assumptions made to by-pass this lack of knowledge are plausible, but need investigation to be made more credible and to be refined to take into account the geometry of the obstruction. Nevertheless, the comparison between analysis and measurement would indicate that the approach has merit.

The relationship obtained for the time history of clear-water scour at an abutment should not be considered much more than speculation. Although a number of the assumptions leading to this, and the other tentative relationships may be questionable, the analyses should be useful in planning further investigations of clear-water scour problems.

APPENDIX.—NOTATION

The following symbols have been adopted for use in this paper:

B = width of flow section, in feet (subscripts 1 and 2 refer to uncontracted and contracted widths, respectively);

b = width of pier, in feet;

$\bar{c}$ = concentration of sediment, percent by weight;

D = mean diameter of sediment, in feet;

d_s = limiting depth of scour in contraction or at pier or abutment, in feet;

d_t = depth of scour at time t during active phase of scour, in feet;

F = Froude number $(V/\sqrt{g\,y})$;

g = gravitational acceleration, in feet per second squared;

H_L = head loss at contraction, in feet;

K = head loss coefficient $\left[H_L/\left(V_2^2/2g - V_1^2/2g\right)\right]$;

l = length of embankment, in feet;

n = Manning coefficient;

Q = water discharge, in cubic feet per second;

Q_s = sediment load, in cubic feet per minute;

r = ratio of depth of scour at pier or abutment to depth of scour in equivalent long contraction;

t = time, in minutes, (subscript 1/2 refers to time required to attain one-half the limiting depth of scour);

V = velocity of flow, in feet per second (subscripts 1 and 2 refer to velocity in uncontracted and contracted widths, respectively);

V_n = nominal velocity in contraction $(Q/y_1 B_2)$;

x = ratio of transient depth of scour to limiting depth of scour (d_t/d_s);

y = depth of flow, in feet (subscripts 1 and 2 refer to depths in uncontracted and contracted widths, respectively, and subscript 0 refers to depth of approach to pier or abutment);

τ_c = critical tractive force, in pounds per square foot;

τ_t = boundary shear in scour hole at time t during active phase of scour, in pounds per square foot; and

τ_0' = intensity of shear at boundary associated with sediment particles, in pounds per square foot.

ƐY WORDS: bridge abutments; bridges; hydraulics; piers; scour; sediment

ƁSTRACT: Based on the proposition that the limit of clear-water scour is a boundary
ear equal to the critical tractive force, analytical relationships are obtained for the
our in a long contraction, at an abutment, and around a pier. The pier and abutment
lutions make use of the assumption that the flow beyond the scour hole can be ignored
d that the depth of scour at the pier or abutment is a multiple r of the scour in the
uivalent long contraction. An expression for the active phase of scour is obtained
ing a simplified transport equation. Comparison of predictions with measurements
om several laboratories is reasonably satisfactory.

ƐFERENCE: "Analysis of Relief Bridge Scour," by Emmett M. Laursen, Journal of
Hydraulics Division, ASCE, Vol. 89, No. HY3, Proc. Paper 3516, May, 1963. pp.
-118.

THE TOTAL

V	mean velocity, q/y_0, fps
w	fall velocity of sediment particle, fps
y	elevation above stream bed, ft
y_0	depth of flow, ft
z	measured exponent for concentration distribution
z_1	theoretical exponent for concentration distribution
β	coefficient of proportionality between ϵ_s and ϵ_m
γ	specific weight of water, lb/ft^3
δ'	thickness of laminar sub-layer, ft
ϵ_m	mixing coefficient for momentum
ϵ_s	mixing coefficient for sediment
κ	coefficient in logarithmic velocity distribution
ν	kinematic viscosity of water, ft^2/sec
ρ	density of water, γ/g, lb sec^2/ft^4
τ	intensity of shear, lb/ft^2
τ_c	critical tractive force for beginning of sediment movement
τ_0	boundary shear, or tractive force, at stream bed, $\gamma y_0 S$
τ_0'	boundary shear associated with sediment particles

Journal of the
HYDRAULICS DIVISION
Proceedings of the American Society of Civil Engineers

THE TOTAL SEDIMENT LOAD OF STREAMS[1]

Emmett M. Laursen,[2] A.M. ASCE
(Proc. Paper 1530)

ABSTRACT

Relationships are proposed which give both the quantity and quality of the total, suspended, and bed loads as functions of the stream and sediment characteristics. In the process of empirically defining the relationships, an encouraging correlation of laboratory and field data (including some original experiments) was obtained.

SYNOPSIS

Although a rigorous analysis of the general problem of sediment transportation is not yet possible, by means of a descriptive analysis the various factors involved have been isolated and the relationships among them qualitatively indicated. Parameters linking the hydraulic characteristics of the flow and the characteristics of the bed material were then formed through the use of appropriate approximations. The relationships between these parameters, however, could only be defined empirically.

The experimental data which were used for this purpose included original experiments conducted at the Iowa Institute of Hydraulic Research and published data from other sources. Empirical curves were drawn for the total, suspended, and bed loads, and a computation procedure devised to consider the non-uniformity of the bed material. The correlation of laboratory data which was finally achieved was good considering the probable errors of the measurements. The proposed relationships were also used to predict the

Note: Discussion open until July 1, 1958. A postponement of this closing date can be obtained by writing to the ASCE Manager of Technical Publications. Paper 1530 is part of the copyrighted Journal of the Hydraulics Division, Proceedings of the American Society of Civil Engineers, Vol. 84, No. HY 1, February, 1958.

1. A dissertation submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in the Department of Mechanics and Hydraulics of the State University of Iowa.
2. Research Engr., Iowa Inst. of Hydr. Research, Iowa City, Iowa.

sediment-transporting characteristics of three natural streams with encouraging results.

INTRODUCTION

That the flow in a channel will exert a tangential force on the boundary of the channel is well known. If the boundary is composed of discrete particles, it is readily apparent that those particles may thus be set in motion. Not as apparent is the functional relationship which must exist between the flow and the particle movement. The paramount example of the general problem is the transport of sediment in alluvial streams. Despite the fact that rivers and canals have been the concern of man since the dawn of history, even today their behavior cannot be predicted with complete satisfaction or certainty.

Most investigations in the past have been concerned with the particle movement at the bed—i.e., the bed load. As a result, many formulas for the rate of bed-load transportation have been proposed. None, however, has achieved general acceptance. More recently, the phenomenon of sediment suspension has been investigated both experimentally and analytically. Aside from certain secondary, but nevertheless material, considerations that are as yet unresolved, the distribution of suspended sediment has thereby been satisfactorily formulated. The latest studies, several contemporaneous with the one of which this analysis is a part, have now progressed to the most general phase of the sediment transport problem—i.e., the combined bed load and suspended load, or total load. The experimental phase of the investigation of the total sediment load conducted at the Iowa Institute of Hydraulic Research is only included briefly herein. It is reported in detail, including all pertinent experimental data, in a final report to the Office of Naval Research, the sponsors of the study.

Qualitative Analysis

Bed Load

The total sediment load can be divided into two parts: the bed load, in which the particles move essentially in contact with the fixed (or semi-fixed) boundary, and the suspended load, in which the particles move entirely surrounded by and at essentially the velocity of the water. Although under some conditions the bed load may be only a small fraction of the suspended load, the bed load is fundamentally the more important, because it is necessary to the existence of the suspended load.

In order to set any individual particle of bed sediment into motion, the flow will have to exert upon it a finite force of some certain magnitude which depends on the shape, size, and density of the particle and its placement among the particles that surround it. The force exerted on this particular particle will depend on the average tangential force per unit area exerted by the flow, the position of the particle with respect to other particles in the neighborhood, and, if the flow is turbulent, on the particular instant of observation. Unless, however, one subscribes to the notion that turbulent fluctuations can be infinitely large, there is a limiting mean-flow condition below which this particle will not move. More importantly, there is a limit below

which none of the particles making up this semi-fixed boundary will move. Usually this limiting flow condition is characterized by a "critical tractive force," $\tau_c = (\gamma y_0 S)_c$ (see List of Symbols), which, although real, is difficult to define except subjectively.

If the tractive force on the boundary is greater than the critical, some of the particles on the surface will move; the number, the size, and the velocity of the particles in motion will then determine the rate of bed-load transportation. The mode of movement will be rolling and sliding over other stationary particles, although, because the surface is rough, the moving particles could conceivably lose contact with the boundary briefly. The force exerted by the flow on the moving particles must be transmitted to the stationary particles forming the fixed boundary. This notion does not ignore the acceleration and deceleration experienced by the particle, but recognizes that in a statistical steady-state condition of bed-load transport the total tractive force of the flow must eventually be transmitted to the stationary boundary.

Many equations have been suggested for the prediction of the rate of bed-load transportation as a function of the flow conditions and the sediment properties. In several of the equations the total tractive force, $\tau_0 = \gamma y_0 S$, is the only flow characteristic considered; in almost all of the others either the total tractive force or the slope is included, together with either the velocity or the discharge. Although some degree of similarity can be demonstrated among these various equations(1) (see References), the difference in rate of transport as predicted by them is fully as great as one might expect from one's first impression of the dissimilarity of the formulations of the different equations.(2) Unfortunately, none of the equations can be shown to be better than the others on grounds of either expediency or principle. All fit some data, but none fits all data. Little or no basis in theory is claimed for most of the suggested equations. Rather, they are relationships which have been found by cut-and-try curve fitting or more or less arbitrary equation forms together with experimentally determined coefficients.

Two equations for the rate of bed-load transportation, proposed comparatively recently by A. A. Kalinske(3) and H. A. Einstein(4,5) have the distinction of being considered theoretical. To the extent that both could be derived, logically, from explicit, but different, sets of assumptions, they can be so considered. Both, however, can be criticized on a number of counts, especially as to the implications of some of the necessary assumptions. For example, Kalinske at an intermediate step in his presentation equated the number of particles in motion to the number of particles on the bed. This assumption implies that the number of particles in motion is constant no matter what the rate of transport. The principal objections that can be raised in respect to Einstein's development cannot be as simply stated (elsewhere they have been reviewed in detail),(6) but they are equally damaging to the confidence that can be placed in his equation. Unfortunately, therefore, the "theoretical" equations are no more acceptable than the "empirical" equations. In a subsequent section still another bed-load equation will be presented which is basically empirical, although it has some rationality—its merit being that it is an integral part of the total load relationship.

Suspended Load

A suspension of particles heavier than the fluid (and too large for Brownian movement) is possible because of the mixing action of the turbulent flow.

In a turbulent flow there is a constant exchange of fluid masses, or volumes, across planes everywhere in the field of flow. For reasons of continuity equal volumes of fluid must move up and down past any horizontal plane. It is readily apparent that, since the particles are falling with respect to the fluid surrounding them whether the fluid is moving up or down, the mixing action can offset the action of gravity only if there are more particles in the fluid volume moving up than there are in the fluid volume moving down. That is to say, that there must be a concentration gradient such that the concentration is greater at lower levels if the mixing action is to maintain a statistically steady-state condition.

The equation expressing the equilibrium state of a sediment suspension is

$$w\,c = -\epsilon_s \frac{dc}{dy} \tag{1}$$

the left-hand side of the equation representing the rate at which sediment is falling per unit area across a horizontal plane under the influence of gravity, and the right-hand side the net rate at which sediment is being lifted by the mixing action of the turbulent flow at the same elevation y. The mixing coefficient ϵ_s is obviously a measure of the rate of fluid exchange, at least as a first approximation.

Similarly, the equation for the apparent shear in turbulent flow can be written as

$$\tau/\rho = \epsilon_m \frac{dv}{dy} \tag{2}$$

If one disregards secondary effects such as viscous stress and virtual mass, it would seem that the two mixing coefficients ϵ_s and ϵ_m should be the same. It is possible, however, that the mixing concept as represented by these two equations is oversimplified and that in moving a finite vertical distance the fluid does not transfer, on the average, the temporal mean conditions of its point of origin. That is to say that a natural-selection process may actually occur in the mixing action.

The usual expression for the distribution of sediment in the vertical is obtained by assuming proportionality of the mixing coefficients ($\epsilon_s = \beta\epsilon_m$), linear variation of the shear, and logarithmic velocity distribution.(7) The differential equation for the sediment suspension can then be integrated to give

$$\frac{c}{c_a} = \left(\frac{y_0 - y}{y}\,\frac{a}{y_0 - a}\right)^z \tag{3}$$

where $z = w/\beta\kappa\sqrt{\tau_0/\rho}$. Only a relative distribution is thereby obtained, as the concentration at any level y is expressed in ratio to the concentration c_a at some level a. Obviously, if the concentration c_a could be specified by some other means, the average suspended-load concentration $\bar{c}_s$ could be obtained by integration of the expression

$$\bar{c}_s = 265\,\frac{q_{ss}}{q} = \frac{\int_0^{y_0} c\,v\,dy}{\int_0^{y_0} c\,dy} \tag{4}$$

The two factors which will dominate in determining the average concentration of the suspended load are c_a and $\sqrt{\tau_0/\rho}/w$. The range of variation of other factors such as β and κ is small in comparison to the possible variation in these two.

As can be seen from Eq. (3) the ratio of the shear velocity to the fall velocity is the primary factor determining the degree of uniformity of the concentration. Given a value of c_a at the lower-most level of suspension, the more uniform the dispersion of the sediment (i.e., the larger the value of $\sqrt{\tau_0/\rho}/w$) the greater will be the value of the mean concentration $\bar{c}_s$. Although perhaps not observable, there should be a level, or zone, near the bed where the concentration becomes less dependent on the mixing action of the turbulent flow than on the rate at which particles are cast up, or entrained, from the bed.

The assumptions made by Einstein in expanding his bed-load function to permit the calculation of the total load(5) were that the reference level could be taken as twice the diameter of the sediment particle and that the concentration in this zone was proportional to the rate of bed-load transport divided by the shear velocity associated with the sediment particle and the distance from the reference level to the bed. Implicitly he also assumed that the mechanism of entrainment was the mixing action of the turbulent flow which is so effective at higher levels, and that the logarithmic velocity distribution (and, therefore, the concentration distribution) was valid as close as this to the bed.

Earlier Lane and Kalinske(8) had suggested that, for the sediment particles of a size class represented by a mean fall velocity w, the concentration at the "bottom" was proportional to the percentage of the bed of that particular size and a function of the shear-velocity/fall-velocity ratio. Experimental investigations under their direction resulted in modifications of the originally proposed relationship. Pien(9) simply adopted a distance from the bottom of 0.1 the depth of flow as the reference level. Hsia,(10) because he used a very fine sediment which was almost uniformly dispersed in the flow, considered the bottom concentration equal to the mean concentration, but found that a second parameter, $d\sqrt{\tau_0/\rho}/\gamma$, proportional to the ratio of the particle diameter to the thickness of the laminar sublayer, was needed. The basic concept in both the original and modified relationships was that the vertical turbulent fluctuations "picked up" material from the bottom.

Interchange Between the Bed and the Flow

A sediment particle which is suspended in the turbulent flow will follow an erratic path which will depend on the velocity of the fluid about it from instant to instant. Each of the many particles which pass through any point in the field of flow will, of course, follow a different path. Although the paths of the individual particles cannot be predicted, in the aggregate a pattern of diffusion of the particles passing through the given point will obtain. Even if the particles were of the same density as the fluid, within a nominal distance some of them would reach the bed of the stream because of the turbulent mixing action. In order to maintain a steady state of suspension within the field of flow for every particle that returns to the bed, a particle must, by some means, be removed from the bed and injected into the flow.

Considering the importance of the interchange phenomenon to the understanding of the phenomenon of the suspended load, there has been surprisingly little speculation as to the mechanism whereby the sediment particles are removed from the bed. The two notions that have been advanced are (1) that the flow around the particle results in a lift force greater than the weight of the particle and that the particle consequently moves up from the bed into the flow,[11,12] and (2) that the mixing action of the turbulent flow is sufficiently strong at the bed level to remove the particle from the bed.[12,13]

It cannot be disputed that there might be lift forces on a particle on the bed, because the particle is not in a uniform or even symmetrical flow field but in a velocity gradient, because the pressure may approach the stagnation magnitude under some particles, or because the velocity possesses locally a component in the upward direction. No matter how this possible lift force may come into being, however, there will also be a drag force on the particle such that the particle will begin to move or, if already moving, will move faster parallel to the bed—i.e., in the direction of the drag force. If the lift became almost equal to the weight of the particle, any small drag force would be sufficient to roll or slide the particle. Since the forces exerted by the flow on the particle are the result of the relative motion of particle and fluid, they could be expected to decrease once the particle is accelerated. How a lift force greater than the weight of the particle could develop on a particle that is free to move is, therefore, difficult to envision, except possibly through the action of vertical components of large scale turbulence.

This possibility is, of course, the same as the notion that the mixing action of the turbulent flow extends to the level of the bed. For a smooth, solid bed it is readily apparent that at least the vertical turbulent fluctuations will decrease with proximity to the boundary to the limit of zero at the boundary. Although a rough, porous boundary such as is formed by the sediment might either result in, or permit, larger turbulent fluctuations than have been found in the vicinity of smooth boundaries, one would still expect the mixing action of the turbulent flow to grow weaker as the boundary is approached. Although the process cannot be categorically ruled out, it would seem that the turbulent exchange which can be quite effective in the interior of the flow would be minimal and rather inadequate at the boundary.

A more plausible explanation of how a sediment particle can leave the bed can be based on Newton's first law of motion—that a body in motion will move uniformly in a straight line until acted upon by some force. Consider now a particle moving up the weather slope of a dune. The forces on the particle are the propelling forces of pressure and shear due to the flow around the particle, the force of gravity, and the reaction force of the bed that can be resolved into a normal, support force and a tangential, resistance force. Just before the crest of the dune all of these forces will be acting on the particle; just after the crest of the dune the bed forces supporting the particle and resisting the motion will be lost, but the fluid forces and the gravitational force will continue to act on the particle (although the fluid forces may change because the flow around the particle changes). The motion of the particle will then depend on the velocity (both direction and magnitude) of the particle as it leaves the crest of the dune and the forces which thereafter act upon it.

If the velocity of the particle and the propelling forces are small and the weight of the particle under water is large, the particle will merely roll over the crest and down the lee slope of the dune. This, of course, is a description of bed-load movement. If the gravitational force is small in comparison to the momentum (mass times velocity) of the particle, the particle will only gradually deviate from its initial direction of motion and will tend to move in the parabolic path of a projectile. This ideal path, of course, will be modified by the effect of the fluid forces on the particle.

Although the prediction of the path of the particle may not be possible, it should be clear that, if the velocity of the particle moving as bed load is great enough, the particle can leave the immediate vicinity of the bed in what might be described as a self-launching action. Moreover, the factors which will govern the motion of the particle, other than the properties of the fluid and the particle itself, are the form of the bed, the flow pattern (including the velocity magnitude), the gravitational force, and the velocity of the particle at the crest of the dune. The amount of material involved in this action will be related to the number of particles in motion and the size of the particles. Since the product of the number, size, and velocity of the particles is the volume rate of bed-load transportation, it is immediately apparent that the bed load will play a primary role in the determination of the suspended load.

If the bed is covered with dunes, this self-launching action is easy to visualize. Even more to the point, it can be observed. If there are no dunes—i.e., if the bed is plane—the same action is still possible, because the bed-load particles must move over roughness elements of their own size. As a result, the direction of motion of the particle can be away from the bed and, if its velocity is great enough, the particle can escape from the immediate neighborhood of the boundary. If the moving particle in the case of the duned bed can be likened to a ski-jumper, in the case of the plane bed it can be likened to a hot grounder in a sand-lot baseball game.

The secondary motion, or large-scale vortices, of the turbulent flow can also result in a movement of the particles in contact with the bed whereby they can be self-launched into the flow. This can be demonstrated simply in a glass jar filled with water with a bed of fine sand on the bottom. By moving a pencil in a small orbit an almost irrotational vortex can be induced. Because of the secondary motion, the sand particles move inward in spiral paths and a small, sharp-pointed, cusped dune is formed. If the velocity of flow near the bed, and consequently, the velocity of the particles, is sufficiently great, the same self-launching action from the peak of the dune is observable. (The importance of macroturbulence such as this in the movement of sediment has been noted by Matthes[14] although the mechanism whereby the particles are removed from the bed is not described. A similar phenomenon has also been noted by Lane).[15] In a stream or a laboratory flume this action is quite impossible of observation. It can be readily observed every autumn, however, with dust and fallen leaves as the sediment particles. On a large scale, of course, it is the tornado of the western plains.

Observations

Equipment and Technique

Since most previous investigations of sediment transportation have been concerned with bed-load movement, very few data on the rate of transport of total load, or even suspended load, were available at the time this study was initiated. An experimental phase of the investigation, therefore, was essential to the determination of quantitative information relative to this most general case of sediment movement. Because the data which was gathered were so important in the development of the relationships for the sediment load, a brief description of the experimental phase is included herein.

The principal item of experimental equipment was a recirculating, tilting flume with an overall length of 105 feet, shown in Fig. 1. The test section was 90 feet long and had a cross section 36 inches wide and 18 inches deep. In order to permit visual observation of the flow, the sides of the test section were made of 1/4-inch plate glass.

Because the resistance to flow will depend in large measure on the roughness of the self-formed alluvial bed, the discharge and the depth of flow (and hence the velocity) are usually fixed in experiments such as these, and the slope is allowed to develop as required. In order to speed up the deposition-erosion process whereby the slope is established, it is expedient to be able to tilt the entire flume during operation. The mechanism which was used to accomplish this was a system of interconnected cams.

The rate of flow was controlled by the speed of the pumps and measured by calibrated, segmental orifices in each line in conjunction with an air-water manometer. The depth of flow was determined by the volume of water in the system and the water-surface elevation was measured at 10-foot intervals along the test section by piezometers connected to an open manometer board. The water-surface and sand-bed elevations relative to the flume could be determined by means of point gages mounted on carriages which in turn slid on stainless-steel rails which were parallel to the flume bottom. The slope of the flume itself was obtained by measuring the travel of the lower end with a vernier and scale made from a standard point gage.

Values of velocity and concentration at a point were obtained by means of the combination instrument shown in Fig. 2. Two Pitot tubes, modified to produce a larger differential reading, were mounted on each side of a rectangular sampler nozzle. The differential-head indication was obtained with an air-water manometer. A small variable-speed pump was used to establish any desired rate of flow into the sampler-intake nozzle. The rate of flow was measured by a calibrated Venturi meter and air-water manometer. The pumped sample was collected at first in quart milk bottles; later large glass tubes were obtained to simplify the handling of the sample. The amount of sediment in the samples was determined by weighing in calibrated pycnometers. For small concentrations a standard 10-ml pycnometer with a fitted stopper having a capillary hole was used and for large concentrations a 500-ml flask with a narrow, graduated neck.

In order to measure the total load, a low submerged weir was installed at the downstream end of the test section. The weir was formed of a quarter cylinder cut from a nominal 4-inch brass pipe, and placed so that the convex side was upstream and a tangent to the top edge, or crest, of the weir was parallel to the flume bottom. The crest of the weir was approximately an inch above the mean elevation of the sand bed and slightly above the crest of the dunes.

In a study such as this the operation and the measuring techniques must be designed first to establish normal equilibrium transport conditions and then to obtain a reliable and complete description of those conditions. Although equilibrium here refers specifically to a state of flow in which there is neither aggradation nor degradation of the bed, it also implies that there is no change in state with time. Because the rate of change from a non-equilibrium to an equilibrium state will depend primarily upon the degree of nonequilibrium, the desired steady state will tend to be approached asymptotically.

Arbitrary, but judicious, operation of a sediment-transport flume can materially reduce the time required to attain a normal equilibrium condition.

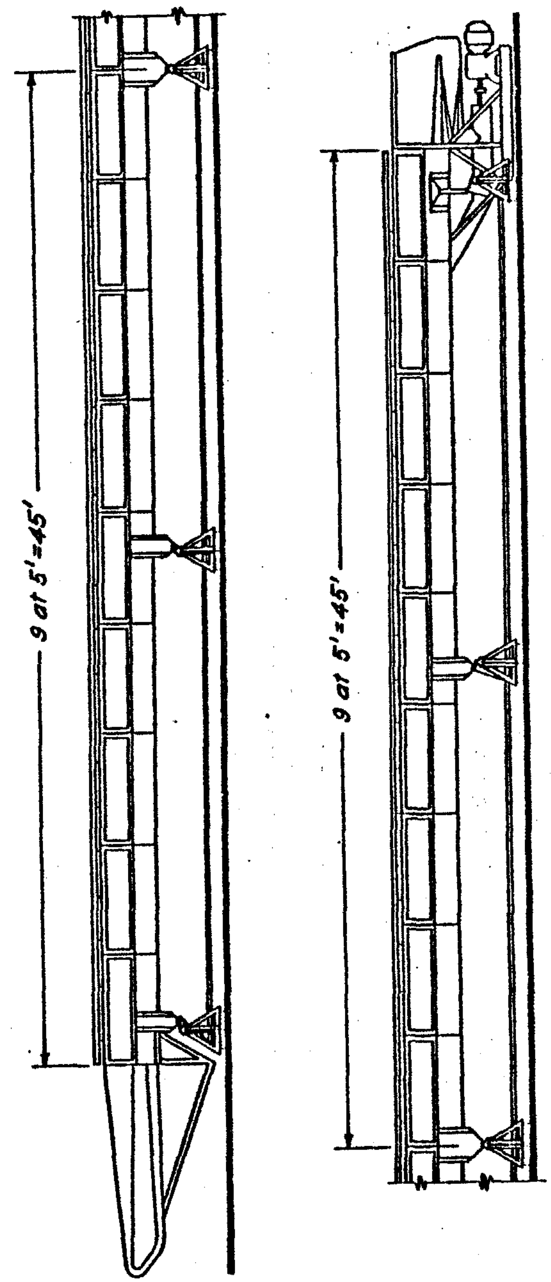


Fig. 1. Flume.

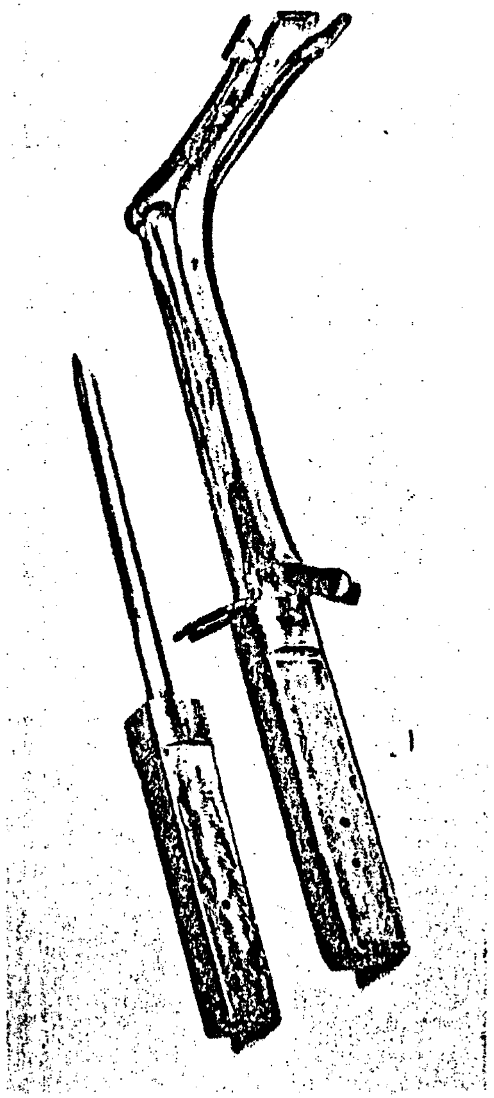


Fig. 2. Combination Pitot tube and sampler.

To confirm the establishment of equilibrium, however, repeated measurements over considerable periods of time are necessary. To verify the uniformity of a reach, measurements at successive sections are required.

Given a width of section (which is fixed by the construction of the test section) and a bed material (which can be chosen arbitrarily) only two of the three primary flow characteristics (discharge, depth, and slope) can be considered to be independent variables. In a natural stream the slope and the discharge are the imposed conditions and the depth is conditional upon these and upon the configuration of the stream. Operation of a recirculating, tilting flume is simpler if the discharge and depth are chosen as the imposed conditions and the other characteristics of the flow are allowed, or helped, to attain their natural value or form.

In the first runs measurements were taken of all pertinent characteristics of the flow as the condition of equilibrium was approached. The slope proved to be the most sensitive index, as it was affected by both the non-uniformity of the flow and the roughness of the bed. As deposition and scour occurred, remolding the overall bed so that the rate of sediment transport at every section was the same, there was a concomitant change in the water-surface slope. As the natural ripple or dune pattern developed, the roughness of the bed changed. Therefore, the resistance to flow changed, and, consequently, the slope of the water surface. The velocity distribution and concentration distribution were affected by the position of the dunes relative to the measuring section—especially, of course, near the bed. As a result, the spatial variation of these measurements was of the same order as the temporal variation, unless the flow was adjusting itself quite rapidly toward the normal equilibrium condition.

For the coarser of the two sands used, the slope proved to be a sufficient indicator of the attainment of the normal equilibrium condition. When the bed had been remolded and its configuration established, the slope of the water surface no longer varied with time. (Tilting the flume, in effect, simply reduced the time necessary for remolding the bed.) A reach of approximately 50 feet, from about 35 feet to about 85 feet from the entrance, would then exhibit uniformity—as defined by a constant slope and constant depth of flow.

For the finer sand the water-surface slope alone was not found to be an adequate measure of the state of flow. This was because the length of the reach displaying uniformity was only about 30 feet, and, therefore, the measurement of the slope could not be as precise. Measurements of the sand-bed slope and elevation were found to be necessary also. In order to measure the sand bed the flow had to be stopped and the run interrupted. Therefore, the sand-bed measurements were only used as a final check after the water-surface slope indicated stability.

Fortunately, because the total time required for a single complete run made continuous operation impractical, interruption of a run did not affect materially either the attainment or the state of equilibrium. If care was taken in stopping and then resuming a run, the state of flow at the time of interruption was reestablished within an hour at the most. As has been mentioned before, the criterion for normal equilibrium flow is that the characteristics of flow do not change with time. Since this state is approached asymptotically, it is obvious that the final determination of whether the desired state has been reached must rest on the judgment of the operator. When normal equilibrium had been attained—in the operator's judgment—measurements were made of water-surface elevations, velocity distribution, concentration distribution, and sand-bed elevations.

For the coarser sand, only centerline distributions of velocity and concentration were determined. For the finer sand, complete traverses during several runs were made across the flow section in the uniform zone and over the low quarter-circle weir. Measurements of the dune size could be obtained in the case of the coarser sand. However, for the very fine sand the material settling out of suspension almost obliterated the dune forms.

Water-surface and sand-bed elevations could be read to 0.001 foot. Although errors in any single determination could be greater, the error which might be expected in the average of repeated measurements should not exceed this value. The depth of flow ranged from about 0.25 to about 1.00 foot. At the smallest depths the percentage error, therefore, could be of the order of 1%. The slope ranged from about 0.0004 to about 0.0018 over the 50-foot reach of the coarser sand and from about 0.0008 to about 0.0012 over the 30-foot reach of the finer sand. Errors as large as 10%, therefore, are possible.

Few of the individual velocity determinations in the interior of the flow should be in error by more than 2 or 3%. Near the sand bed, however, the possible error in the measurements became larger as the boundary was approached, because the velocity became smaller. Moreover, the position of the dune relative to the measuring section was unknown, so that the meaning of the measurements near the sand bed was somewhat indefinite.

The samples which were taken for the determinations of point concentrations were large enough to give a good time average. Fig. 3 shows the long-period fluctuations of the concentration at a point as a function of time. The short-period fluctuations have been averaged out because each successive determination of concentration was made from a 1-quart sample, each quart being equivalent to a stream tube almost the length of the test section. In the first runs 5-quart samples were taken to reduce this source of error in the concentration measurements. The slightly larger 5000-cc tubes, later substituted for the milk bottles, were equivalent in volume, and the procedure of transferring the sediment sample to the pycnometer was simpler.

Experimental Results

The size-frequency distribution of the two sands which were used is shown in Fig. 4. Despite the fact that the finer sand is within what is generally considered the silt range, it is a true sand. Both sands were fractions of the Ottawa deposit (the coarsest fraction of which is the standard concrete testing sand) and are almost pure silica.

The experimental results for the coarser sand are summarized in Table I. A more detailed summary is included in the report to the sponsor.[16] In regard to the basic problem being investigated—the correlation between the sediment load and the flow and sediment characteristics—little can be learned from comparing the pertinent values for different runs, even with the aid of simple plots. An empirical relationship describing this correlation was found only after a qualitative analysis of the problem had indicated parameters which could be taken as expressing the factors governing the phenomena involved. This qualitative analysis and the relationships finally obtained are the subject of the following section. Several general characteristics of interest, however, are immediately apparent from the tabulated summary.

The recorded dune heights h and l show that within the range of conditions represented here, the size of the dunes did not vary greatly—except that in one run a plane bed was formed. (A number of other attempts to obtain a

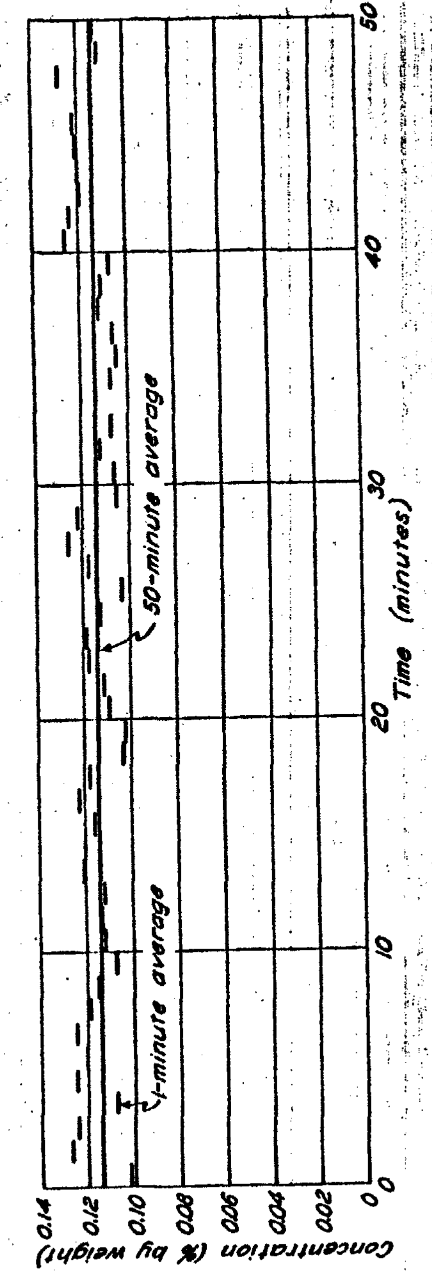


Fig. 3. Variation of concentration with time.

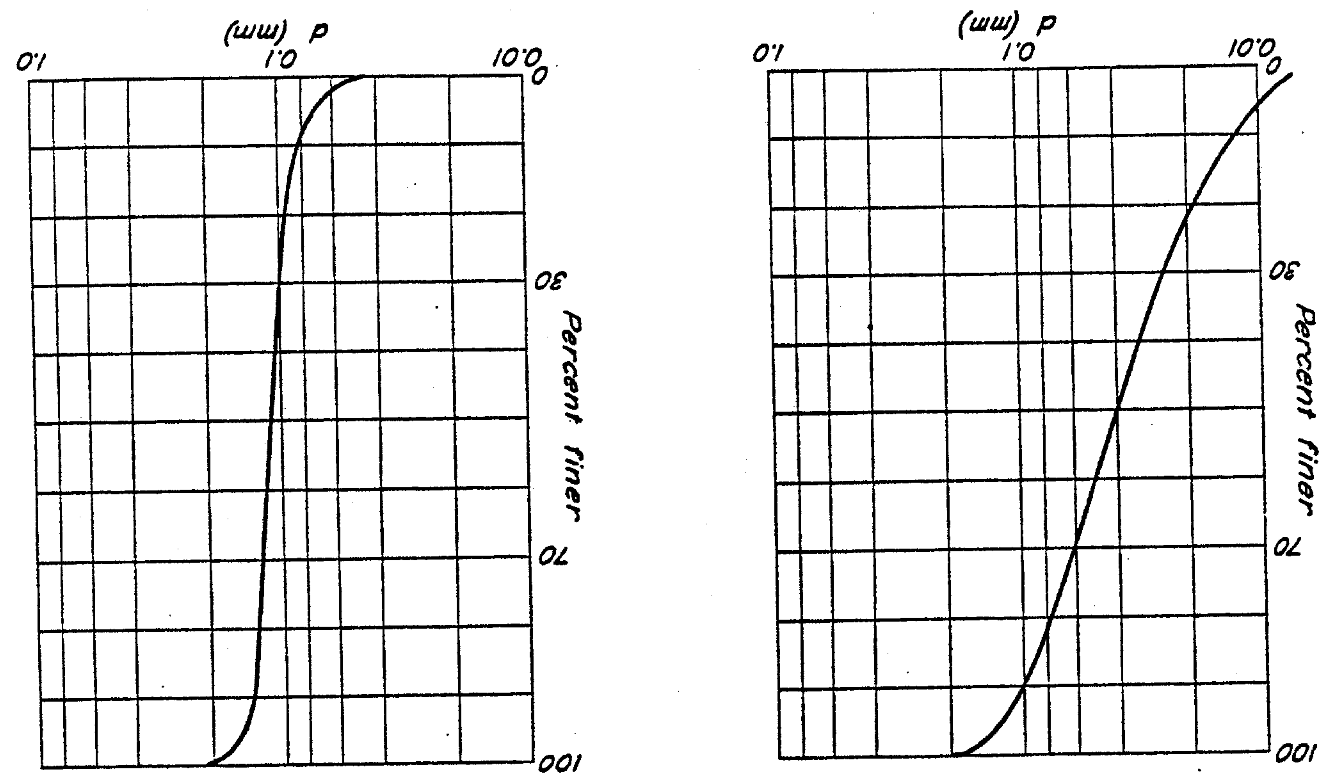


Fig. 4. Size-frequency distribution of sands.

Table I. Summary of results for 0.1-mm sand

Run	$\bar{c}$	V	y_0	S	T	h	l	$2y_0/h$	f	1/m	κ	s	s_1	β
1	0.225	1.89	0.561	0.00122	20.6	0.083	0.48	13.6	0.062	3.85	0.32	0.40	0.58	1.40
2	0.029	1.36	0.751	0.00055	21.8	0.077	0.48	19.6	0.058	4.35	0.33	0.32	0.72	2.25
3	0.014	1.33	0.928	0.00043	22.8	0.074	—	25.0	0.058	4.75	0.41	0.45	0.72	1.60
4	0.270	2.31	0.927	0.00101	24.0	0.084	0.51	22.0	0.045	4.15	0.38	0.63	0.47	0.75
5	0.424	1.92	0.514	0.00152	21.3	0.094	—	11.0	0.055	3.85	0.30	0.41	0.54	1.32
6	0.313	1.70	0.518	0.00144	21.6	0.100	0.48	10.4	0.066	3.45	0.29	0.40	0.53	1.32
7	0.066	1.32	0.533	0.00106	23.3	0.108	—	9.8	0.084	3.45	0.31	0.42	0.61	1.45
8	0.156	1.73	0.756	0.00092	21.6	0.100	0.56	15.2	0.060	3.45	0.31	0.42	0.55	1.31
9	0.061	1.59	0.995	0.00058	26.5	0.110	0.38	18.0	0.058	3.35	0.29	0.44	0.61	1.39
10	0.272	1.55	0.381	0.00186	24.9	0.101	0.44	7.6	0.076	3.15	0.28	0.42	0.55	1.31
11	0.055	1.07	0.311	0.00160	26.4	0.109	0.44	5.8	0.112	3.35	0.36	0.49	0.65	1.33
12	0.103	1.28	0.382	0.00150	21.5	0.080	0.45	9.6	0.090	3.45	0.29	0.40	0.61	1.52
13	0.131	1.69	0.725	0.00080	23.7	0.075	0.45	19.4	0.052	4.00	0.32	0.40	0.61	1.52
14	0.143	1.15	0.250	0.00210	19.8	0.063	0.45	8.0	0.102	3.25	0.31	0.34	0.63	1.85
15	0.515	3.36	0.472	0.00120	22.8	Plane		(2640)	0.013	4.75	0.17	0.97	0.61	0.63
16	0.305	2.20	0.709	0.00107	23.2	0.072	—	19.8	0.040	3.25	0.25	0.68	0.53	0.78

plane bed were unsuccessful. The combinations of velocity and depth giving Froude numbers of about 0.8 resulted in very unstable flow conditions in the flume.) As can be seen in the respective tabulations, the relative roughness $2y_0/h$ of the duned bed exhibited a greater variation than did the absolute roughness h.

In Fig. 5 the Weisbach resistance coefficient is plotted as a function of the relative roughness together with the relationship for fully rough-pipe flow based on the Nikuradse experiments with sand-roughened pipes. Despite the scatter of the experimental points a very good agreement is evident between the two kinds of roughness in the two flow systems. In fact, one should probably interpret the superposition of the points on the curve as due to a fortuitous choice of a typical roughness length until further evidence is available as to the effect of roughness shape and spacing on the resistance coefficient. One can conclude, however, that the bed configuration is of major importance in the resistance to flow of alluvial streams.

Similarly the roughness has an effect on the velocity distribution, which—except for the plane bed—is in accord with the Nikuradse experiments. Fig. 6 shows the power-law exponent as a function of the relative roughness together with the Nikuradse relationship. The values of the exponent m were obtained from log-log plots of the velocity distribution. Semi-logarithmic plots of the velocity distribution were also made and the values of κ obtained therefrom are listed in the table. No systematic correlation for the variation of κ was found with either the concentration or the roughness.

Similarly, no pattern of variation was found for the values of β listed in the table. The value of β was taken as the ratio of the theoretical exponent of the concentration distribution z_1 to the measured exponent z as obtained from log-log plots of the concentration distribution. In the theoretical evaluation of z_1 the fall velocity obtained in a bottom-withdrawal-tube determination of the size-frequency distribution and the nominal value of 0.4 for κ were used. The effect of the concentration would be to decrease the fall velocity and, therefore, z_1 by less than 10%. Use of the experimental values of κ would increase z_1 and, therefore, β. For most of the runs this increase would be of the order of 30%.

Table II summarizes the results for the finer sand. From the tabulated f values in this table it can be seen that the resistance to the flow for these runs with the finer sand is about the same as for those with the coarser sand. Although meaningful measurements of the roughness could not be obtained because of the deposition of the suspended load, it could be observed that the size of the dunes was approximately the same as for the previous series of runs. The velocity distribution characterized by the exponent m as obtained from log-log plots is also indicative of a rough boundary similar to the runs with the coarser sand. The values of κ from semi-logarithmic plots listed in Table II are slightly smaller for this series. A comparison of theoretical and measured concentration distribution by taking a ratio of the appropriate z values results again in β values greater than unity, and about the same as those found for the coarser sand. Correction of the fall velocity for the concentration effect would decrease β by as much as 40%, but use of the experimentally determined values of κ would almost double the value of β.

Of perhaps greater significance is the fact that the total load measured over the weir was almost the same as the suspended load measured in the normal sections. In fact, the errors in measurement of the suspended load were larger than the bed load. It is estimated that the bed load for these runs was probably not more than 1% of the total load.

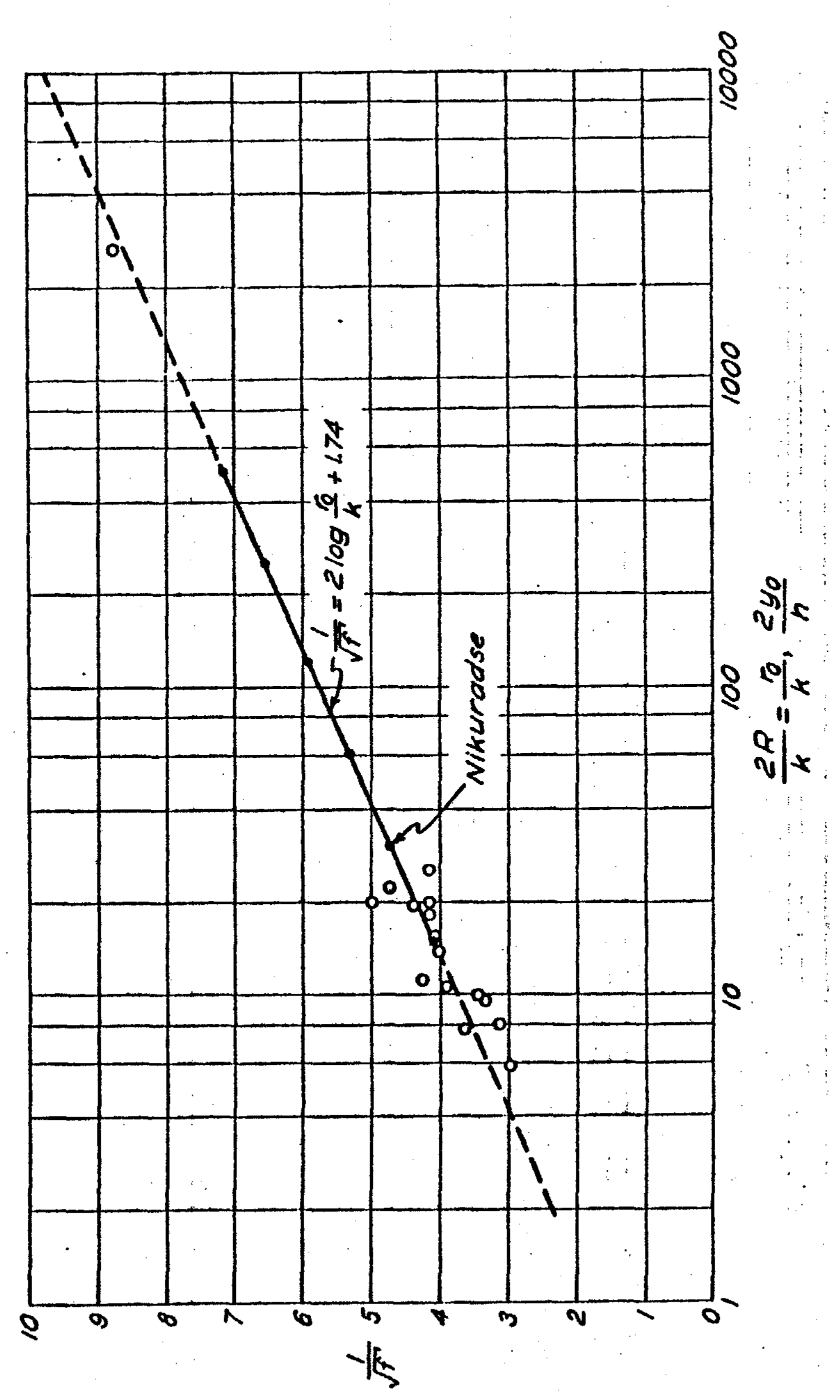


Fig. 5. Effect of roughness on resistance to flow.

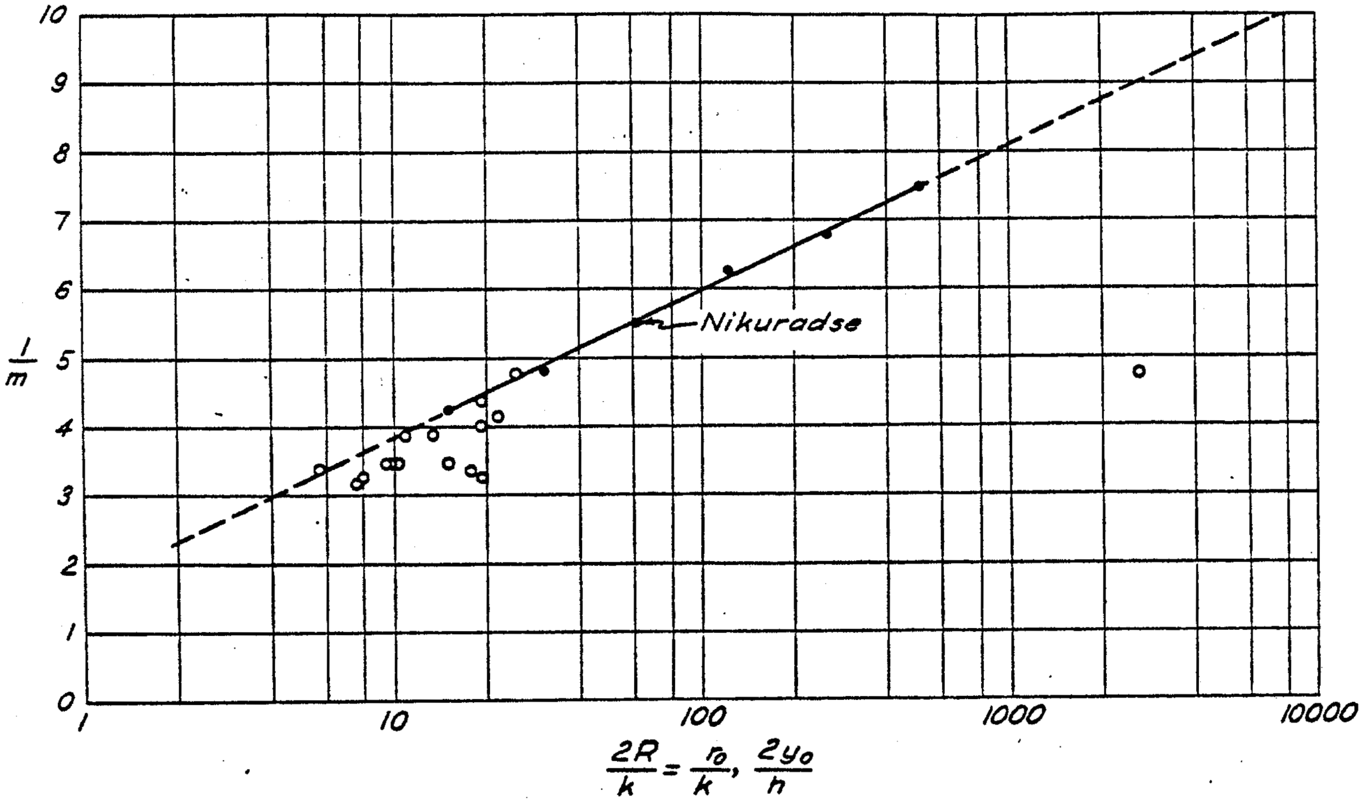


Fig. 6. Effect of roughness on power-law exponent.

Table II. Summary of results for 0.04-mm sand

Run	$\bar{c}_t$	$\bar{c}_s$	V	y_0	S	f	1/m	κ	z	z_1	β
101	8.34	6.90	1.87	0.566	0.00101	0.042	2.94	0.19	0.076	0.088	1.16
102	8.20	6.72	1.74	0.462	0.00117	0.046	3.44	0.27	0.065	0.091	1.40
103	5.84	5.76	1.91	0.674	0.00086	0.057	3.22	0.24	0.062	0.088	1.42
104	9.70	9.48	2.13	0.478	0.00107	0.029	2.63	0.18	0.092	0.095	1.03
105	8.34	8.76	2.61	0.565	0.00114	0.024	5.00	0.11	0.102	0.097	0.95
106	3.03	3.07	1.23	0.542	0.00081	0.075	3.12	0.25	0.050	0.098	1.96
107	0.73	0.75	0.85	0.380	0.00078	0.107	5.00	0.25	0.030	0.124	4.10
108	9.81	9.99	2.42	0.663	0.00100	0.029	4.75	0.25	0.083	0.095	1.14

Relationships for the Sediment Load

Development of the Relationships

Given a flow condition (mean velocity, depth, and slope), and a sediment (mean size, frequency distribution, specific gravity, and shape), there will be a corresponding rate of sediment transport, partly as bed load, partly as suspended load. The qualitative discussion of the mechanism of sediment movement in the preceding sections was for the purpose of selecting the factors which determine the rate of transport. If the hydraulic and sediment characteristics can be linked in parameters descriptive of these factors, then at least empirical relationships for the sediment load may be obtained. (At the present time any attempt at a rigorous analysis is defeated at the outset because the fluid forces on the individual particles cannot be specified in sufficient detail.) One of the desired parameters is readily apparent—the shear-velocity/fall-velocity ratio, $\sqrt{\tau_0/\rho}/w$. This ratio expresses in large measure the effectiveness of the mixing action of the turbulence and, of a certainty, should be included in the relationship for the suspended-sediment load.

The other desired parameters cannot be as confidently stated. Especially needed is a clear understanding of the forces acting on the particles in motion as bed load. For the present it is practically necessary to relate the forces causing motion to the tractive force, and the forces resisting motion to the critical tractive force for the beginning of movement. Only in the case of the plane bed, however, can the total tractive force be considered significant. If there are dunes on the bed (the more common condition) the part of the total tractive force which is resisted by the tangential component of the integrated pressure distribution along the duned boundary cannot be considered effective in causing the sediment particles to move. An approximate measure of the remainder of the tractive force can be obtained by the use of the Manning formula and Strickler's expression for n as a function of the sediment diameter:[17]

$$\tau_0' = \frac{V^2\, d^{1/3}}{30\, y_0^{1/3}} \tag{5}$$

Although this expression may be only a crude approximation for the shear in the rather complex flow along the weather slope of the dune, it has the advantage of being computationally simple.

An expression for the critical tractive force can be written as

$$\tau_c = C\, d \tag{6}$$

where C is a coefficient dependent on the sediment characteristics and the flow conditions near the boundary. Both τ_0' and τ_c are representative of the average shear per unit area of the bed, rather than the force on the particle which is moving or about to move. Although neither is a true measure, even in approximate form, of the desired forces, one might conjecture that each is related to these forces in much the same manner. Thus, the ratio τ_0'/τ_c can be assumed to be a significant parameter descriptive of bed-load transportation. Moreover, if the rate of bed-load transport is a primary factor in the interchange phenomenon, as seems likely, this ratio would also be important to the suspended-load movement.

A third parameter, which is also suggested by the discussion of the preceding sections, is the ratio of the velocity of the sediment particles moving as bed load to their fall velocity. This ratio would be a measure of the tendency for self-launching and would be important for suspended-load rather than bed-load movement. Similarly, the form of the bed might be descriptive of the opportunity for self-launching. In the final analysis of the data, the influence of such a self-launching parameter could not be assessed—partly because of the scatter due to experimental errors, and partly because it would necessarily resemble the other parameters.

To test the significance of the ratio τ_0'/τ_c, the data obtained in this study were plotted as $\bar{c} = 265\, q_s/q$ against $(\tau_0'/\tau_c)-1$, and an almost linear variation was found for each sand. For reasons more intuitive than rational, the factor

$$\frac{\sqrt{\tau_0'/\rho}\; d}{V\, y_0} \propto \left(\frac{d}{y_0}\right)^{7/6} \tag{7}$$

was included in the relationship and a plot made of

$$\frac{\bar{c}}{\left(\frac{d}{y_0}\right)^{7/6}\left(\frac{\tau_0'}{\tau_c} - 1\right)} = f\left(\frac{\sqrt{\tau_0/\rho'}}{w}\right) \tag{8}$$

using the data of this study and other published data. The resulting relationship is shown in Fig. 7. The solid curve drawn through the points represents the total load and the dashed curve the bed load. Table III summarizes the source and the pertinent information for the various data plotted in Fig. 7. Only those runs for which the particle shear approached or was less than the critical shear were rejected. A simple ratio like τ_0'/τ_c, of course, cannot be expected to describe conditions at this limit.

For the data of Toch and Hsia, C values for the critical tractive force of 8 and 16, respectively, were used; otherwise a value of 4 was employed. These values are plotted together with Shields' curve in Fig. 8. The values used are approximately in agreement with the recommendations of White,[18] and for values of d/δ' above 0.2 with Shields[19] as well. For smaller values, where the laminar sub-layer completely envelops the particles, the variation of the critical tractive force with the diameter of the particle is not clear. It should be noted that Shields' data did not include values of d/δ' much less than 0.2.

It was noted, especially on the larger work sheet from which Fig. 7 was prepared, that there was a systematic scatter to the points for any series as identified by the different symbols. It was surmised that this tendency for the sets of points to cross the mean curve might be due to the non-uniformity of the sediments, and a computational procedure was devised to consider the composition of the bed material. This computational procedure is represented by the equation

$$\bar{c} = \Sigma p \left(\frac{d}{y_0}\right)^{7/6} \left(\frac{\tau_0'}{\tau_c} - 1\right)\; f\left(\frac{\sqrt{\tau_0/\rho'}}{w}\right) \tag{9}$$

wherein the contributions of each fraction p of size d are summed to obtain the mean contribution. Values of τ_c and w were determined for the mean diameter d of the fraction, but the same values of τ_0' and $\sqrt{\tau_0/\rho'}$ were used for all fractions, τ_0' being determined from the mean diameter of the total sediment sample.

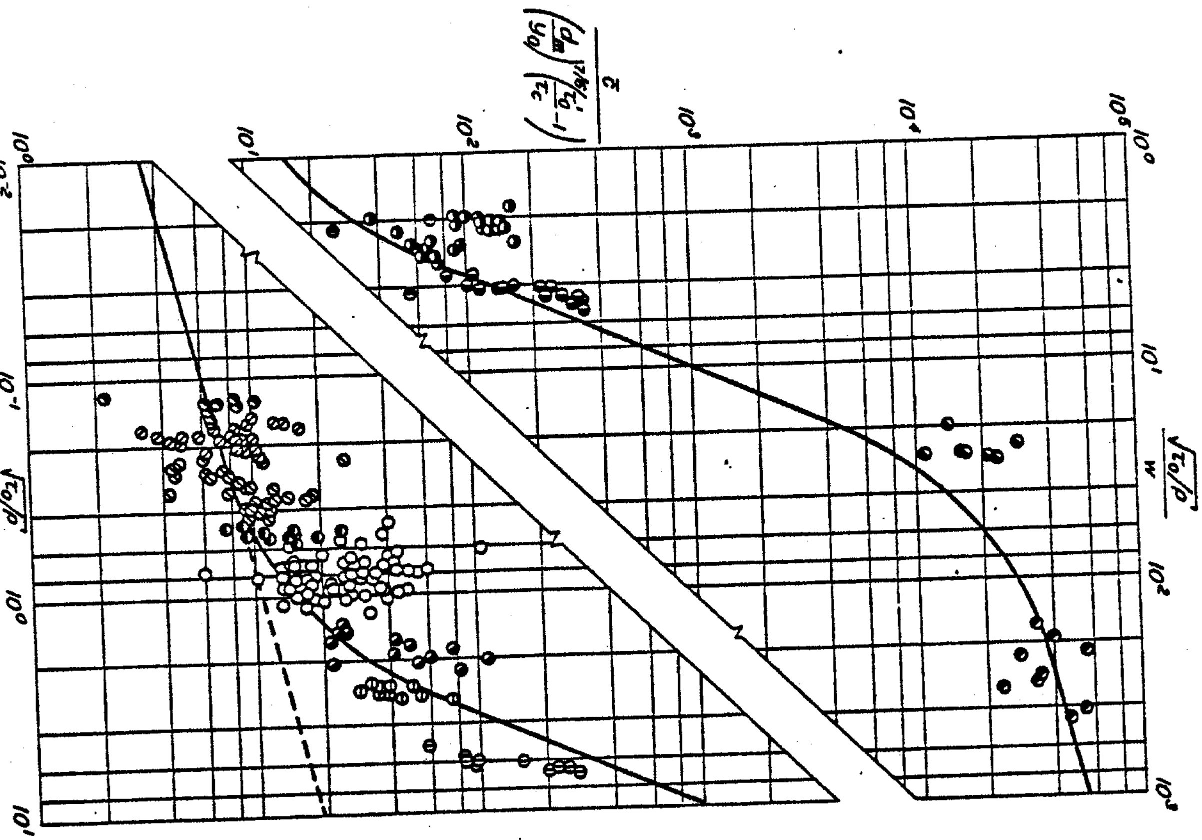


Fig. 7. Correlation of mean concentration with hydraulic and sediment characteristics.

Table III. Summary of data in Figs. 7 and 9

Investigator	Symbol	d_m(mm)	$\bar{c}$	$\sqrt{\tau_0/\rho}/w$	τ_0'/τ_c	Remarks
Lin, Rand (Table I) [16]	•	0.11	0.014 - 0.52	3.6 - 5.4	2.77 - 23.8	Susp
Toch (Table II) [16]	•	0.04	0.73 - 9.99	20.2 - 28.4	1.96 - 11.3	Total
Hsia [10]	•	0.011	0.64 - 11.1	168 - 451	1.89 - 18.0	Total
Pien [9]	•	0.18	0.013 - 0.24	2.0 - 4.3	4.0 - 10.9	Susp
MacDougall [22]	•	1.44	0.012 - 0.13	0.15 - 0.22	1.11 - 2.43	Bed
	•	0.66	0.018 - 0.12	0.23 - 0.43	1.17 - 2.99	Bed
O'Brien [22]	○	0.37	0.001 - 0.13	0.43 - 1.21	1.11 - 8.85	Bed
Brooks [23]	•	0.088	0.019 - 0.53	5.0 - 6.9	2.01 - 14.0	Total
	•	0.145	0.020 - 0.24	2.5 - 3.1	1.86 - 9.5	Total
Lin, Barton [24]	•	0.18	0.003 - 0.37	1.8 - 3.1	1.45 - 19.3	Total
WES [25]	•	0.20	0.004 - 0.08	1.4 - 2.2	1.26 - 8.2	Total
	•	0.50	0.006 - 0.05	0.46 - 0.57	1.30 - 2.91	Bed
	•	4.08	0.002 - 0.03	0.12 - 0.15	1.02 - 1.20	Bed

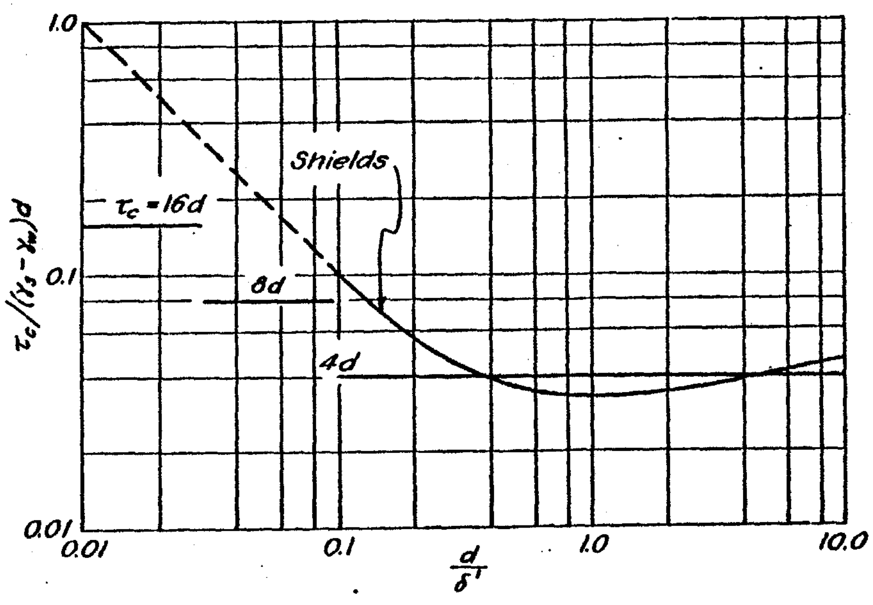


Fig. 8. Critical tractive force.

Fig. 9 shows a comparison between the measured mean concentration and the concentration as computed in this manner. A tendency for the plane beds to have a measured concentration less than computed could be noted, as well as a hint that there might be a systematic scatter with the ratio τ_0'/τ_0. It is readily apparent that this ratio could be taken as a measure of the roughness of the bed; together with the other parameters which have been introduced it might also be taken as descriptive of the ratio of the velocity of the bed-load particles to their fall velocity. The unsystematic scatter due probably to errors in measurement, was too great to assess the possible influence of these two factors. Actually a rather good correlation is indicated over the ten-thousand-fold range in concentration represented in this plot.

Equation (9) also allows the prediction of the composition of the sediment load (bed load, suspended load, or total load depending on the function of the shear-velocity/fall-velocity ratio employed). Fig. 10 shows a comparison with measurement of such a prediction for several runs.

Although the data used in Figs. 7 and 9 represent a large variation in sediment size and concentration, the measurements were all made in laboratory flumes under controlled conditions. Therefore, an attempt was made to predict the sediment load under field conditions using the data published by H. A. Einstein for Mountain Creek in South Carolina and West Goose Creek in Mississippi,[20] and by the USGS for the Niobrara River near Cody, Nebraska.[21] The results are shown in Figs. 11, 12, and 13, respectively.

In the upper part of the figures the velocity and slope data are plotted together with the mean curves which were used in conjunction with Eq. (9) to obtain a predicted sediment load. On the left a curve, or curves, for the predicted load and the data of the measured load are plotted as a function of the discharge. On the right are plotted the predicted composition of the sediment load and the measured composition of the bed material.

In the case of Mountain Creek the predicted load was in general less than measured. It may be noted, however, that artificial flash floods were used to obtain much of these data (Fig. 11), with a difference in conditions between rising and falling stage as indicated in the figure. The measured points of the largest flood are connected in the figure by light lines indicating the temporal continuity. The measured difference between the rising and falling stages is roughly comparable to the predicted difference. Of perhaps greater significance is the discrepancy between the predicted and measured composition of the bed load. The predicted mean size of the bed load was 0.6 mm as compared to a measured value the same as the bed material, or 0.9 mm. If a smaller value for τ_c had been used, the predicted load would, of course, have been greater and the composition of the bed load would have more closely approached that of the bed material.

The predicted bed load for West Goose Creek (Fig. 12) agrees very well with that measured for lower discharges, but it is considerably too large for higher discharges. It is noteworthy that the discrepancy is pronounced only at velocities of flow above 2.5 fps. Based on the experience of the writer, it seems doubtful that a slot 2 feet in width such as was used would capture much more than half the load at this high a velocity with so fine a sand (0.3 mm). The predicted size of the bed load for West Goose Creek is only slightly smaller than the size of the bed material, and is well within the scatter of the measured composition of the bed load.

The excellent prediction of the sediment load for the gaging-station section of the Niobrara River (Fig. 13) does not include some fifty-nine measurements

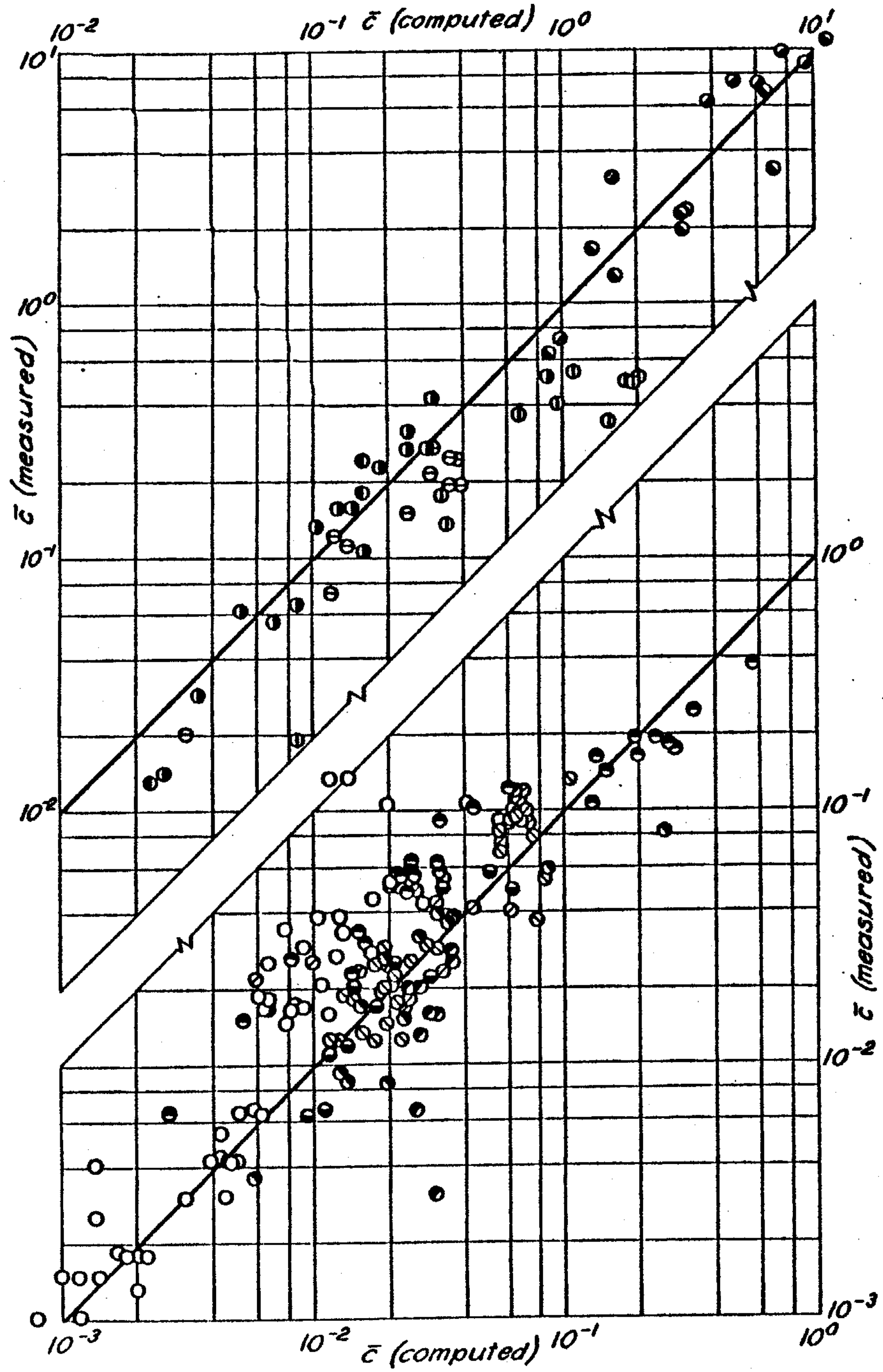


Fig. 9. Comparison of computed and measured concentrations.

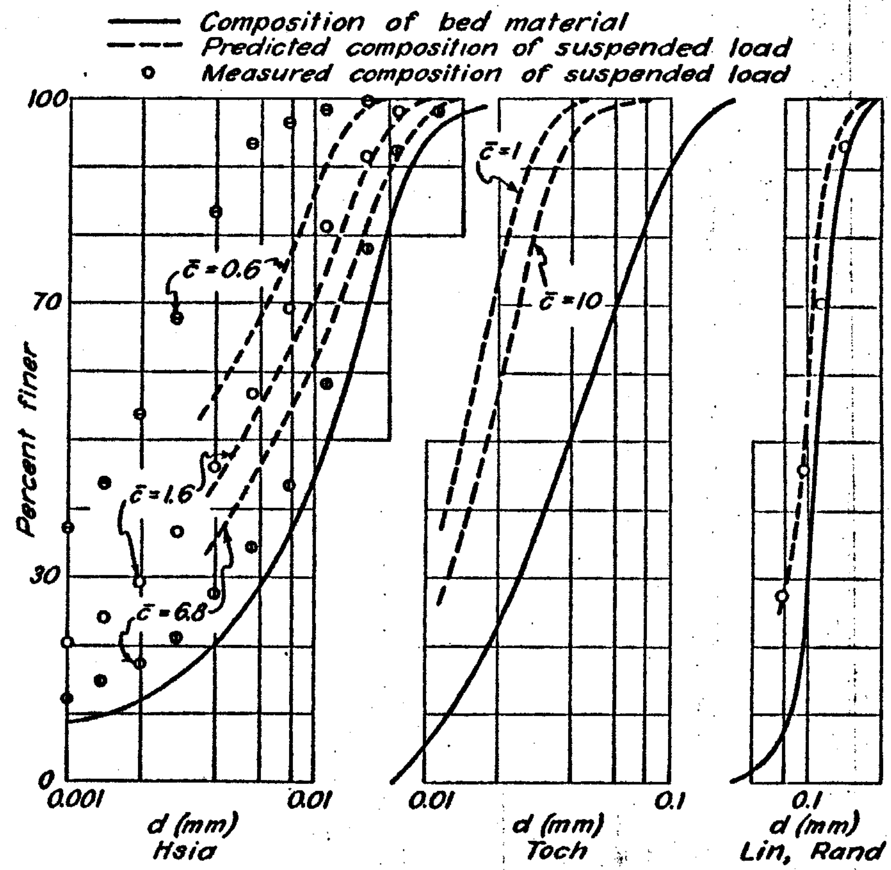


Fig. 10. Predicted composition of suspended load.

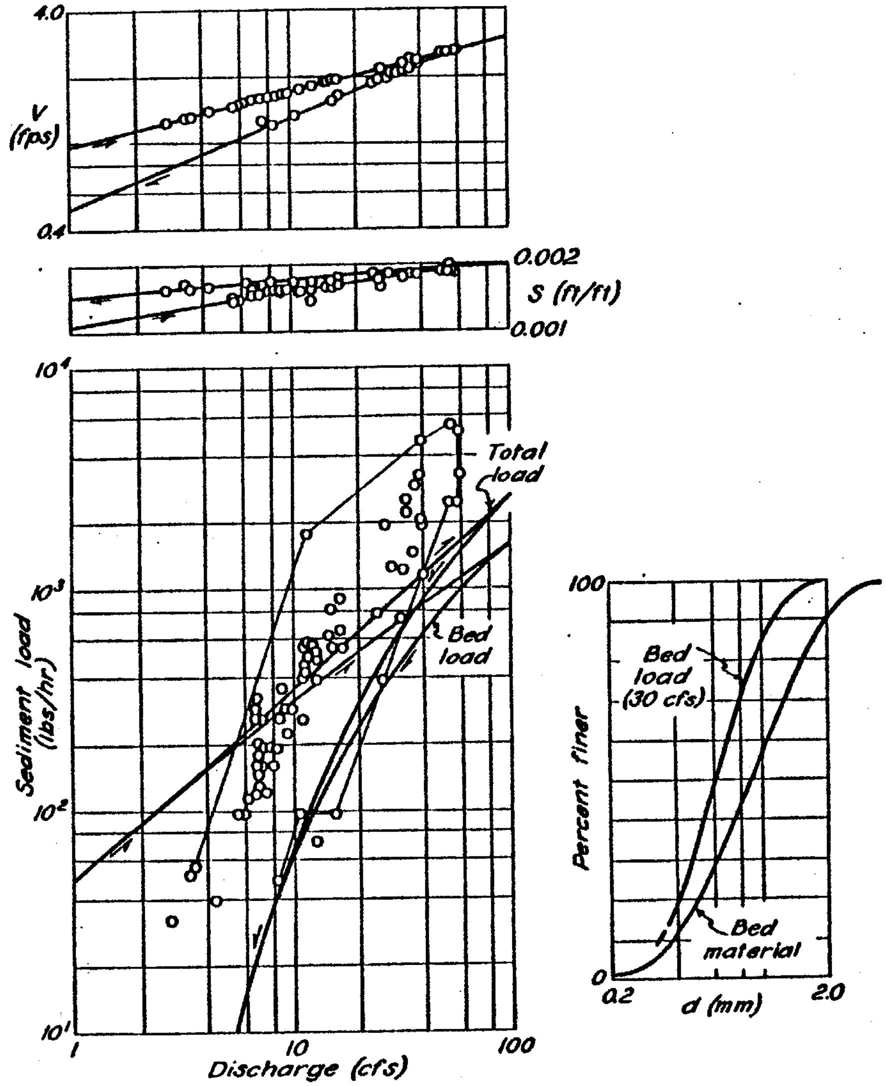


Fig. 11. Prediction of sediment load of Mountain Creek.

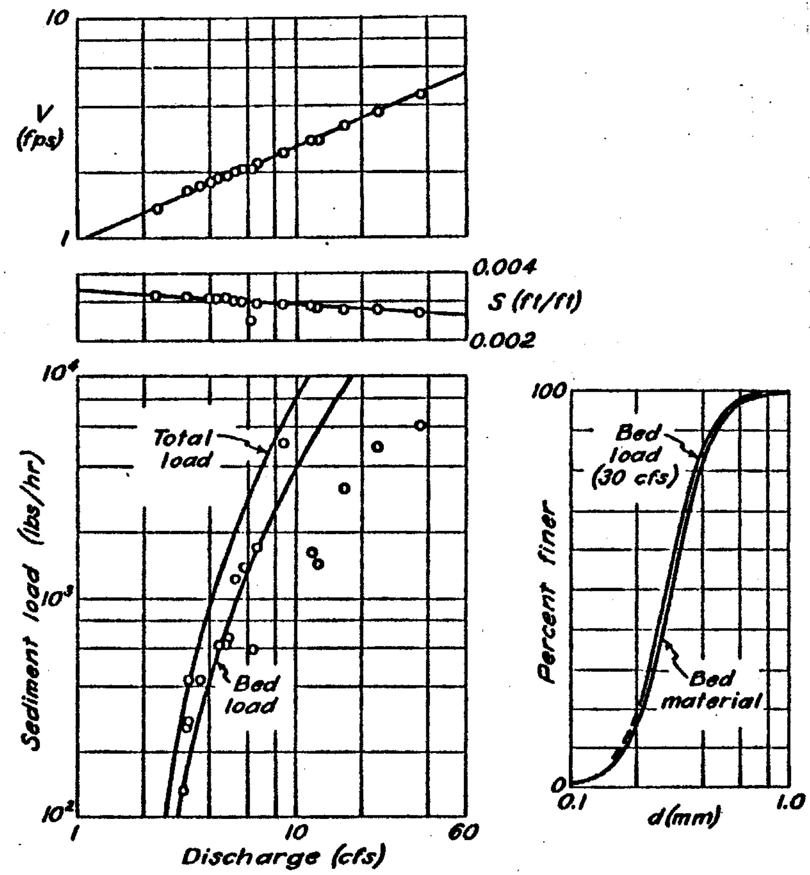


Fig. 12. Prediction of sediment load of West Goose Creek.

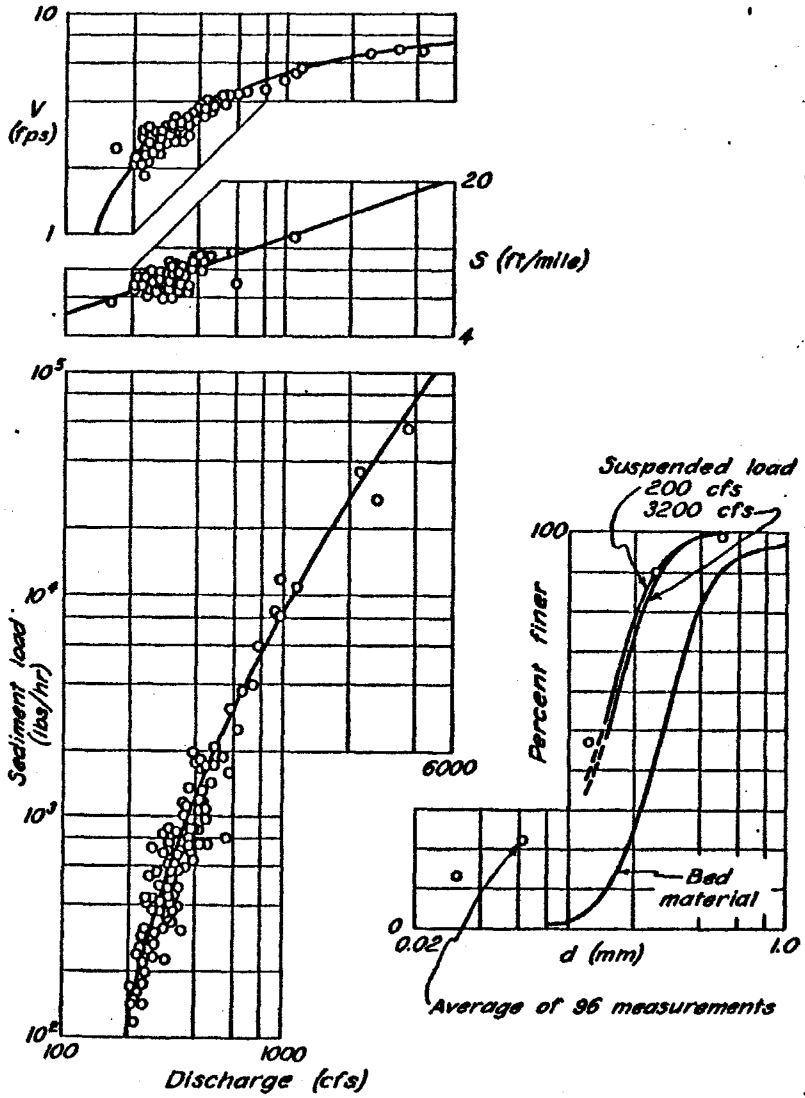


Fig. 13. Prediction of sediment load of Niobrara River.

in the range of 200 to 2000 tons per day which would fall on the computed curve or on points next to the curve. In order to compare the predicted and measured composition of the suspended load, seventeen measurements which included a large amount of fine material (the result of surface erosion) were rejected. It should be noted, however, that the measured values of concentration have not been corrected for the inherent sampling error due to the fact that the sampler does not traverse the entire depth.

Applicability of the Proposed Relationships

The proposed rate-of-transport relationships, Eq. (9) and Fig. 14, give both the quantity and the quality of the total, suspended, and bed loads as functions of the basic hydraulic characteristics of the stream and the characteristics of the bed sediment. Thus, they can be applied directly and simply to the prediction of the sediment-transporting characteristics of a stream from other measurable or computable characteristics. However, it should be kept in mind that the relationships are basically empirical and hence can be used with confidence only within the range of conditions for which they have been tested against actual measurements.

In order to increase the reliance that can be placed on their applicability, the proposed relationships must be tested against reliable measurements over an ever wider range of conditions. Special field studies would be particularly valuable—not only to extend the scale to include large rivers, but also to assess the effect of such stream characteristics as channel shape and alignment. Such continued testing could result in the addition of secondary parameters as well as some modification of the present relationships between the basic parameters.

Because approximations had to be made in the formulation of those parameters, continued study of the factors entering into the sediment-transport relationships is also indicated. For example, from Fig. 8 it is quite evident that the critical tractive force relationship for fine materials is not adequately understood. An investigation of the magnitude and distribution of the forces on a boundary of composite roughness such as a duned bed is also needed. It is readily apparent that studies such as these would permit better formulations of the parameters governing the sediment load. Other investigations that would be desirable concern the mutual interference of particles of various sizes and the formation of dunes and ripples.

Unfortunately, the need for a means of predicting the sediment-transporting characteristics of a stream cannot wait on the completion of all desirable research. Since considerable rationality can be ascribed to the parameters of the proposed relationships, and since their ability to correlate measured laboratory and field data has been demonstrated, it would seem reasonable and proper to attempt to apply the relationships to field problems—so long as there is full realization that they are new and comparatively untried.

For any stream which has been gaged for a period of years the hydraulic factors needed for the prediction of the sediment load should be available. Information as to the characteristics of the normal bed material should then permit the construction of load-discharge relations such as those in Figs. 11, 12, and 13. Sheet erosion during heavy rains will contribute a temporary fine fraction to the normal bed material. The "wash" load which results therefrom cannot be predicted from the proposed relationships unless the temporary fine fraction can be estimated. For streams of which the gaging

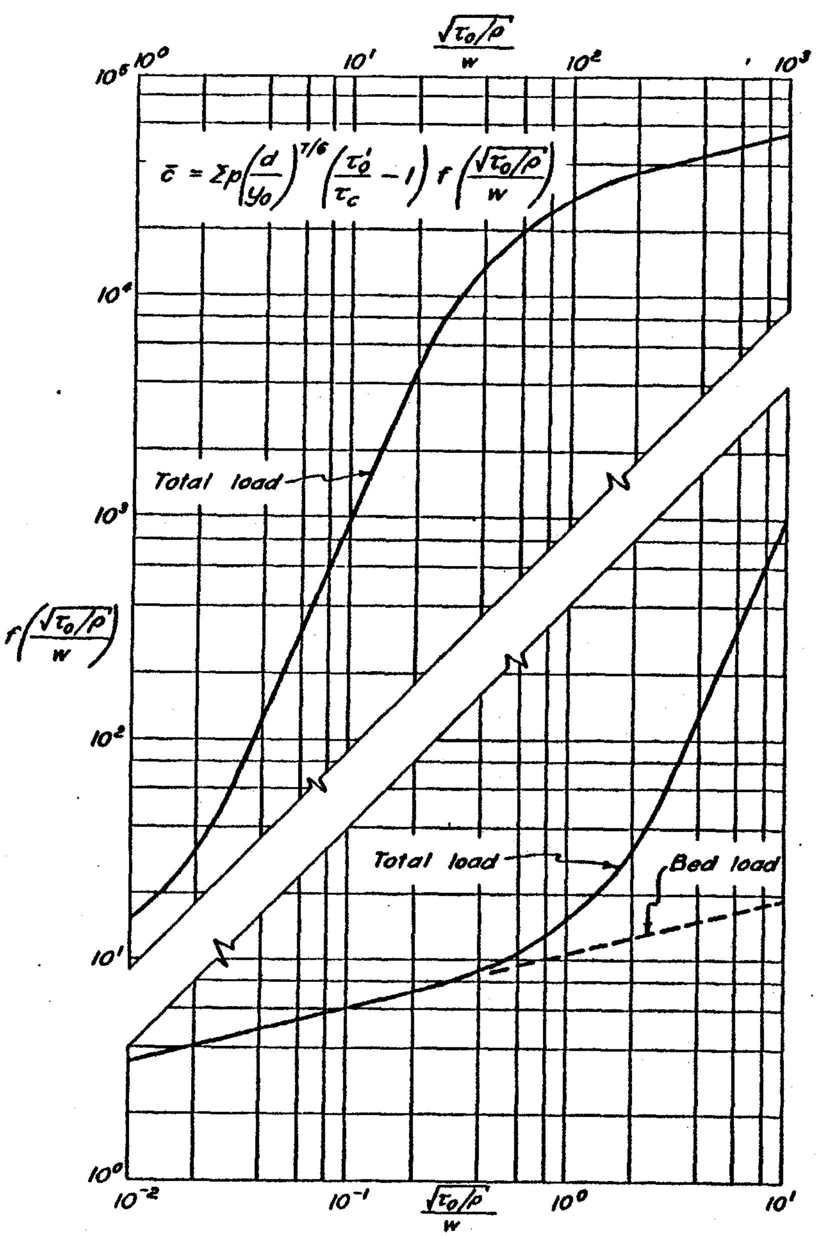


Fig. 14. Relationships for sediment load.

program has included suspended-sediment measurements it may be possible with the aid of the proposed relationships to correlate the wash load with the watershed and rainfall characteristics such as soil type, cultivation, basin slopes, season, intensity of rain, antecedent rain, etc. The utility of such a correlation of unmeasured streams is obvious.

In the case of degradation studies, the influence of the wash load should be small; since the bed material can be considered known, the rate and location of the degradation should be predictable. The case of aggradation studies is more difficult, because the source of the excess sediment load will determine its composition. In this respect the aggradation problem is similar to the problem of wash load on poised streams.

CONCLUSIONS

Through the use of (1) a qualitative analysis, (2) original experiments of a specialized nature, and (3) supplementary data from other sources, empirical relationships for the primary aspects of the sediment load have been obtained. These relationships permit the composition and the rate of transport of the total load, the suspended load, and the bed load to be evaluated from the hydraulic characteristics of the stream (mean velocity and depth of flow and energy gradient) and the characteristics of the bed sediment (frequency distribution of size and fall velocity).

In the process of defining the relationships between the parameters which were found to govern the sediment loads, a correlation of laboratory data representing a ten-thousand-fold range in rate of transport was obtained with a scatter probably not much greater than the error in the experimental observations. The extent to which the proposed relationships could predict field conditions was demonstrated for three natural streams. The degree of approximation was especially encouraging, since the ultimate goal of the search for a general sediment-transport function is its application to practical engineering problems.

ACKNOWLEDGEMENTS

The experiments reported herein were performed by Messrs. Pin-Nam Lin, Walter Rand, and Arthur Toch, Research Associates of the Iowa Institute of Hydraulic Research, with the aid of various Research Assistants. Their work was conducted under the immediate supervision of the writer and the general supervision of Dr. Hunter Rouse, Director of the Institute. The investigation was partially supported by the Office of Naval Research under Contract NSonr-500(02).

REFERENCES

1. Laursen, E. M., "The Application of Sediment-Transport Mechanics to Stable-Channel Design," Jour. Hyd. Div. ASCE, Vol. 82, No. HY 4, Aug. 1956.

2. Lara, J. M., discussion of "The Present Status of Research on Sediment Transport" by Ning Chien, Trans. ASCE, Vol. 121, 1956, pp. 871-6.

3. Kalinske, A. A., "The Movement of Sediment as Bed Load in Rivers," Trans. AGU, Vol. 28, 1947, pp. 615-20.

4. Einstein, H. A., "Formulas for the Transportation of Bed Load," Trans. ASCE, Vol. 107, 1942, pp. 561-77.

5. Einstein, H. A., "The Bed Load Function for Sediment Transportation in Open Channel Flows," Tech. Bulletin No. 1026, U. S. Dept. of Agriculture, Soil Conservation Service, 1950.

6. Laursen, E. M., discussion of "Application of the Modified Einstein Procedure for Computation of Total Sediment Load" by K. B. Schroeder and C. H. Hembree, Trans. AGU, October 1957.

7. Rouse, H., "Modern Conceptions of the Mechanics of Fluid Turbulence," Trans. ASCE, Vol. 102, 1937, pp. 463-543.

8. Lane, E. W. and Kalinske, A. A., "The Relation of Suspended to Bed Material in Rivers," Trans. AGU, Vol. 20, 1939, pp. 637-41.

9. Pien, C-L., "Investigation of Turbulence and Suspended Material Transportation in Open Channels," Ph.D. Dissertation, State University of Iowa, 1941.

10. Hsia, C. H., "A Study of Transportation of Fine Sediments by Flowing Water," Ph.D. Dissertation, State University of Iowa, 1943.

11. Jeffrys, H., "On the Transportation of Sediment by Streams," Proc. Cambridge Phil. Soc., Vol. 25, 1929, pp. 272-6.

12. Rubey, W. W., "The Force Required to Move Particles on a Stream Bed," U. S. Geol. Survey Professional Paper 189-E, 1938.

13. Rouse, H., "An Analysis of Sediment Transportation in the Light of Fluid Turbulence," U. S. Dept. of Agriculture, Soil Conservation Service, SCS-TP-25, 1939.

14. Matthes, G. H., "Macroturbulence in Natural Stream Flow," Trans. AGU, Vol. 28, 1937, pp. 255-61.

15. Lane, E. W., "A New Method of Sediment Transportation," Trans. AGU, Vol. 25, 1944, p. 900.

16. Laursen, E. M., "An Investigation of the Total Sediment Load," Final Report to the Office of Naval Research, Iowa Institute of Hydraulic Research, State University of Iowa, June, 1957.

17. Einstein, H. A. and Barbarossa, N. L., "River Channel Roughness," Trans. ASCE, Vol. 117, 1952, pp. 1121-46.

18. White, C. M., "Equilibrium of Grains on Bed of Stream," Proc. Royal Society (London), Vol. 174A, 1940, pp. 322-34.

19. "Sediment Transportation," Chap. XII, Engineering Hydraulics, ed. Hunter Rouse, John Wiley and Sons, 1950.

20. Einstein, H. A., "Bed-Load Transportation in Mountain Creek," U. S. Dept. of Agriculture, Soil Conservation Service, SCS-TP-55, 1944.

21. Colby, B. R. and Hembree, C. H., "Computations of Total Sediment Discharge, Niobrara River near Cody, Nebraska," U. S. Geol. Survey Water

Supply Paper 1357, 1955.

22. Johnson, J. W., "Laboratory Investigations on Bed-Load Transportation and Bed Roughness," U. S. Dept. of Agriculture, Soil Conservation Service, SCS-TP-50, 1943.

23. Brooks, N. H., "Mechanics of Streams with Movable Beds of Fine Sand," Proc. ASCE, Vol. 81, Separate No. 668, April, 1955.

24. Barton, J. R. and Lin, P-N., "A Study of Sediment Transport in Alluvial Channels," Research Report, Civil Engineering Department, Colorado A and M College, 1955.

25. "Studies of River Bed Materials and Their Movement with Special Reference to the Lower Mississippi River," U. S. Waterways Experiment Station Paper 17, Jan. 1935.

List of Symbols

a	elevation of reference concentration c_a, ft
c	sediment concentration at a point, percent by weight
$\bar{c}$	mean concentration, 265 q_s/q (subscripts s, b, t refer to suspended, bed, and total load, respectively), percent by weight
c_a	concentration at reference level a, percent by weight
C	coefficient relating critical tractive force to sediment size
d	diameter of sediment particle (mean diameter of fraction p of bed material), ft
d_m	mean diameter of bed material, ft
f	Weisbach resistance coefficient
F	Froude number, $V/\sqrt{gy_0}$
h	dune height, ft
l	dune length, ft
m	exponent in power-law velocity distribution
n	Manning roughness coefficient
p	fraction of bed material of diameter d
q	rate of flow per unit width, Vy_0, cfs/ft
q_s	volume rate of sediment transport per unit width (subscripts s, b, t refer to suspended, bed, and total load, respectively), cfs/ft
R	hydraulic radius, ft
$\mathcal{R}$	Reynolds number, $2Vy_0/\nu$
S	energy gradient, ft/ft
v	velocity of flow at a point, fps
v_s	surface velocity obtained by extrapolating power-law velocity distribution, fps